

Analysis and Design of Cooperative BICM–OFDM Systems

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Abstract—In this paper, we propose a novel cooperative diversity scheme for wireless systems employing the combination of bit–interleaved coded modulation (BICM) and orthogonal frequency division multiplexing (OFDM). The proposed scheme utilizes an amplify–and–forward protocol where relays are assigned to multiple groups. Relays in the same group transmit concurrently over disjoint sets of sub–carriers and relays in different groups transmit in different time slots. We derive closed–form expressions for the asymptotic worst–case pairwise error probability and the diversity gain of the proposed cooperative BICM–OFDM scheme. Based on the derived analytical results we develop design guidelines for sub–carrier allocation, relay grouping, and relay selection. Simulation results corroborate the derived analytical results and confirm the effectiveness of the developed optimization framework.

Index Terms—BICM, OFDM, Cooperative Diversity.

I. INTRODUCTION

IN recent years, cooperative diversity techniques have attracted considerable research interest due to their possible use in future cellular, ad–hoc, and sensor networks [1]. Because of its simplicity and low complexity, amplify–and–forward (AF) relaying is one of the most popular cooperative diversity techniques. The performance of cooperative AF relay systems has been extensively studied in the literature, cf. e.g. [2]– [4], and various power allocation and relay selection strategies have been proposed in [2], [3], [5], [6]. In [2]– [6], and in the majority of the existing literature, frequency–flat fading channels and either uncoded transmission or channel capacities are considered. However, practical broadband wireless communication systems are typically affected by frequency–selective fading and employ non–ideal channel coding schemes. A widely used approach to overcome the negative effects of frequency–selectivity is orthogonal frequency division multiplexing (OFDM). OFDM based AF cooperative diversity systems have been studied in [7]– [9]. However, the impact of practical channel coding schemes on

performance and system design was not investigated in [7]– [9].

For point–to–point transmission, the combination of bit–interleaved coded modulation (BICM) [10] and OFDM is a popular approach to exploit the inherent diversity offered by frequency–selective fading channels. In particular, it was shown in [11] that BICM–OFDM can extract the full diversity offered by the wireless channel assuming that the adopted code has a sufficiently large free distance. As a result, BICM–OFDM forms the basis of the IEEE 802.11 and 802.16 families of standards and most emerging wireless standards will also adopt this technique. Hence, BICM–OFDM based cooperative diversity schemes have tremendous potential for consideration in next generation broadband wireless communication systems. To the best of our knowledge, the analysis, design, and optimization of cooperative BICM–OFDM systems has not been considered in the literature yet.

In this paper, we propose a BICM–OFDM based cooperative diversity scheme, where the available relays are divided into multiple groups. In the first time slot, after coding, interleaving, mapping, and modulation, the source transmits a data packet to the relays and the destination. The relays adopt an AF protocol to forward the received signals to the destination where relays in different groups transmit in different time slots but relays in the same group transmit concurrently over disjoint sets of OFDM sub–carriers. The destination combines the signals received in the different time slots and performs standard Viterbi decoding. We derive a closed–form expression for the asymptotic worst–case pairwise error probability (PEP) of the proposed cooperative diversity scheme. This expression provides significant insight into the impact of system parameters such as the sub–carrier allocation, the number of relay groups, the number of relays in a group, the free distance of the code, and the frequency diversity of the links in the network. The derived asymptotic PEP expression is exploited to develop guidelines for sub–carrier allocation within a relay group, optimal relay grouping, and relay selection. A related power allocation problem was considered in [12]. Simulation results confirm the validity of the derived analytical results and show the effectiveness of the proposed optimization techniques.

The remainder of this paper is organized as follows. In Section II, the system model for cooperative BICM–OFDM is introduced. The analysis and optimization of the proposed scheme are presented in Section III and Section IV, respectively. In Section V, simulation results are provided, and some conclusions are drawn in Section VI.

Notation: In this paper, $\mathcal{E}\{\cdot\}$, $[\cdot]^T$, $[\cdot]^H$, $|\cdot|$, $\|\cdot\|$, and

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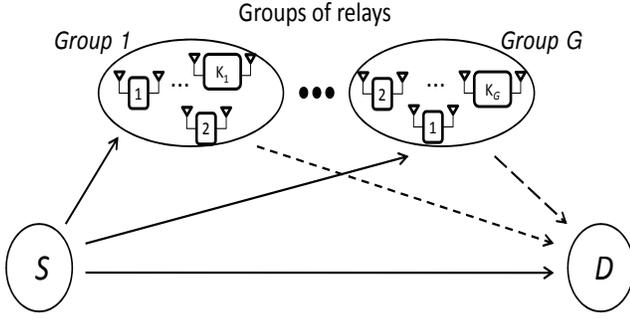


Fig. 1. Schematic block diagram of proposed cooperative diversity system. Different line styles for the arrows indicate transmission in different time slots. Note that relays within the same group do not have to be geographically close to each other and can be placed arbitrarily.

$\det(\cdot)$ denote statistical expectation, transposition, Hermitian transposition, the magnitude of a scalar or the cardinality of a set, the L_2 -norm of a vector, and the determinant of a matrix, respectively. $\lambda_m(\mathbf{X})$, $1 \leq m \leq \text{rank}\{\mathbf{X}\}$, denotes the non-zero eigenvalues of matrix \mathbf{X} and \mathbf{I}_M is the $M \times M$ identity matrix. $\lceil \cdot \rceil$ denotes rounding to the closest integer value and a function $f(x)$ is $o(g(x))$ if $\lim_{x \rightarrow 0} f(x)/g(x) = 0$.

II. SYSTEM MODEL

The considered system consists of one source (S) terminal, G groups of relays with K_ν relays in the ν th group, $1 \leq \nu \leq G$, and one destination (D) terminal, cf. Fig. 1. In the following, we denote the set of groups by $\mathcal{G} \triangleq \{1, 2, \dots, G\}$ and the set of relays in the ν th group by \mathcal{K}_ν . In this section, we describe the processing required for cooperative BICM-OFDM at the source, the relays, and the destination, cf. Fig. 2.

A. Signal Model

The adopted relaying protocol comprises $G + 1$ time slots. In the first time slot, the source transmits and both the relays and the destination receive unless stated otherwise. In the $(\nu + 1)$ th time slot, the K_ν relays of the ν th group transmit concurrently over disjoint sets of sub-carriers, where $\nu \in \mathcal{G}$. The source employs conventional BICM-OFDM [11], i.e., the output bits $c_{k'}$, $0 \leq k' < \log_2(M)N$, of a binary convolutional encoder with minimum free distance d_f are interleaved and Gray mapped onto symbols $X[k] \in \mathcal{X}$, $k \in \mathcal{N}$, $\mathcal{N} \triangleq \{0, 1, \dots, N - 1\}$, where \mathcal{X} denotes an M -ary symbol alphabet such as M -ary phase-shift keying (M -PSK) or M -ary quadrature amplitude modulation (M -QAM) and N is the number of data sub-carriers in one OFDM symbol. The transmitted symbols are assumed to have unit average energy, i.e., $\mathcal{E}\{|X[k]|^2\} = 1$. The effect of the interleaver can be modeled by the mapping $k' \rightarrow (k, i)$, where k' denotes the original index of coded bit $c_{k'}$, and k and i denote the index of symbol $X[k]$ and the position of $c_{k'}$ in the label of $X[k]$, respectively. Assuming the worst-case error event of the code spans $d \geq d_f$ consecutive bits, the interleaver ensures that at least any d consecutive bits at the output of the encoder are mapped to different sub-carriers [11].

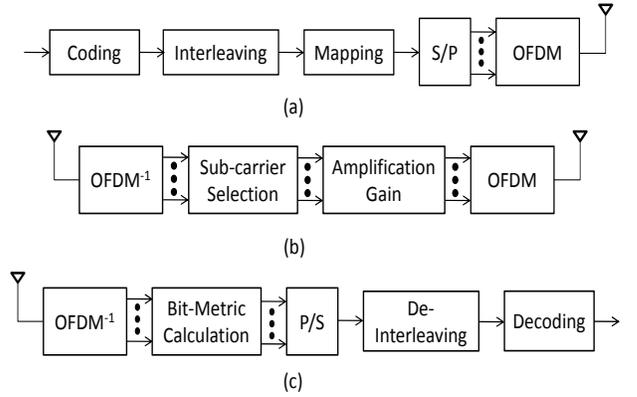


Fig. 2. Block diagram of (a) source, (b) relay, and (c) destination.

