SLNC for Multi–source Multi–relay BICM–OFDM Systems

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***Abstract*—In this paper, we study the application of bit–interleaved coded modulation (BICM) and orthogonal frequency division multiplexing (OFDM) to reap the benefits of wireless multiuser network coding in practical frequency–selective fading channels. We propose a mapping based symbol level network coding (SLNC) scheme for a cooperative diversity system comprising multiple sources, multiple relays, and one common destination. A simple cooperative maximum–ratio combining scheme is used at the destination and is shown to successfully exploit both the full spatial and the full frequency diversity offered by the channel for arbitrary numbers of sources, arbitrary numbers of relays, and arbitrary linear modulation schemes. To gain analytical insight for system design, we derive a closed–form upper bound for the asymptotic worst–case pairwise error probability (PEP) and obtain the diversity gain of the considered SLNC scheme for BICM–OFDM systems. These analytical results reveal the influence of the various system parameters, such as the number of sources, the free distance of the code, and the frequency diversity of the involved links, on performance. Furthermore, we propose two different relay selection schemes for the considered system: a) bulk selection, i.e., a single best relay is selected to transmit on all sub–carriers, and b) per–subcarrier selection, where a best relay is selected on each sub–carrier. Last but not least, we exploit the derived PEP expression for selecting a subset of sources from the set of active sources when the number of active sources is larger than the number of available orthogonal relay channels. We study the achievable diversity gain for the proposed relay and source subset selection schemes. Numerical results corroborate the derived diversity gain expressions and confirm the performance gains.**

***Index Terms*—Cooperative Diversity, Network Coding, Relay Selection, Source Subset Selection, BICM–OFDM.**

I. Introduction

Cooperative diversity (CD) techniques can achieve high diversity gains in distributed wireless networks, where nodes are allowed to cooperate by relaying each other's signal, and have attracted considerable research interest due to their possible use in future cellular, ad–ho, and sensor networks [1], [2]. Conventional relay cooperation protocols rely on either amplify–and–forward (AF) or decode–and–forward (DF) operations [1], [3], [4]. However, as the number of sources grows, the traditional diversity achieving cooperation schemes incur throughput loss, because one relay is typically limited to serve only a single source at a particular time [5], [6]. To circumvent this problem, wireless network coding has been recently considered for cooperative diversity systems [7], [8].

Several wireless network coding schemes such as physical–layer network coding (PLNC) for two way relaying [9], and bit level network coding (BLNC) [7], [10], complex field network coding (CFNC) [11], and compute–and–forward (CPF) network coding [12] for general multi–source cooperative diversity systems have been proposed recently. In this work, we consider a relay aided multi–source cooperation framework based on symbol level network coding (SLNC) [13]. At the relays, we form an SLNC symbol based on the symbols received from the different sources. In particular, the set of received source symbols is mapped to a new extended signal constellation, i.e., the relay transmit symbols are taken from a larger signal constellation than the source symbols. Hence, the proposed SLNC scheme can be considered as a special case of non–linear network coding [14], [15], [16]. Compared to BLNC, CFNC and SLNC exploit more degrees of freedom by encoding information in both the complex field and GF(2) rather than GF(2) alone, and hence result in a higher coding gain [17]. However, in CFNC [11], the relay transmit symbols, which also belong to a larger constellation compared to the source symbols, do not adhere to a regular constellation. Hence, by properly selecting the signal constellation, SLNC can achieve a larger minimum Euclidean distance for the relay transmit symbols compared to CFNC. Furthermore, in CFNC and CPF [12], all sources transmit concurrently. Thus, perfect synchronisation among all sources' transmissions is required, which is difficult to achieve in practice. In addition, CFNC requires complex multiuser detection at the relays. In the proposed SLNC scheme, the sources transmit over orthogonal channels to the relays, which decreases the throughput compared to CFNC, but allows for simple single user decoding at the relays and relaxed synchronization requirements compared to CFNC.

Critical to the diversity gain achieved by multi–source network coding schemes is the processing performed at the relays. In this paper, we assume that the relay nodes cannot afford analog processing and storage, and thus decode the received signals. For DF protocols, diversity can be achieved with conventional maximum ratio combining at the destination only if the forwarded packets are error free [18]. Thus, several works on multi–source multi–relay network coding assume that the relays do not forward erroneous packets [19]. However, relaying packets selectively leads to the dismissal of entire packets even when the number of erroneously decoded bits within a packet is small, ultimately affecting the error performance. Surprisingly, the decoding performance at the destination can be improved

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by also forwarding erroneous packets [11], [20] provided that the effect of error propagation via the relay link is mitigated by a channel aware combining scheme at the destination. To this end, cooperative maximum–ratio combining (C–MRC), which was proposed for uncoded DF relaying in [20], can be adopted. C–MRC has low complexity compared to maximum–likelihood (ML) decoding but achieves a similar performance [20].

Moreover, bit–interleaved coded modulation combined with orthogonal frequency division multiplexing (BICM–OFDM) is a popular approach to exploit the inherent diversity offered by frequency–selective channels [21] and forms the basis for many wireless standards (e.g., IEEE 802.16x, LTE). The fundamental limits and properties of BICM have been analyzed mostly in the context of point-to-point transmission, see e.g., [22], [23], [24]. The few works that have studied the application of BICM in multi–node communication systems, such as relay and network–coded systems, mostly considered flat–fading links [25], [26], [27]. Furthermore, most existing wireless network coding schemes assume frequency–flat fading links and/or uncoded transmission [7], [10], [11], [19], [28]. Hence, these existing methods are not general enough to cope with frequency–selective fading. As frequency–selective channels are commonly encountered in practice, it is of both theoretical and practical interest to investigate the performance of cooperative diversity systems employing network coding and BICM–OFDM jointly. Recently, the performance of cooperative BICM–OFDM with AF and DF relays was studied in [29] and [30], respectively. However, the analysis and design guidelines given in [29] and [30] are not applicable to multi–source cooperative BICM–OFDM communication systems employing network coding. Furthermore, the authors in [31] studied the combination of BLNC and BICM–OFDM for single relay networks and showed that with BLNC, the diversity gains of different sources are mutually dependent, i.e., the maximum achievable diversity gain of a source can be limited by the frequency diversity of the direct links of other sources. Although it is known that for uncoded systems and flat fading, symbol based network coding (e.g., CFNC [11], SLNC) and bit based network coding (e.g., BLNC [7], [10]) achieve identical diversity gains [11], it is not clear from the reported literature if this is also true for coded systems and frequency–selective fading. Hence, one of the objectives of this paper is to analyze the error rate performance and the diversity gains of systems employing SLNC and BICM–OFDM. Moreover, the literature on relay selection for cooperative OFDM systems is very sparse. In [32], the authors proposed relay selection schemes for cooperative AF OFDM systems but a diversity gain analysis was not provided. The relay selection scheme proposed in [29] for AF BICM–OFDM is based on the average error rate and does not yield any additional diversity gain. Furthermore, if the number of available orthogonal relay channels is smaller than the number of active sources, only a subset of the sources can transmit. However, to the best of the authors' knowledge, there is no reported work on the analysis of source subset selection in the context of network coding and relaying, even for uncoded flat–fading links. Hence, the design and evaluation of relay and source subset selection schemes for BICM–OFDM systems employing wireless network coding is an open research problem.

In this paper, we consider SLNC based cooperative diversity schemes with the aim to achieve the maximum diversity gain offered by the channel. Standard BICM–OFDM employing arbitrary M–ary modulation is adopted at the sources. We assume that the source–relay multiple access channel is non–ideal and the relay may forward error–prone packets to the destination. Furthermore, we assume a two phase communication protocol: In Phase I, multiple sources transmit over orthogonal channels and in Phase II, the relays concurrently transmit network coded symbols over disjoint sets of OFDM sub–carriers. The relays first decode the signal received from the multiple sources and then obtain the network coded symbol by mapping the set of decoded source symbols to a larger constellation. The destination combines the source signals received from direct transmission and the relayed network coded symbols into a C–MRC bit metric for decoding. This metric can be considered as an extension of the C–MRC scheme in [20]. Below, we summarize the original contributions of this paper:

1) We propose a mapping based SLNC scheme and provide a mathematical framework for the analysis of the asymptotic worst–case pairwise error probability (PEP) and the diversity gain of the considered multi–source multi–relay BICM–OFDM system for high signal–to–noise ratio (SNR). The PEP expressions and the diversity gain provide important insight into the influence of the different system parameters (such as the free distance of the code, the frequency diversity of the links, and the average SNR) on performance. We also show that in cooperative BICM–OFDM systems, SLNC achieves a higher diversity gain than BLNC [31], if the fading is frequency–selective.
2) We exploit the derived instantaneous PEP expressions for optimization of the network via relay selection and source subset selection. We consider two different relay selection schemes: a) bulk selection where one relay is selected to transmit over all sub–carriers and b) per sub–carrier selection where the best relay is selected on each sub–carrier. We show analytically that both schemes achieve identical diversity gains. Furthermore, we consider source subset selection and propose a selection scheme that jointly selects the best relay and the best subset of sources. The resulting diversity gain is also derived.

The remainder of this paper is organized as follows. In Section II, the considered multi–user multi–relay system model is presented and the proposed C–MRC bit metric is introduced. The asymptotic PEP upper bound and the diversity gain of the considered system are derived in Section III. In Sections IV and V, relay selection and source subset selection are considered, respectively. Finally, supporting simulation results are provided and conclusions are drawn in Sections VI and VII, respectively.

Notation: In this paper, $\mathcal{E}\{\cdot\}$, $[\cdot]^T$, and $|\cdot|$ denote statistical expectation, transposition, and the magnitude of a scalar or the cardinality of a set, respectively. $\lambda_m(\boldsymbol{X})$, $1 \leq m \leq \mathrm{rank}\{\boldsymbol{X}\}$, denotes the non–zero eigenvalues of matrix $\boldsymbol{X}$ and $\Re\{\cdot\}$ denotes the real part of a complex number. $\Gamma(\cdot)$ denotes the Gamma function, $\doteq$ denotes asymptotic equivalence, and $\circledast$ denotes convolution.

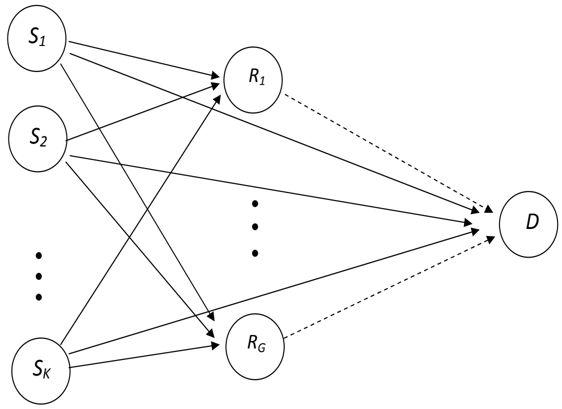

Fig. 1. System model for multi–source multi–relay network coded cooperation. Solid and dashed lines indicate transmission in Phase I and II, respectively.

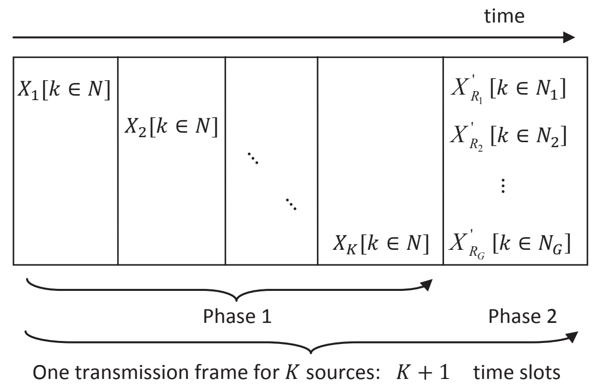

Fig. 2. Structure of the two–phase model with K sources and G relays participating in data transmission.

II. System Model

The considered system consists of K source terminals, S_j, $j \in \mathcal{S} := \{1, \ldots, K\}$, G relays, R_u, $u \in \mathcal{R} := \{1, \ldots, G\}$, and one destination terminal D. The adopted relaying protocol comprises two phases, cf. Fig. 1. In Phase I, the sources S_j transmit their symbols to relay R_u and destination D over K orthogonal channels, which is in contrast to CFNC [11] and CPF [12]. In Phase II, the relays R_u transmit the network coded symbols over disjoint sets of sub–carriers to the destination. In the following, we explain the two phase signal model and the decoding operation. For the considered system, we assume perfect frequency synchronization.