Throughout this paper we assume conventional OFDM processing at the source, the relays, and the destination and a sufficiently long cyclic prefix (CP) to avoid interference between sub-carriers. Thus, the received signal at the destination in the first time slot and the k th sub-carrier can be modeled as

$$Y_0[k] = \sqrt{P_0}H_0[k]X[k] + N_0[k], \quad k \in \mathcal{N}, \quad (1)$$

where P_0 is the average transmit power in each sub-carrier at the source, $N_0[k]$ is zero-mean complex additive white Gaussian noise (AWGN) with variance σ^2 , and $H_0[k]$ is the frequency response of the $S \rightarrow D$ channel. The frequency response is given by $H_0[k] \triangleq \mathbf{w}_0^H[k]\mathbf{h}_0$, where $\mathbf{w}_0[k] \triangleq [1, w^k, \dots, w^{(L_0-1)k}]^T$, $w \triangleq e^{-j2\pi/N_t}$ (N_t : total number of sub-carriers, including both data and pilot sub-carriers), and column vector \mathbf{h}_0 ($\sigma_0^2 \triangleq \mathcal{E}\{\mathbf{h}_0^H \mathbf{h}_0\}$) contains the L_0 channel impulse response (CIR) coefficients of the $S \rightarrow D$ channel.

The received signal at the j th relay of group ν , R_{ν_j} , $j \in \mathcal{K}_\nu$, $\nu \in \mathcal{G}$, in the k th sub-carrier in the first time slot can be modeled as

$$U_{\nu_j}[k] = \sqrt{P_0}H_{1,\nu_j}[k]X[k] + N_{\nu_j}[k], \quad k \in \mathcal{N}, \quad (2)$$

where $N_{\nu_j}[k]$ is zero-mean complex AWGN with variance σ^2 and $H_{1,\nu_j}[k] \triangleq \mathbf{w}_{1,\nu_j}^H[k]\mathbf{h}_{1,\nu_j}$ is the frequency response of the $S \rightarrow R_{\nu_j}$ link. Here, column vector \mathbf{h}_{1,ν_j} ($\sigma_{1,\nu_j}^2 \triangleq \mathcal{E}\{\mathbf{h}_{1,\nu_j}^H \mathbf{h}_{1,\nu_j}\}$) contains the L_{1,ν_j} CIR coefficients of the $S \rightarrow R_{\nu_j}$ link and $\mathbf{w}_{1,\nu_j}[k] \triangleq [1, w^k, \dots, w^{(L_{1,\nu_j}-1)k}]^T$.

In the $(\nu + 1)$ th time slot, relay R_{ν_j} selects a set $\mathcal{N}_{\nu_j} \subseteq \mathcal{N}$ of sub-carriers, amplifies the signal received on these sub-carriers in the first time slot and sets the signals in the other sub-carriers to zero, cf. Fig. 2(b). The sets of sub-carriers are chosen such that $\mathcal{N}_{\nu_j} \cap \mathcal{N}_{\nu_i} = \emptyset$, $i \neq j$, and $\sum_{j=1}^{K_\nu} |\mathcal{N}_{\nu_j}| = N$. The sub-carrier selection will be discussed more in detail in Section IV-A. The signal received at the destination in sub-carrier $k \in \mathcal{N}_{\nu_j}$ in the $(\nu + 1)$ th time slot is given by

$$Y_\nu[k] = A_{\nu_j}[k]H_{2,\nu_j}[k]U_{\nu_j}[k] + N_\nu[k], \quad j \in \mathcal{K}_\nu, \nu \in \mathcal{G}, \quad (3)$$

where $A_{\nu_j}[k]$ denotes the amplification gain applied at relay R_{ν_j} in sub-carrier $k \in \mathcal{N}_{\nu_j}$, $N_\nu[k]$ is zero-mean complex AWGN with variance σ^2 , and $H_{2,\nu_j}[k] \triangleq \mathbf{w}_{2,\nu_j}^H[k]\mathbf{h}_{2,\nu_j}$ is

the frequency response of the $R_{\nu_j} \rightarrow D$ link. Here, column vector \mathbf{h}_{2,ν_j} ($\sigma_{2,\nu_j}^2 \triangleq \mathcal{E}\{\mathbf{h}_{2,\nu_j}^H \mathbf{h}_{2,\nu_j}\}$) contains the L_{2,ν_j} CIR coefficients of the $R_{\nu_j} \rightarrow D$ channel and $\mathbf{w}_{2,\nu_j}[k] \triangleq [1, w^k, \dots, w^{(L_{2,\nu_j}-1)k}]^T$.

B. Amplification Gain

For the amplification gain, $A_{\nu_j}[k]$, several choices have been proposed in the literature. The most widely used gain is [2]

$$A_{\nu_j}[k] = \sqrt{\frac{P_{\nu_j}}{P_0 |H_{1,\nu_j}[k]|^2 + \sigma^2}}, \quad k \in \mathcal{N}_{\nu_j}, j \in \mathcal{K}_{\nu}, \nu \in \mathcal{G}, \quad (4)$$

which maintains a constant average transmit power P_{ν_j} at relay R_{ν_j} in each sub-carrier k . As this choice does not usually lead to a tractable mathematical analysis, it is customary in the literature, e.g. [2], to approximate this relay gain as

$$A_{\nu_j}[k] = \sqrt{\frac{P_{\nu_j}}{P_0 |H_{1,\nu_j}[k]|^2}}, \quad k \in \mathcal{N}_{\nu_j}, j \in \mathcal{K}_{\nu}, \nu \in \mathcal{G}, \quad (5)$$

for performance analysis. It is well-known that for the high SNR regime the gains in (4) and (5) yield practically identical performances [2]. In this paper, we will also adopt the gain in (4) but resort to the gain in (5) for the performance analysis presented in Section III.

C. Diversity Combining and Decoding

We assume perfect synchronization, channel estimation, and demodulation at the relays and the destination. From (1)–(3), the received signal at the destination in sub-carrier k and time slot ν can be modeled as

$$Y_{\nu}[k] = \Psi_{\nu}[k]X[k] + \tilde{N}_{\nu}[k], \quad k \in \mathcal{N}, \nu \in 0 \cup \mathcal{G}, \quad (6)$$

where, for $j \in \mathcal{K}_{\nu}$,

$$\Psi_{\nu}[k] \triangleq \begin{cases} \sqrt{P_0}H_0[k], & k \in \mathcal{N}, \nu = 0, \\ \sqrt{P_0}A_{\nu_j}[k]H_{1,\nu_j}[k]H_{2,\nu_j}[k], & k \in \mathcal{N}_{\nu_j}, \nu \in \mathcal{G}, \end{cases} \quad (7)$$

and

$$\tilde{N}_{\nu}[k] \triangleq \begin{cases} N_0[k], & k \in \mathcal{N}, \nu = 0, \\ A_{\nu_j}[k]H_{2,\nu_j}[k]N_{\nu_j}[k] + N_{\nu}[k], & k \in \mathcal{N}_{\nu_j}, \nu \in \mathcal{G}. \end{cases} \quad (8)$$

Here, $\tilde{N}_{\nu}[k]$ is the zero-mean effective noise at the destination in sub-carrier k and time slot ν with variance

$$\sigma_{\nu}^2[k] \triangleq \begin{cases} \sigma^2, & k \in \mathcal{N}, \nu = 0, \\ (|A_{\nu_j}[k]|^2 |H_{2,\nu_j}[k]|^2 + 1)\sigma^2, & k \in \mathcal{N}_{\nu_j}, \nu \in \mathcal{G}. \end{cases} \quad (9)$$

Following [10], [11], the bit metric for the i th bit in the label of $X[k]$ is calculated as

$$m_k^i[c_{k'}] = \min_{X \in \mathcal{X}_{c_{k'}}^i} \left\{ \sum_{\nu=0}^G \frac{|Y_{\nu}[k] - \Psi_{\nu}[k]X|^2}{\sigma_{\nu}^2[k]} \right\}, \quad (10)$$

where \mathcal{X}_b^i denotes the subset of all symbols $X \in \mathcal{X}$ whose label has value $b \in \{0, 1\}$ in position i . The bit metrics are

de-interleaved and Viterbi decoded as usual [10], [11], cf. Fig. 2(c).

In some applications, the direct $S \rightarrow D$ link is not exploited because of heavy attenuation or because the destination does not receive in the first time slot. In this case, the first term ($\nu = 0$) is omitted in the sum in (10).