A. *Phase I*

Each source S_j employs conventional BICM–OFDM[1], i.e., the output bits $c_{j,k'}$, $0 \leq k' < \log_2(M)N$, of a binary convolutional encoder with minimum free distance d_f are interleaved and mapped (via constellation mapping function $\mathcal{M}_x$) onto symbols $X_j[k] \in \mathcal{X}$, $k \in \mathcal{N}$, $\mathcal{N} \triangleq \{0, 1, \ldots, N-1\}$, where $\mathcal{X}$ denotes an M–ary symbol alphabet and N is the number of data sub–carriers in one OFDM symbol. The effect of the interleaver can be modeled by the mapping $k' \to (k, i)$, where k' denotes the original index of coded bit $c_{j,k'}$, and k and i denote the index of symbol $X_j[k]$ and the position of $c_{j,k'}$ in the label of $X_j[k]$, respectively. The interleaver is designed such that consecutive coded bits are: 1) mapped onto different symbols; 2) transmitted over different sub–carriers; and 3) interleaved within one OFDM symbol to keep the decoding delay at the receiver low. The transmitted symbols are assumed to have unit average energy, i.e., $\mathcal{E}\{|X_j[k]|^2\} = 1$. Throughout this paper we assume conventional OFDM processing at the sources, the relay, and the destination and a sufficiently long cyclic prefix (CP) to avoid interference between sub–carriers.

In Phase I, the received signal at D from S_j on sub–carrier $k \in \mathcal{N}$ can be modeled as

$$Y_{S_jD}[k] = \sqrt{P} H_{S_jD}[k] X_j[k] + N_{S_jD}[k], \quad \forall j, k, \quad (1)$$

where P is the average transmit power in each sub–carrier, $N_{S_jD}[k]$ is complex additive white Gaussian noise (AWGN) with variance $\sigma^2_{n_{S_jD}}$, and $H_{S_jD}[k]$ is the gain on sub–carrier k of the $S_j \to D$ channel.

The received signal at R_u from S_j on the kth sub–carrier can be modeled as

$$Y_{S_jR_u}[k] = \sqrt{P} H_{S_jR_u}[k] X_j[k] + N_{S_jR_u}[k], \quad \forall j, k, u, \quad (2)$$

where $N_{S_jR_u}[k]$ is complex AWGN with variance $\sigma^2_{n_{S_jR_u}}$ and $H_{S_jR_u}[k]$ is the gain on sub–carrier k of the $S_j \to R_u$ channel.

To decode the bits transmitted by S_j, R_u computes the BICM bit metric for the ith bit in the label of symbol $X_j[k]$ as

$$\zeta^i_{k,u}[c_{j,k'}] = \min_{X_j \in \mathcal{X}^i_{c_{j,k'}}} \left\{ |Y_{S_jR_u}[k] - \sqrt{P} H_{S_jR_u}[k] X_j|^2 \right\}, \quad (3)$$

where $\mathcal{X}^i_b$ denotes the subset of all symbols $X \in \mathcal{X}$ whose label has value $b \in \{0, 1\}$ in position i, and in general $|\mathcal{X}^i_b| = |\mathcal{X}|/2$. The bit metrics are de–interleaved and Viterbi decoded at R_u.

B. *Phase II*

In Phase II, relay R_u selects a set $\mathcal{N}_u \subseteq \mathcal{N}$ of sub–carriers and transmits the network coded version of the K source symbols to the destination. The sets of sub–carriers are chosen such that $\mathcal{N}_u \cap \mathcal{N}_\nu = \emptyset$, $u \neq \nu$, and $\sum_{u=1}^{G} |\mathcal{N}_u| = N$, cf. Fig. 2. The sub–carriers are uniformly distributed among the relays.

Furthermore, in the proposed SLNC scheme, the relay transmit symbol is obtained as $X'_{R_u}[k] \triangleq \mathcal{M}_R(X'_1[k], \ldots, X'_K[k])$, $k \in \mathcal{N}_u$, where $X'_j[k] \in \{X_j[k], \hat{X}_{j,u}[k]\} \in \mathcal{X}$, $\hat{X}_{j,u}[k]$ denotes an erroneously detected symbol at R_u, $\mathcal{M}_R(\cdot)$ is a bijective function which maps the set of $\{X'_j[k]\}$, $\forall j$, into an arbitrary constellation of size M^K. The mapped symbol $X'_{R_u}[k]$ can be interpreted as a non–linear combination of the source symbols, i.e., the adopted SLNC scheme is a form of non–linear network coding [14] where the relay transmit symbols are obtained by mapping. For comparison, in CFNC [11], the relay transmit symbol is obtained from a linear combination of the source symbols and is given by $X'_{R_u}[k] \triangleq \sum_{j=1}^{K} \theta_j X'_j[k]$, $k \in \mathcal{N}_u$, where coefficients θ_j, $\forall j$, are drawn from the complex field such that $X'_{R_u}[k]$ is unique for every possible set of source symbols (i.e., constellation size for CFNC symbols is also M^K). The set of θ_j, $\forall j$, is not unique and the coefficients originally designed for linear constellation precoding for co–located multi–antenna systems in [34] are a possible choice [11]. Due to the bijective mapping, there is a one–to–one correspondence between the set of decoded source symbols

[1]We note that OFDM is an efficient approach to cope with frequency–selective fading [33], and BICM facilitates the exploitation of frequency diversity if OFDM is applied [21].

and the SLNC and CFNC symbols. Although this one–to–one mapping reduces the minimum Euclidean distance of the SLNC and CFNC constellation as the constellation size of the source symbols and the number of sources grow, it is necessary for extraction of the full diversity offered by the channel, as will be discussed in Section III–B. We show an example for SLNC and CFNC constellations for $M = 4$ and $K = 2$ in Fig. 3, where the θ_j for CFNC are chosen as in [34]. From Fig. 3, we observe that the minimum Euclidean distance of the SLNC symbols is larger than that of the CFNC symbols. We provide simulation results for the constellations shown in Fig. 3 in Section VI. We note that although the SLNC constellation shown in Fig. 3 is a regular one, irregular constellations can be obtained by choosing appropriate mapping functions $\mathcal{M}_R(\cdot)$. Hence, the CFNC constellation may be interpreted as a special case of SLNC.

Phase II comprises just one time slot and the signal received at D in sub–carrier k from R_u is given by

$$Y_{RD}[k] = \sqrt{P} H_{R_u D}[k] X'_{R_u}[k] + N_{R_u D}[k], \quad k \in \mathcal{N}_u, \quad (4)$$

where $N_{R_u D}[k]$ is complex AWGN with variance $\sigma^2_{n_{R_u D}}$ and $H_{R_u D}[k]$ is the frequency response of the $R_u \to D$ channel.

C. *Decoding at* D

Due to possible decision errors at the relay, conventional MRC at the destination does not achieve full diversity and optimal ML decoding entails a very high complexity. Therefore, we adopt C–MRC, originally proposed for uncoded DF relaying, and combine it with the conventional BICM decoding metric.

At D, the bit metric for source S_j for the ith bit in the label of symbol $X_j[k]$, $k \in \mathcal{N}_u$, is given by $m^i_k[c_{j,k'}]$

$$= \min_{X_j \in \mathcal{X}^i_{c_{j,k'}}, X_l \in \mathcal{X}, l \neq j} \Bigg\{ \sum_{l=1}^{K} \frac{|Y_{S_l D}[k] - \sqrt{P} H_{S_l D}[k] X_l|^2}{\sigma^2_{n_{S_l D}}} + \lambda_u[k] \frac{|Y_{R_u D}[k] - \sqrt{P} H_{R_u D}[k] X'_{R_u}|^2}{\sigma^2_{n_{R_u D}}} \Bigg\}, \quad (5)$$