III. PERFORMANCE ANALYSIS

In this section, we derive an upper bound on the asymptotic worst-case PEP of cooperative BICM-OFDM and investigate the diversity gain of the system. The insights gained from the proposed analysis will be exploited for system optimization in Section IV. For the presented analysis, it is convenient to define the average sub-carrier SNRs of the $S \rightarrow D$, $S \rightarrow R_{\nu_j}$, and $R_{\nu_j} \rightarrow D$ links as $\bar{\gamma}_0 \triangleq P_0\sigma_0^2/\sigma^2$, $\bar{\gamma}_{1,\nu_j} \triangleq P_0\sigma_{1,\nu_j}^2/\sigma^2$, and $\bar{\gamma}_{2,\nu_j} \triangleq P_{\nu_j}\sigma_{2,\nu_j}^2/\sigma^2$, respectively, where $j \in \mathcal{K}_{\nu}$ and $\nu \in \mathcal{G}$. Furthermore, we assume mutually independent Rayleigh fading for all links and introduce the normalized correlation matrices $\mathbf{C}_0 \triangleq \mathcal{E}\{\mathbf{h}_0\mathbf{h}_0^H\}/\sigma_0^2$, $\mathbf{C}_{1,\nu_j} \triangleq \mathcal{E}\{\mathbf{h}_{1,\nu_j}\mathbf{h}_{1,\nu_j}^H\}/\sigma_{1,\nu_j}^2$, and $\mathbf{C}_{2,\nu_j} \triangleq \mathcal{E}\{\mathbf{h}_{2,\nu_j}\mathbf{h}_{2,\nu_j}^H\}/\sigma_{2,\nu_j}^2$, which are all assumed to have full rank.

A. Asymptotic PEP

In this subsection, we analyze the worst-case PEP for $\bar{\gamma}_0, \bar{\gamma}_{1,\nu_j}, \bar{\gamma}_{2,\nu_j} \rightarrow \infty$, $j \in \mathcal{K}_{\nu}$, $\nu \in \mathcal{G}$. For this purpose, we first define vectors $\mathbf{h}_{1,\nu} \triangleq [\mathbf{h}_{1,\nu_1}^T, \dots, \mathbf{h}_{1,\nu_{K_{\nu}}}^T]^T$, $\mathbf{h}_{2,\nu} \triangleq [\mathbf{h}_{2,\nu_1}^T, \dots, \mathbf{h}_{2,\nu_{K_{\nu}}}^T]^T$, $\mathbf{h}_1 \triangleq [\mathbf{h}_{1,1}^T, \dots, \mathbf{h}_{1,G}^T]^T$, and $\mathbf{h}_2 \triangleq [\mathbf{h}_{2,1}^T, \dots, \mathbf{h}_{2,G}^T]^T$. Assuming a code with free distance d_f , the worst-case PEP of two codewords \mathbf{c} and $\hat{\mathbf{c}}$ conditioned on $\mathbf{h}_0, \mathbf{h}_1, \mathbf{h}_2$, can be expressed as [11]

$$P(\mathbf{c}, \hat{\mathbf{c}}|\mathbf{h}_0, \mathbf{h}_1, \mathbf{h}_2) = \Pr \left\{ \sum_{k,d_f} m_k^i[c_{k'}] \geq \sum_{k,d_f} m_k^i[\hat{c}_{k'}] \right\}, \quad (11)$$

where the sum in (11) is over d_f distinct sub-carriers $\{k_1, k_2, \dots, k_{d_f}\}$, which are determined by the interleaver, and $c_{k'}$ and $\hat{c}_{k'}$ denote bits in the codewords \mathbf{c} and $\hat{\mathbf{c}}$, respectively. The PEP in (11) can be upper bounded as [11]

$$P(\mathbf{c}, \hat{\mathbf{c}}|\mathbf{h}_0, \mathbf{h}_1, \mathbf{h}_2) \leq \frac{1}{2} \exp \left(-\frac{d_{\min}^2}{4} \sum_{k,d_f} \sum_{\nu=0}^G \frac{|\Psi_{\nu}[k]|^2}{\sigma_{\nu}^2[k]} \right), \quad (12)$$

where d_{\min} denotes the minimum Euclidean distance of the signal constellation \mathcal{X} . Averaging (12) over $\mathbf{h}_0, \mathbf{h}_{1,\nu}, \mathbf{h}_{2,\nu}, \nu \in \mathcal{G}$, we obtain the unconditional PEP in (13) as shown at the top of the next page.

For convenience, let $\xi \triangleq d_{\min}^2/4$. Using (7) and (9), I can be written as [13]

$$\begin{aligned} \text{I} &= \frac{1}{\det(\mathbf{I}_{L_0} + \xi \bar{\gamma}_0 \mathbf{W}_0 \mathbf{C}_0)} \\ &= \frac{1}{(\xi \bar{\gamma}_0)^{r_0} \prod_{m=1}^{r_0} \lambda_m(\mathbf{W}_0 \mathbf{C}_0)} + o\left(\frac{1}{\bar{\gamma}_0^{r_0}}\right), \end{aligned} \quad (14)$$

where $\mathbf{W}_0 \triangleq \sum_{k,d_f} \mathbf{w}_0[k] \mathbf{w}_0^H[k]$ and $r_0 \triangleq \text{rank}\{\mathbf{W}_0 \mathbf{C}_0\} =$

$$P(\mathbf{c}, \hat{\mathbf{c}}) \leq \frac{1}{2} \underbrace{\mathcal{E}_{\mathbf{h}_0} \left\{ \exp \left(-\frac{d_{\min}^2}{4} \sum_{k, d_f} \frac{|\Psi_0[k]|^2}{\sigma_0^2[k]} \right) \right\}}_I \prod_{\nu=1}^G \underbrace{\mathcal{E}_{\mathbf{h}_{1,\nu}, \mathbf{h}_{2,\nu}} \left\{ \exp \left(-\frac{d_{\min}^2}{4} \sum_{k, d_f} \frac{|\Psi_\nu[k]|^2}{\sigma_\nu^2[k]} \right) \right\}}_{\Pi_\nu}. \quad (13)$$

$\min\{d_f, L_0\}$ [11]. From (13), Π_ν can be written as

$$\Pi_\nu = \mathcal{E}_{\mathbf{h}_{1,\nu}, \mathbf{h}_{2,\nu}} \left\{ \exp \left(-\xi \sum_{j=1}^{K_\nu} \sum_{k, d_f, \nu_j} \frac{|\Psi_\nu[k]|^2}{\sigma_\nu^2[k]} \right) \right\}, \quad (15)$$

where d_{f,ν_j} denotes the number of bits belonging to the considered error event sent by relay R_{ν_j} , and consequently, $\sum_{j=1}^{K_\nu} d_{f,\nu_j} = d_f$. Using (5), (7), and (9), we get (16) as shown at the top of the next page from (15), where we exploited the definitions of the average link SNRs and the fact that the CIR coefficients of different links are mutually independent. Although deriving an exact closed-form expression for Π_{ν_j} in (16) does not seem feasible, adopting a similar approach as was used for uncoded transmission and frequency-flat fading in [2, Appendix], we derive the following asymptotic upper bound in the Appendix

$$\begin{aligned} \Pi_{\nu_j} &\leq \frac{1}{(\xi \bar{\gamma}_{1,\nu_j})^{r_{1,\nu_j}} \prod_{m=1}^{r_{1,\nu_j}} \lambda_m(\mathbf{W}_{1,\nu_j} \mathbf{C}_{1,\nu_j})} \\ &+ \frac{1}{(\xi \bar{\gamma}_{2,\nu_j})^{r_{2,\nu_j}} \prod_{m=1}^{r_{2,\nu_j}} \lambda_m(\mathbf{W}_{2,\nu_j} \mathbf{C}_{2,\nu_j})}, \end{aligned} \quad (17)$$

where $\mathbf{W}_{1,\nu_j} \triangleq \sum_{k, d_f, \nu_j} \mathbf{w}_{1,\nu_j}[k] \mathbf{w}_{1,\nu_j}^H[k]$, $\mathbf{W}_{2,\nu_j} \triangleq \sum_{k, d_f, \nu_j} \mathbf{w}_{2,\nu_j}[k] \mathbf{w}_{2,\nu_j}^H[k]$, $r_{1,\nu_j} \triangleq \text{rank}\{\mathbf{W}_{1,\nu_j} \mathbf{C}_{1,\nu_j}\} = \min\{d_{f,\nu_j}, L_{1,\nu_j}\}$, and $r_{2,\nu_j} \triangleq \text{rank}\{\mathbf{W}_{2,\nu_j} \mathbf{C}_{2,\nu_j}\} = \min\{d_{f,\nu_j}, L_{2,\nu_j}\}$.

Combining (13), (14), (16), and (17) and assuming the high SNR regime, $\bar{\gamma}_0, \bar{\gamma}_{1,\nu_j}, \bar{\gamma}_{2,\nu_j} \rightarrow \infty$, $j \in \mathcal{K}_\nu$, $\nu \in \mathcal{G}$, an asymptotic upper bound for the worst-case PEP is obtained in (18) as shown at the top of the next page. We note that if the direct $S \rightarrow D$ link is not exploited, (18) remains valid if we set the term outside the double product equal to one.