where X'_{R_u} is obtained by applying $\mathcal{M}_R(\cdot)$ to the trial source symbols X_l, $\lambda_u[k]$ is a weight factor which accounts for the relative quality of the $S_j \to R_u$, $\forall j$, and $R_u \to D$ links on sub–carrier k. In particular, the weight $\lambda_u[k] \triangleq \gamma_{eq,u}[k]/\gamma_{R_u D}[k]$ accounts for the bottleneck link of relay R_u, where $\gamma_{eq,u}[k] \triangleq \min\{\{\min_j \gamma_{S_j R_u}[k]\}, \gamma_{R_u D}[k]\}$. Here, $\gamma_{S_j R_u}[k] \triangleq P|H_{S_j R_u}[k]|^2/\sigma^2_{n_{S_j R_u}}[k]$ and $\gamma_{R_u D}[k] \triangleq P|H_{R_u D}[k]|^2/\sigma^2_{n_{R_u D}}[k]$. In other words, (5) is the conventional MRC decoding metric if the $R_u \to D$ link is weaker than all $S_j \to R_u$ links (i.e., $\gamma_{S_j R_u}[k] \geq \gamma_{R_u D}[k]$, $\forall j$). If any of the $S_j \to R_u$ links is weaker than the $R_u \to D$ link, the second part of the metric in (5) is attenuated by $\lambda_u[k] = \min_j \gamma_{S_j R_u}[k]/\gamma_{R_u D}[k] < 1$ to limit the impact of possible decision errors at R_u. For the following, we define $\bar{\gamma}_Z \triangleq P\mathcal{E}\{|H_Z[k]|^2\}/\sigma^2_{n_Z} = P\sigma^2_{h_Z}/\sigma^2_{n_Z}$, $Z \in \{S_j D, S_j R_u, R_u D\}$, and note that frequency response $H_Z[k]$, $Z \in \{S_j D, S_j R_u, R_u D\}$, can be expressed as $H_Z[k] = \boldsymbol{w}^H_Z[k]\boldsymbol{h}_Z$, where $\boldsymbol{w}_Z[k]$ is the discrete Fourier transform vector of length L_Z (frequency diversity of link Z) on sub–carrier k, and $\boldsymbol{h}_Z$ is a vector containing the complex Gaussian channel impulse response (CIR) coefficients of link Z.

III. Performance Analysis

In this section, we derive an upper bound on the asymptotic worst–case PEP of source S_j using C–MRC combined with BICM decoding as in (5). Since the channel sub–carrier gains belonging to one error event are correlated in general due to the non–ideal interleaving, cf. Section II-A, it is extremely difficult, if not impossible, to accurately predict the coding gain of BICM–OFDM. Hence, we focus on the analysis of the diversity gain in this paper, as the error performance mainly depends on the diversity gain at high SNR. A similar approach was adopted for point–to–point systems in the original BICM–OFDM paper [21]. We denote the transmitted codeword by $\boldsymbol{c}_j$ and the detected codeword at the destination by $\tilde{\boldsymbol{c}}_j$. For a code with free distance d_f, $\boldsymbol{c}_j$ and $\tilde{\boldsymbol{c}}_j$ differ in d_f positions for the worst–case error event. The subset of sub–carriers containing the erroneous bits is denoted by $\mathcal{K}_j \triangleq \{k_1, k_2, \cdots, k_{d_\mathrm{f}}\}$. The corresponding transmitted and detected symbols for each source are collected in vectors $\boldsymbol{x}_l \triangleq \{X_l[k] | k \in \mathcal{K}_j\}$ and $\tilde{\boldsymbol{x}}_l \triangleq \{\tilde{X}_l[k] | k \in \mathcal{K}_j\}$, respectively, where $l \in \{1, \ldots, K\}$. Note that one or more source symbols may be received in error at R_u, $\forall u$. However, considering multiple errors at the relay in the PEP analysis results in higher order terms which decay faster with increasing SNR compared to the cases of single errors and no errors at the relays. Consequently, we assume that among the transmit symbols $X_l[k]$, $\forall l$, at most one is received in error at R_u. Here, $\hat{X}_{R_u,q}[k]$ denotes the transmit symbol of relay R_u when the signal from source S_q is received in error at R_u, and hence $X'_{R_u}[k]$ can be modeled as $X'_{R_u}[k] \in \{X_{R_u}[k], \hat{X}_{R_u,q}[k]\}$. The relay transmit symbols corresponding to the sub–carriers in $\mathcal{K}_j$ are collected in vector $\boldsymbol{x}'_R \triangleq \{X'_{R_u}[k] | k \in \mathcal{K}_{j,u}\}$, where the sub–carrier set $\mathcal{K}_{j,u}$ is allocated to R_u and $\mathcal{K}_j = \bigcup_{u \in \mathcal{G}} \mathcal{K}_{j,u}$. The sets $\mathcal{K}_{j,u}$, $\forall u$, are disjoint and $\mathcal{K}_{j,u}$ denotes the set of $d_{\mathrm{f},u}$ sub–carriers containing $d_{\mathrm{f},u}$ bits of the error event of S_j. As $\mathcal{K}_j = \bigcup_{u \in \mathcal{G}} \mathcal{K}_{j,u}$ holds, $\sum_u d_{\mathrm{f},u} = d_\mathrm{f}$ is valid as well. We also define $\boldsymbol{x} \triangleq [\boldsymbol{x}^T_1, \ldots, \boldsymbol{x}^T_K, \boldsymbol{x}'^T_R]^T$ and $\tilde{\boldsymbol{x}} \triangleq [\tilde{\boldsymbol{x}}^T_1, \ldots, \tilde{\boldsymbol{x}}^T_K, \tilde{\boldsymbol{x}}^T_R]^T$, where $\tilde{\boldsymbol{x}}_R \triangleq \{\tilde{X}_{R_u}[k] | k \in \mathcal{K}_{j,u}\}$ and $\tilde{\boldsymbol{x}}$ is a vector containing the detected symbols at D. Note that $\bigcap_{u \in \mathcal{G}} \mathcal{K}_{j,u} = \emptyset$. Both $\boldsymbol{x}$ and $\tilde{\boldsymbol{x}}$ contain $K+1$ elements, where each element itself is a $d_\mathrm{f} \times 1$ vector.