B. Diversity Gain

To get more insight into the system performance, we investigate the diversity gain of cooperative BICM-OFDM. Considering the case $\bar{\gamma}_0 = \bar{\gamma}_{1,\nu_j} = \bar{\gamma}_{2,\nu_j} = \bar{\gamma}$, we define the diversity gain as the negative slope of the PEP in (18) as a function of $\bar{\gamma}$ on a double-logarithmic scale. From (18) we obtain the diversity gain as

$$\begin{aligned} G_d &= r_0 + \sum_{\nu=1}^G \sum_{j=1}^{K_\nu} \min\{r_{1,\nu_j}, r_{2,\nu_j}\} \\ &= \min\{d_f, L_0\} + \sum_{\nu=1}^G \sum_{j=1}^{K_\nu} \min\{d_{f,\nu_j}, L_{1,\nu_j}, L_{2,\nu_j}\}. \end{aligned} \quad (19)$$

Eq. (19) reveals that the maximum diversity gain of the proposed system is limited by either the free distance of the code, the frequency diversity offered by the channel, or both.

In particular, for channels that are rich in frequency diversity, i.e., $L_0 \geq d_f$ and $\min\{L_{1,\nu_j}, L_{2,\nu_j}\} \geq d_{f,\nu_j}$, $j \in \mathcal{K}_\nu$, $\nu \in \mathcal{G}$, we obtain $G_d = (G+1)d_f$. Eq. (19) also gives important insight for system design. For example, if there is one group with only one relay and the $S \rightarrow R_1$ and $R_1 \rightarrow D$ channels are rich in diversity with $\min\{L_{1,1}, L_{2,1}\} \geq d_f$ (we drop index ν for convenience), the system achieves the maximum diversity gain $G_d = r_0 + d_f$ with this single relay. On the other hand, if the $S \rightarrow R_1$ and $R_1 \rightarrow D$ channels are not rich in diversity and $\min\{L_{1,1}, L_{2,1}\} < d_{f,1} = d_f$, we can improve the diversity gain by adding a second relay which transmits every second bit of the coded bit stream. In doing so, we decrease $d_{f,1}$ by a factor of two (assuming $d_{f,1}$ is even) and we may achieve the maximum diversity gain provided that the new $d_{f,1}$ and $d_{f,2} = d_f - d_{f,1}$ do not exceed $\min\{L_{1,1}, L_{2,1}\}$ and $\min\{L_{1,2}, L_{2,2}\}$, respectively. Roughly speaking, by adding more relays to the ν th group, we decrease d_{f,ν_j} , $j \in \mathcal{K}_\nu$, and as a result make up for missing frequency diversity by adding more spatial diversity. For example, in the extreme case, where all $S \rightarrow R_{\nu_j}$ and $R_{\nu_j} \rightarrow D$ channels are frequency flat, d_f relays are needed in each group to achieve the maximum diversity gain possible with a code with free distance d_f . On the other hand, by adding an additional group, we can increase the diversity gain by up to d_f at the expense of a decrease in spectral efficiency since an additional time slot is needed for transmission.

Finally, we note that for the case where the direct $S \rightarrow D$ link is not used, (19) remains valid if we set $r_0 = L_0 = 0$.

IV. DESIGN OF COOPERATIVE BICM-OFDM SYSTEMS

In this section, we exploit the analytical results from Section III for the design and optimization of cooperative BICM-OFDM systems. In particular, we discuss sub-carrier allocation, relay grouping, and relay selection. While the proposed sub-carrier allocation scheme is based on the insight gained from the diversity analysis in Section III-B, the other optimization problems directly exploit the upper bound on the asymptotic worst-case PEP. However, the PEP in (18) depends on the sub-carriers involved in a particular error event since \mathbf{W}_0 , \mathbf{W}_{1,ν_j} , and \mathbf{W}_{2,ν_j} depend on the sub-carriers. Since this dependence is cumbersome for optimization, we first find

$$\Phi_0 \triangleq \min_{\mathbf{W}_0 \in \mathcal{W}_0} \prod_{m=1}^{r_0} \lambda_m(\mathbf{W}_0 \mathbf{C}_0), \quad (20)$$

$$\Phi_{1,\nu_j} \triangleq \min_{\mathbf{W}_{1,\nu_j} \in \mathcal{W}_{1,\nu_j}} \prod_{m=1}^{r_{1,\nu_j}} \lambda_m(\mathbf{W}_{1,\nu_j} \mathbf{C}_{1,\nu_j}), \quad (21)$$

$$\Phi_{2,\nu_j} \triangleq \min_{\mathbf{W}_{2,\nu_j} \in \mathcal{W}_{2,\nu_j}} \prod_{m=1}^{r_{2,\nu_j}} \lambda_m(\mathbf{W}_{2,\nu_j} \mathbf{C}_{2,\nu_j}), \quad (22)$$

$$\Pi_\nu = \prod_{j=1}^{K_\nu} \mathcal{E}_{\mathbf{h}_{1,\nu_j}, \mathbf{h}_{2,\nu_j}} \left\{ \underbrace{\exp \left(-\xi \sum_{k, d_i, \nu_j} \frac{\bar{\gamma}_{1,\nu_j} \bar{\gamma}_{2,\nu_j} |H_{1,\nu_j}[k]|^2 |H_{2,\nu_j}[k]|^2}{\bar{\gamma}_{1,\nu_j} |H_{1,\nu_j}[k]|^2 + \bar{\gamma}_{2,\nu_j} |H_{2,\nu_j}[k]|^2} \right)}_{\Pi_{\nu_j}} \right\}, \quad (16)$$

$$P(\mathbf{c}, \hat{\mathbf{c}}) \leq \frac{1}{(\xi \bar{\gamma}_0)^{r_0} \prod_{m=1}^{r_0} \lambda_m(\mathbf{W}_0 \mathbf{C}_0)} \prod_{\nu=1}^G \prod_{j=1}^{K_\nu} \left(\frac{1}{(\xi \bar{\gamma}_{1,\nu_j})^{r_{1,\nu_j}} \prod_{m=1}^{r_{1,\nu_j}} \lambda_m(\mathbf{W}_{1,\nu_j} \mathbf{C}_{1,\nu_j})} + \frac{1}{(\xi \bar{\gamma}_{2,\nu_j})^{r_{2,\nu_j}} \prod_{m=1}^{r_{2,\nu_j}} \lambda_m(\mathbf{W}_{2,\nu_j} \mathbf{C}_{2,\nu_j})} \right). \quad (18)$$

where \mathcal{W}_0 , \mathcal{W}_{1,ν_j} , and \mathcal{W}_{2,ν_j} are the sets of all possible matrices \mathbf{W}_0 , \mathbf{W}_{1,ν_j} , and \mathbf{W}_{2,ν_j} , $j \in \mathcal{K}_\nu$, $\nu \in \mathcal{G}$, respectively. These sets are defined by the sub-carrier allocation at the relays and the interleaver at the source and can be easily determined. Using Φ_0 , Φ_{1,ν_j} , and Φ_{2,ν_j} in (18) implies a further upper bounding of the worst-case PEP.

A. Sub-carrier Allocation and Interleaver Design

The results on diversity in Section III-B show that the interleaver and the sub-carrier allocation should be designed such that d_f, ν_j and $\min\{L_{1,\nu_j}, L_{2,\nu_j}\}$ are matched to each other for all relay groups and any d consecutive bits at the output of the encoder, where $d \geq d_f$ denotes the length of the worst-case error event. Within these d consecutive bits, two codewords corresponding to the worst-case error event differ in d_f bits. While there exist many different designs that guarantee full diversity, we propose here two simple sub-carrier allocation schemes, which can be combined with a conventional rectangular interleaver with $N_{\text{row}} = N$ rows and $N_{\text{col}} = \log_2 M$ columns, i.e., the interleaver is chosen such that coded bits $c_{k'_1}$ and $c_{k'_2}$ with $|k'_1 - k'_2| \leq d$ are mapped onto different symbols. Note that the interleaver is independent of the number of groups and independent of the number of relays in a particular group which makes the design simple. We assume in the following that $K_\nu \leq d_f$, $\nu \in \mathcal{G}$, because having more than d_f relays in a group cannot increase the diversity gain of the system.

In the proposed sub-carrier allocation schemes, the set of data sub-carriers \mathcal{N} is divided into a number of subsets which are referred to as *chunks*. We consider two schemes for the allocation of these chunks to the relays: 1) Uniform allocation and 2) Non-uniform allocation.

1) Uniform Allocation: In this scheme, the relays uniformly share the sub-carriers carrying d consecutive bits. For the ν th group with K_ν relays, the set of data sub-carriers \mathcal{N} is divided into K_ν chunks $\mathcal{C}_{\nu_j}^u$, $1 \leq j \leq K_\nu$, where each chunk contains $N_{c,\nu} = N/K_\nu$ sub-carriers (we assume that K_ν , $\nu \in \mathcal{G}$, is a factor of N). The chunk assigned to relay R_{ν_j} contains sub-carriers $\mathcal{C}_{\nu_j}^u \triangleq \{j-1, j-1+K_\nu, j-1+2K_\nu, \dots, j-1+(N_{c,\nu}-1)K_\nu\}$, $j \in \mathcal{K}_\nu$, $\nu \in \mathcal{G}$. Note that the number of chunks and the chunk size may be different for each group.

2) Non-uniform Allocation: Here, sub-carriers are allocated to the relays in a group according to the frequency diversity of the relay links. In this scheme, the set of data sub-carriers \mathcal{N} is divided into d chunks, where we assume that d is a factor of N and each chunk contains $N_c = N/d$

sub-carriers. The chunks are defined as $\mathcal{C}_i \triangleq \{i-1, i-1+d, i-1+2d, \dots, i-1+(N_c-1)d\}$, $1 \leq i \leq d$. Since only d_f out of d consecutive bits at the output of the encoder contribute to the diversity of the system, we first consider the allocation of d_f out of d chunks. Considering the ν th group, we first assign each relay one chunk and the remaining $d_f - K_\nu$ chunks are assigned to the relays according to the diversity orders of their respective channels. For this purpose, we compute

$$\zeta_{\nu_j} = \frac{\min(L_{1,\nu_j}, L_{2,\nu_j})}{\sum_{\kappa=1}^{K_\nu} \min(L_{1,\nu_\kappa}, L_{2,\nu_\kappa})}, \quad j \in \mathcal{K}_\nu, \quad (23)$$

which reflects how strong the $S \rightarrow R_{\nu_j}$ and $R_{\nu_j} \rightarrow D$ links of relay R_{ν_j} are in terms of frequency diversity compared to the links of the other relays in group ν . Next, for simplicity, we order the relays according to their ζ_{ν_j} values and re-label them such that R_{ν_1} is the relay with the largest value (ζ_{ν_1}) and so on. Now, we are ready to determine the number of chunks assigned to the relays according to their rank. In particular, relay R_{ν_1} receives additional $N_{\nu_1}^d \triangleq \lceil \zeta_{\nu_1}(d_f - K_\nu) \rceil$ chunks, and has a total of $N_{\nu_1} = 1 + N_{\nu_1}^d$ chunks. Similarly, relay R_{ν_2} receives additional $N_{\nu_2}^d \triangleq \lceil \zeta_{\nu_2}(d_f - K_\nu) \rceil$ chunks, and has a total of $N_{\nu_2} = 1 + N_{\nu_2}^d$ chunks. This procedure is continued until $\sum_{j=1}^{K_\nu} N_{\nu_j} = d_f$. The remaining $d - d_f$ chunks are allocated in such away as to ensure that the maximum diversity order is achieved for any d consecutive bits and the N_{ν_j} , $1 \leq j \leq K_\nu$, are incremented accordingly. This typically means that these $d - d_f$ chunks have to be allocated to the relay(s) whose links offer the most frequency diversity. Which relay should transmit which chunk(s) depends on the exact locations of the d_f bits that determine the free distance of the code within the span of the d consecutive bits of the error event. These locations are known *a priori* from the trellis structure of the code.

Example: To better illustrate the interleaving and sub-carrier allocation among relays we consider an example. We assume $N = 60$ data sub-carriers, one group with $K = 3$ relays R_1, R_2 , and R_3 (we drop group index $\nu = 1$ for convenience), 16-ary modulation, and a rate $1/2$ convolutional code with generator polynomials $(7, 5)_8$, $d = 6$, and $d_f = 5$. The total number of coded bits is $N \log_2 M = 240$. At the output of the interleaver, bits are read as $0, 60, 120, 180, 1, 61, \dots, 239$. For uniform allocation, each relay is assigned a chunk of $N/K = 20$ sub-carriers with $\mathcal{C}_1^u = \{0, 3, 6, \dots, 57\}$, and so on, cf. Fig. 3. For non-uniform allocation, we have $d = 6$ chunks \mathcal{C}_i , $1 \leq i \leq 6$, where $\mathcal{C}_1 = \{0, 6, 12, \dots, 54\}$, $\mathcal{C}_2 = \{1, 7, 13, \dots, 55\}$, and so on. Furthermore, assuming $\min(L_{1,1}, L_{2,1}) = 5$,

	R₁	R₂	R₃
bit index →	0,60,120,180	1,61,121,181	2,62,122,182
subcarrier index →	0	1	2
	3,63,123,183	4,64,124,184	5,65,125,185
	3	4	5
	⋮	⋮	⋮
	57,117,177,237	58,118,178,238	59,119,179,239
	57	58	59

Fig. 3. Uniform sub-carrier allocation to 3 relays R_1 , R_2 , and R_3 for $d = 6$, $d_f = 5$, 16-ary modulation, and $N = 60$ data sub-carriers in each OFDM symbol.

R₂	R₁	R₃	R₁	R₁	R₁
0,60,120,180	1,61,121,181	2,62,122,182	3,63,123,183	4,64,124,184	5,65,125,185
0	1	2	3	4	5
6,66,126,186	7,67,127,187	8,68,128,188	9,69,129,189	10,70,130,190	11,71,131,191
6	7	8	9	10	11
⋮	⋮	⋮	⋮	⋮	⋮
54,114,174,234	55,115,175,235	56,116,176,236	57,117,177,237	58,118,178,238	59,119,179,239
54	55	56	57	58	59

Fig. 4. Non-uniform sub-carrier allocation to 3 relays R_1 , R_2 , and R_3 for $d = 6$, $d_f = 5$, 16-ary modulation, $N = 60$ data sub-carriers in each OFDM symbol. Here $L_{1,1} = L_{2,1} = 5$, $L_{1,2} = L_{2,2} = 2$, and $L_{1,3} = L_{2,3} = 1$. Bit and sub-carrier indices are similar to those in Fig. 3.

$\min(L_{1,2}, L_{2,2}) = 2$, and $\min(L_{1,3}, L_{2,3}) = 1$, we obtain $N_1 = 4$ (the additional $d - d_f = 1$ chunk is allocated to R_1), $N_2 = 1$, and $N_3 = 1$. For the considered 1/2 rate code, all error events start at even indices k' of the coded bit stream and the bit in the fourth position out of every 6 consecutive bits does not contribute to the free distance of the code. For example, the 1st possible worst-case error event starts at $k' = 0$ with the irrelevant bit at $k' = 3$, the 2nd possible worst-case error event starts at $k' = 2$ with the irrelevant bit at $k' = 5$, and so on. To maximize diversity, we have to allocate the chunks to the relays in such a manner that $\sum_{j=1}^{K_\nu} d_{\nu_j} = d_f$ is satisfied for all possible worst-case error events. For the considered case, this is accomplished by assigning chunks $\mathcal{C}_1^n = \{\mathcal{C}_2, \mathcal{C}_4, \mathcal{C}_5, \mathcal{C}_6\}$, $\mathcal{C}_2^n = \{\mathcal{C}_1\}$, and $\mathcal{C}_3^n = \{\mathcal{C}_3\}$ to R_1 , R_2 , and R_3 , respectively, cf. Fig. 4. With this allocation of chunks, R_1 , R_2 , and R_3 , respectively, carry

3, 1, and 1 bits of the relevant $d_f = 5$ bits that define the free distance of the code for any possible worst-case error event of length $d = 6$ and the resulting diversity gain is $G_d = r_0 + 5$. In contrast, for the uniform allocation, two relays carry two bits and the third relay carries three bits and which relay carries three bits depends on the position of the considered error event in the data packet. Thus, the diversity gain is only $G_d = r_0 + 4$ in this case.

B. Relay Grouping

Relay grouping addresses the following question: Assuming we have K relays available and can afford $G + 1$, $G \geq 2$, time slots for transmission, how should we assign the relays to the G groups? Since the ultimate goal is to minimize the error rate, we base the relay grouping criterion on (18) and (20)–(22), which leads to the following cost function

$$J_{\text{rg}} = \prod_{\nu=1}^G \prod_{j=1}^{K_\nu} \left(\frac{1}{(\xi \bar{\gamma}_{1,\nu_j})^{r_{1,\nu_j}} \Phi_{1,\nu_j}} + \frac{1}{(\xi \bar{\gamma}_{2,\nu_j})^{r_{2,\nu_j}} \Phi_{2,\nu_j}} \right). \quad (24)$$

The optimum relay grouping is obtained from

$$\Theta^* = \arg \min_{\Theta \in \mathcal{F}} J_{\text{rg}}, \quad (25)$$

where \mathcal{F} is the set of all possible groupings of $\sum_{\nu=1}^G K_\nu = K$ relays into G groups, and Θ is one element of \mathcal{F} . For example, if there are $G = 2$ groups and $K = 3$ relays, R_1, R_2, R_3 , then $\mathcal{F} \triangleq \{ \{(R_1), (R_2, R_3)\}; \{(R_2), (R_1, R_3)\}; \{(R_3), (R_1, R_2)\} \}$, where the relays inside parenthesis belong to the same group. Note that the order of the groups does not affect performance, i.e., $\{(R_1), (R_2, R_3)\}$ is equivalent to $\{(R_2, R_3), (R_1)\}$.

C. Relay Selection

Another interesting problem is relay selection, where only a subset of the available relays is selected for transmission in order to reduce complexity. For simplicity, we assume that there is only one relay group in this section and drop the group index $\nu = 1$. Considering again the derived analytical expression for the worst-case PEP, the cost function for relay selection is chosen as

$$J_{\text{rs}}(\mathcal{D}) = \prod_{j \in \mathcal{D}} \left(\frac{1}{(\xi \bar{\gamma}_{1,j})^{r_{1,j}} \Phi_{1,j}} + \frac{1}{(\xi \bar{\gamma}_{2,j})^{r_{2,j}} \Phi_{2,j}} \right), \quad (26)$$

where \mathcal{D} is a subset of \mathcal{K} and, since only the relays in \mathcal{D} transmit, we have now $\sum_{j \in \mathcal{D}} d_{f,j} = d_f$. In the following, we consider two different relay selection problems.

1) Best Relay Subset Selection: In this case, we are interested in finding that subset $\mathcal{D}^* \subseteq \mathcal{K}$ which achieves the optimal performance without limiting the number of relays in \mathcal{D} . The corresponding selection criterion is

$$\mathcal{D}^* = \arg \min_{\mathcal{D} \subseteq \mathcal{K}} \{J_{\text{rs}}(\mathcal{D})\}. \quad (27)$$

The size of \mathcal{D}^* strongly depends on the SNRs and the frequency diversity of the involved links, cf. Section V.

2) Best Relay Selection: In practice, we may want to limit the number of relays that can be chosen in order to limit the

signaling overhead required for synchronization and channel estimation. In the extreme case, we may limit \mathcal{D} in (27) to have only one element, which leads to the best relay selection problem. For high SNR, the best relay selection criterion according to (27) simplifies to a max–min selection criterion

$$j^* = \arg \max_{j \in \mathcal{K}} \{ \min \{ (\xi \bar{\gamma}_{1,j})^{r_{1,j}} \Phi_{1,j}, (\xi \bar{\gamma}_{2,j})^{r_{2,j}} \Phi_{2,j} \} \}, \quad (28)$$

where j^* is the index of the best relay. For the special case of frequency–flat fading, we have $r_{1,j} = r_{2,j} = 1$ and $\Phi_{1,j} = \Phi_{2,j}$, and (28) is equivalent to the conventional max–min criterion [14] developed for uncoded transmission over frequency–flat channels. However, for the general case of frequency–selective fading, (28) achieves a superior performance compared to the conventional max–min criterion.

V. SIMULATION RESULTS

In this section, we present Monte–Carlo simulation results to illustrate the performance of cooperative BICM–OFDM and to support the analytical results and design guidelines developed in Section III and Section IV, respectively. Throughout this section we adopt the rate 1/2 convolutional code with generator polynomials $(7, 5)_8$, worst–case error event length $d = 6$, and free distance $d_f = 5$, 16–QAM modulation with Gray labeling, and $N_t = 64$ sub–carriers of which $N = 60$ are data sub–carriers. The sub–carrier allocation and the interleaver are designed as outlined in Section IV-A. Unless specified otherwise, we employ uniform sub–carrier allocation, assume SNR $\bar{\gamma} = \bar{\gamma}_{1,\nu_j} = \bar{\gamma}_{2,\nu_j}$, $j \in \mathcal{K}_\nu$, $\nu \in \mathcal{G}$, and the direct $S \rightarrow D$ link is *not* exploited. The coefficients of the CIRs of all links are independent, identically distributed (i.i.d.) Rayleigh fading.

We first discuss the diversity gain of the proposed system before we present results for various system design problems. Where appropriate, we drop group index ν for convenience.

Diversity Gain: First, we consider a system with $K = 2$ relays distributed over one group and two groups, respectively. We assume that the CIRs of all $S \rightarrow R_j$ and $R_j \rightarrow D$ links have identical lengths $L_{i,j} = L$ for $i \in \{1, 2\}$, $j \in \{1, 2\}$. Fig. 5 shows the bit error rate (BER) vs. SNR $\bar{\gamma}$ for different CIR lengths L . First, we consider the case where both relays are placed in a single group (solid lines). For the uniform sub–carrier allocation in Section IV-A, consecutive bits in one error event are transmitted via different relays. Thus, of the $d_f = 5$ bits that determine the free distance of the code, one relay carries 3 bits (e.g. $d_{f,1} = 3$) and the other relay carries 2 bits (e.g. $d_{f,2} = 2$). Fig. 5 confirms that for $L = 1$, we obtain a diversity gain of $G_d = \min\{d_{f,1}, L_{1,1}, L_{2,1}\} + \min\{d_{f,2}, L_{1,2}, L_{2,2}\} = 1 + 1 = 2$ as expected from our analysis in Section III-B. For $L = 2$ this gain increases to $G_d = 2 + 2 = 4$ and for $L \geq 3$ the maximum achievable diversity gain (without $S \rightarrow D$ link) of $G_d = d_f = 5$ is attained. The additional performance gain when increasing L from 3 to 4 can be attributed to the lower correlation between sub–carriers for larger CIR lengths resulting in larger eigenvalues $\lambda_m(\cdot)$ in (18). Now, we consider the case where the relays are placed in two different groups (dashed lines). As expected, Fig. 5 shows that potentially a higher diversity gain can be achieved with two groups compared to one group

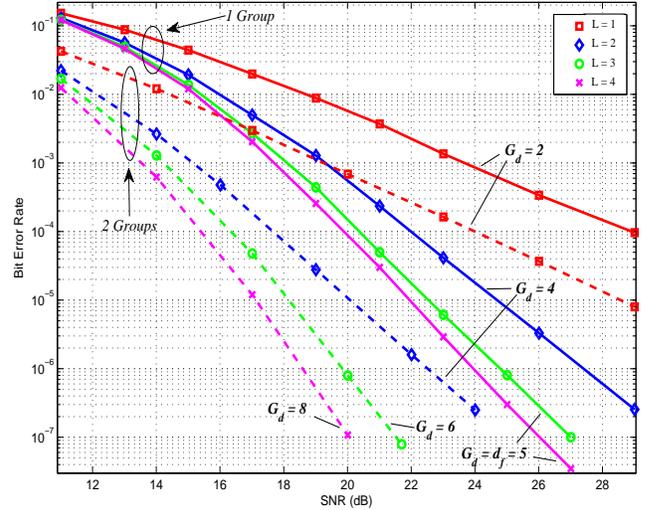


Fig. 5. BER vs. SNR of cooperative BICM–OFDM with two relays in one group and two groups, respectively. No direct $S \rightarrow D$ link, equal SNRs for all links, and uniform sub–carrier allocation.

at the expense of a decrease in throughput. In particular, for $L = 3$ and $L = 4$, G_d is limited to $d_f = 5$ for the single group, but increases to 6 and 8 for two groups, which have a maximum diversity gain of $G_d = 10$, which would be attained for $L \geq 5$ (not shown in the figure). In contrast, for $L = 1$ and $L = 2$, respectively, the single group and the two groups achieve the same diversity gain. Nevertheless, two groups still achieve an SNR gain of about 5 dB compared to one group because of the noise averaging facilitated by the reception of multiple copies of the transmitted signal.

Next, in Fig. 6, we compare the performance of uniform and non–uniform sub–carrier allocation for a system with one group containing two relays. We assume $L_{1,j} = L_{2,j} = L_j$, $j \in \{1, 2\}$, and consider three cases: $L_1 = 1, L_2 = 4$ (Case 1), $L_1 = 2, L_2 = 3$ (Case 2), and $L_1 = L_2 = 3$ (Case 3). First, we consider uniform allocation, where for all three cases, depending on where in the codeword the worst–case error event starts, R_1 carries 2 or 3 bits (i.e., $d_{f,1} = 2$ or $d_{f,1} = 3$) and R_2 carries the remaining $d_{f,2} = d_f - d_{f,1}$ bits. Hence, considering the worst case, we obtain from (19) $G_d = 2 + 1 = 3$, $G_d = 2 + 2 = 4$, and $G_d = 5$ for Cases 1, 2, and 3, respectively. In contrast, for non–uniform sub–carrier allocation, independent from where in the codeword the worst–case error event starts, the bits are assigned such that $d_{f,1} = 1$ and $d_{f,2} = 4$ in Case 1, $d_{f,1} = 2$ and $d_{f,2} = 3$ in Case 2, and $d_{f,1} = 3$ and $d_{f,2} = 2$ or $d_{f,1} = 2$ and $d_{f,2} = 3$ in Case 3, cf. Section IV-A. Thus, non–uniform sub–carrier allocation achieves the maximum diversity order of $G_d = 5$ in all cases. This clearly illustrates the benefits of matching the sub–carrier allocation to the frequency diversity of the channel. For Case 3, the diversity gains for uniform and non–uniform allocation are identical but uniform allocation seems to lead to a lower correlation between sub–carriers resulting in larger eigenvalues $\lambda_m(\cdot)$ in (18) and thus, in a superior BER performance.

Relay Grouping: Next, we consider the problem of assign-

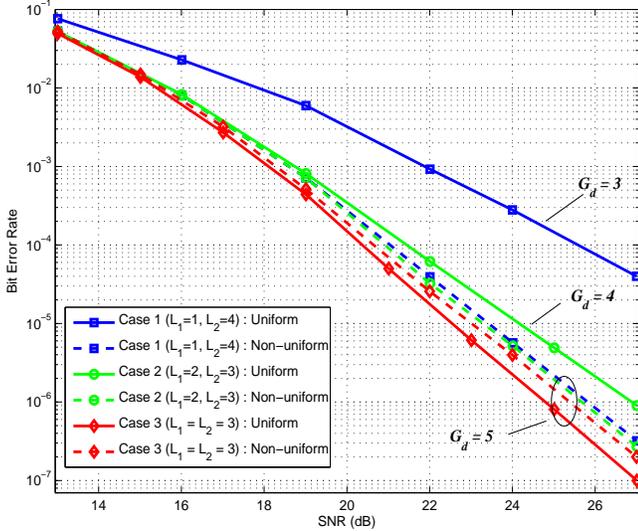


Fig. 6. BER vs. SNR of cooperative BICM-OFDM with two relays in one group. No direct $S \rightarrow D$ link and equal SNRs for all links.

ing five relays R_j , $j \in \{1, \dots, 5\}$, to two groups. We assume $L_{1,i} = L_{2,i} = 1$ for $i \in \{1, 2, 3, 4\}$, $L_{1,5} = L_{2,5} = 5$, and equal SNRs for all links. In Fig. 7, we show the BER for those three assignments which achieve the highest performance. The assignments considered in Fig. 7 are: (R_1, R_2) in group 1 and (R_3, R_4, R_5) in group 2 (Choice 1), (R_1, R_2, R_3) in group 1 and (R_4, R_5) in group 2 (Choice 2), and (R_1, R_2, R_3, R_4) in group 1 and R_5 in group 2 (Choice 3). For this scenario and at SNR = 20 dB, we obtain from (24) $J_{rg} = 10^{-4.99}$, $J_{rg} = 10^{-5.17}$, and $J_{rg} = 10^{-5.88}$ for Choices 1, 2, and 3, respectively. Thus, Choice 3 is adopted according to the criterion in (25), which is also verified by the results shown in Fig. 7. This result is intuitively pleasing since Choice 3 yields highest diversity gain ($G_d = 9$) among all possible relay groupings.

Relay Selection: In Fig. 8, we consider the optimal relay selection problem for a cooperative BICM-OFDM system without direct $S \rightarrow D$ link and with a single group. In particular, we consider the case where the number of selected relays is not fixed *a priori*. We assume that $K = 5$ relays are available for selection with $L_{1,j} = L_{2,j} = 2$, $\bar{\gamma}_{1,j} = \bar{\gamma}_{2,j} = \bar{\gamma}$ (dB) for $j \in \{1, 2, 3\}$ and $L_{1,j} = L_{2,j} = 3$, $\bar{\gamma}_{1,j} = \bar{\gamma}_{2,j} = \bar{\gamma} + 2$ (dB) for $j \in \{4, 5\}$. We assume that a maximum of three relays can be selected for cooperation. In Fig. 8, we show the four selections which achieve the highest performance: (R_2, R_3, R_4) , (R_3, R_4, R_5) , (R_1, R_2, R_3) , and (R_4, R_5) . All these combinations provide $G_d = 5$. Assuming a target SNR of 20 dB, we obtain $J_{rs}(2, 3, 4) = 10^{-3.96}$, $J_{rs}(3, 4, 5) = 10^{-4.53}$, $J_{rs}(1, 2, 3) = 10^{-3.78}$, and $J_{rs}(4, 5) = 10^{-4.55}$, i.e., the proposed relay selection criterion in (27) would indeed select relays R_4 and R_5 , which also yield the best performance according to Fig. 8. Interestingly, in this case, it is preferable to select only two relays for cooperation instead of the maximum allowed three relays, since these two relays enjoy more frequency diversity and a higher SNR

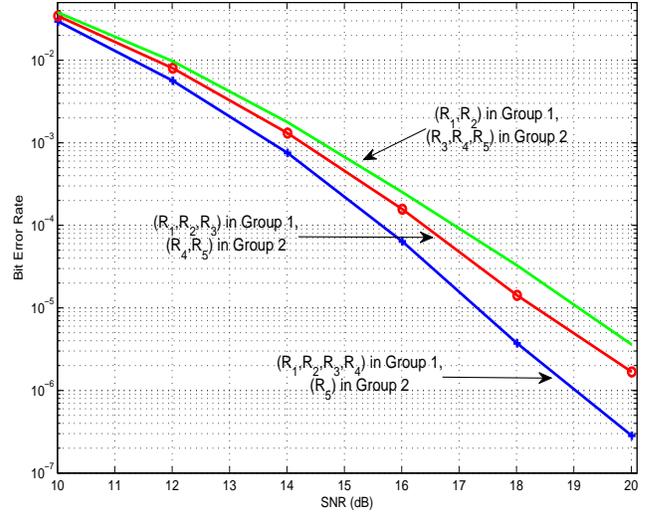


Fig. 7. BER vs. SNR of cooperative BICM-OFDM for different relay groupings. No direct $S \rightarrow D$ link, equal SNRs for all links, uniform sub-carrier allocation, $L_{1,i} = L_{2,i} = 1$ for $i \in \{1, 2, 3, 4\}$, and $L_{1,5} = L_{2,5} = 5$.

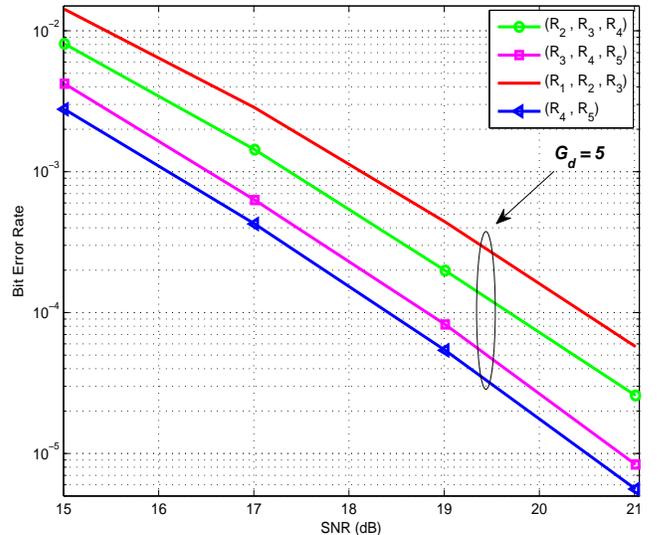


Fig. 8. BER vs. SNR of cooperative BICM-OFDM with selection of three or less of $K = 5$ relays. No direct $S \rightarrow D$ link and uniform sub-carrier allocation.

compared to the remaining three relays. Thus, adding another relay to R_4 and R_5 can only degrade performance since sub-carriers would have to be taken away from the “strong” R_4 and R_5 and given to a “weaker” relay.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we have proposed a novel cooperative BICM-OFDM scheme where groups of relays assist a source in communicating with a destination. Relays in the same group transmit concurrently over disjoint sets of sub-carriers and relays in different groups transmit in different time slots. We have derived closed-form expressions for an upper bound on

$$A \leq \int_{\|\mathbf{h}_{1,\nu_j}\|^2 \leq \epsilon} \int_{\mathbf{h}_{2,\nu_j}} \exp\left(-\xi \sum_{k,d_f,\nu_j} \frac{\bar{\gamma}_{1,\nu_j} \bar{\gamma}_{2,\nu_j} |\mathbf{w}_{1,\nu_j}^H[k] \mathbf{h}_{1,\nu_j}|^2 |\mathbf{w}_{2,\nu_j}^H[k] \mathbf{h}_{2,\nu_j}|^2}{\bar{\gamma}_{1,\nu_j} L_{1,\nu_j} \epsilon + \bar{\gamma}_{2,\nu_j} |\mathbf{w}_{2,\nu_j}^H[k] \mathbf{h}_{2,\nu_j}|^2}\right) p_1(\mathbf{h}_{1,\nu_j}) p_2(\mathbf{h}_{2,\nu_j}) d\mathbf{h}_{1,\nu_j} d\mathbf{h}_{2,\nu_j}. \quad (34)$$

the asymptotic worst-case PEP and the achievable diversity gain of the considered system. Based on these analytical results, we have developed design criteria for sub-carrier allocation to relays, relay grouping, and relay selection. Simulation results have corroborated our analytical findings and confirmed the effectiveness of the proposed design guidelines.

Interesting topics for future work include the design and optimization of such generalized cooperative BICM-OFDM systems, where each relay is allowed to transmit in multiple time slots.¹

APPENDIX

From (16), we obtain

$$\Pi_{\nu_j} = \int_{\mathbf{h}_{1,\nu_j}} \int_{\mathbf{h}_{2,\nu_j}} f(\mathbf{h}_{1,\nu_j}, \mathbf{h}_{2,\nu_j}) p_1(\mathbf{h}_{1,\nu_j}) p_2(\mathbf{h}_{2,\nu_j}) d\mathbf{h}_{1,\nu_j} d\mathbf{h}_{2,\nu_j}, \quad (29)$$

where

$$f(\mathbf{h}_{1,\nu_j}, \mathbf{h}_{2,\nu_j}) = \exp\left(-\xi \sum_{k,d_f,\nu_j} \frac{\bar{\gamma}_{1,\nu_j} \bar{\gamma}_{2,\nu_j} |\mathbf{w}_{1,\nu_j}^H[k] \mathbf{h}_{1,\nu_j}|^2 |\mathbf{w}_{2,\nu_j}^H[k] \mathbf{h}_{2,\nu_j}|^2}{\bar{\gamma}_{1,\nu_j} |\mathbf{w}_{1,\nu_j}^H[k] \mathbf{h}_{1,\nu_j}|^2 + \bar{\gamma}_{2,\nu_j} |\mathbf{w}_{2,\nu_j}^H[k] \mathbf{h}_{2,\nu_j}|^2}\right), \quad (30)$$

and $p_1(\mathbf{h}_{1,\nu_j})$ and $p_2(\mathbf{h}_{2,\nu_j})$ are the probability density functions of \mathbf{h}_{1,ν_j} and \mathbf{h}_{2,ν_j} , respectively. Recall that \mathbf{h}_{1,ν_j} and \mathbf{h}_{2,ν_j} are zero-mean Gaussian random vectors with covariance matrices \mathbf{C}_{1,ν_j} and \mathbf{C}_{2,ν_j} , respectively. For the asymptotic regime of high SNR, $\bar{\gamma}_{1,\nu_j}, \bar{\gamma}_{2,\nu_j} \rightarrow \infty$, decision errors only occur if $\|\mathbf{h}_{1,\nu_j}\|^2 \rightarrow 0$ and/or $\|\mathbf{h}_{2,\nu_j}\|^2 \rightarrow 0$ [15]. Thus, an asymptotic upper bound for Π_{ν_j} is given by

$$\Pi_{\nu_j} \leq A + B, \quad (31)$$

where

$$A \triangleq \int_{\|\mathbf{h}_{1,\nu_j}\|^2 \leq \epsilon} \int_{\mathbf{h}_{2,\nu_j}} f(\mathbf{h}_{1,\nu_j}, \mathbf{h}_{2,\nu_j}) p_1(\mathbf{h}_{1,\nu_j}) p_2(\mathbf{h}_{2,\nu_j}) \times d\mathbf{h}_{1,\nu_j} d\mathbf{h}_{2,\nu_j}, \quad (32)$$

$$B \triangleq \int_{\mathbf{h}_{1,\nu_j}} \int_{\|\mathbf{h}_{2,\nu_j}\|^2 \leq \epsilon} f(\mathbf{h}_{1,\nu_j}, \mathbf{h}_{2,\nu_j}) p_1(\mathbf{h}_{1,\nu_j}) p_2(\mathbf{h}_{2,\nu_j}) \times d\mathbf{h}_{1,\nu_j} d\mathbf{h}_{2,\nu_j}, \quad (33)$$

where $\epsilon \rightarrow 0^+$ denotes a small positive number and the region $\{\mathbf{h}_{1,\nu_j}, \mathbf{h}_{2,\nu_j} : \|\mathbf{h}_{1,\nu_j}\|^2 \leq \epsilon, \|\mathbf{h}_{2,\nu_j}\|^2 \leq \epsilon\}$ is contained in both A and B . Applying Schwarz's inequality, $|\mathbf{w}_{1,\nu_j}^H[k] \mathbf{h}_{1,\nu_j}|^2 \leq L_{1,\nu_j} \epsilon$, in (32), we obtain (34) as shown

at the top of this page. For $|\mathbf{w}_{2,\nu_j}^H[k] \mathbf{h}_{2,\nu_j}|^2 \neq 0$, we have

$$\lim_{\epsilon \rightarrow 0^+} \lim_{\bar{\gamma}_{1,\nu_j}, \bar{\gamma}_{2,\nu_j} \rightarrow \infty} \left(\frac{\bar{\gamma}_{2,\nu_j} |\mathbf{w}_{2,\nu_j}^H[k] \mathbf{h}_{2,\nu_j}|^2}{\bar{\gamma}_{1,\nu_j} L_{1,\nu_j} \epsilon + \bar{\gamma}_{2,\nu_j} |\mathbf{w}_{2,\nu_j}^H[k] \mathbf{h}_{2,\nu_j}|^2} \right) = 1. \quad (35)$$

Exploiting (35) and noting that the contribution of vectors \mathbf{h}_{2,ν_j} fulfilling $|\mathbf{w}_{2,\nu_j}^H[k] \mathbf{h}_{2,\nu_j}|^2 = 0$ to the integral in (34) is zero, we can simplify (34) to

$$A \leq \int_{\mathbf{h}_{1,\nu_j}} \int_{\mathbf{h}_{2,\nu_j}} \exp\left(-\xi \sum_{k,d_f,\nu_j} \bar{\gamma}_{1,\nu_j} |\mathbf{w}_{1,\nu_j}^H[k] \mathbf{h}_{1,\nu_j}|^2\right) \times p_1(\mathbf{h}_{1,\nu_j}) p_2(\mathbf{h}_{2,\nu_j}) d\mathbf{h}_{1,\nu_j} d\mathbf{h}_{2,\nu_j}. \quad (36)$$

Since the integrand in (36) is independent of \mathbf{h}_{2,ν_j} , similar to (14), we obtain

$$A \leq \frac{1}{\det(\mathbf{I}_{L_{1,\nu_j}} + \xi \bar{\gamma}_{1,\nu_j} \mathbf{W}_{1,\nu_j} \mathbf{C}_{1,\nu_j})} = \frac{1}{(\xi \bar{\gamma}_{1,\nu_j})^{r_{1,\nu_j}} \prod_{m=1}^{r_{1,\nu_j}} \lambda_m(\mathbf{W}_{1,\nu_j} \mathbf{C}_{1,\nu_j})} + o\left(\frac{1}{\bar{\gamma}_{1,\nu_j}^{r_{1,\nu_j}}}\right). \quad (37)$$

Similarly, from (33), we obtain

$$B \leq \frac{1}{\det(\mathbf{I}_{L_{2,\nu_j}} + \xi \bar{\gamma}_{2,\nu_j} \mathbf{W}_{2,\nu_j} \mathbf{C}_{2,\nu_j})} = \frac{1}{(\xi \bar{\gamma}_{2,\nu_j})^{r_{2,\nu_j}} \prod_{m=1}^{r_{2,\nu_j}} \lambda_m(\mathbf{W}_{2,\nu_j} \mathbf{C}_{2,\nu_j})} + o\left(\frac{1}{\bar{\gamma}_{2,\nu_j}^{r_{2,\nu_j}}}\right). \quad (38)$$

Combining (31), (37), and (38) yields (17).

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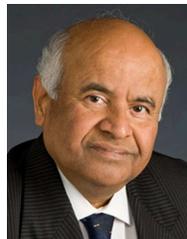
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