A. *Asymptotic PEP*

In this subsection, we analyze the worst–case PEP for $\bar{\gamma}_{S_l D}, \bar{\gamma}_{S_l R}, \bar{\gamma}_{RD} \to \infty$, $\forall l$. For this purpose, we first define vectors $\boldsymbol{h}_{SD} \triangleq [\boldsymbol{h}^T_{S_1 D}, \ldots, \boldsymbol{h}^T_{S_K D}]^T$, $\boldsymbol{h}_{SR} \triangleq [\boldsymbol{h}^T_{S_1 R_1}, \ldots, \boldsymbol{h}^T_{S_K R_1}, \ldots, \boldsymbol{h}^T_{S_1 R_G}, \ldots, \boldsymbol{h}^T_{S_K R_G}]^T$, $\boldsymbol{h}_{RD} \triangleq [\boldsymbol{h}^T_{R_1 D}, \ldots, \boldsymbol{h}^T_{R_G D}]^T$, and $\boldsymbol{h} \triangleq [\boldsymbol{h}^T_{SD}, \boldsymbol{h}^T_{SR}, \boldsymbol{h}^T_{RD}]^T$. Assuming a code with free distance d_f, the worst–case PEP of two codewords $\boldsymbol{c}_j$ and $\tilde{\boldsymbol{c}}_j$ can be bounded as shown in the next page in (6), where $\Theta(i, \mu, q)$

$$= \mathcal{E}_{\boldsymbol{h}} \Big\{ \prod_{p=1}^{i} P_R(\boldsymbol{c}_q, \hat{\boldsymbol{c}}_q | \boldsymbol{h}_{S_q R_{E^\mu_p}}) P_D(\boldsymbol{c}_j, \tilde{\boldsymbol{c}}_j | \boldsymbol{h}, \boldsymbol{x}'_R) \Big\}. \quad (7)$$

Here, i is used to index the relays forwarding erroneous symbols, E^μ (C^μ) is a set containing i ($G-i$) distinct elements from set $\{1, \ldots, G\}$; E^μ_p (C^μ_m) is the pth (mth) element of E^μ (C^μ). E^μ and C^μ are disjoint sets, i.e., $E^\mu \cup C^\mu = \{1, \ldots, G\}$, and $E^\mu \neq E^n$ and $C^\mu \neq C^n$, for any $\mu \neq n$. Furthermore,

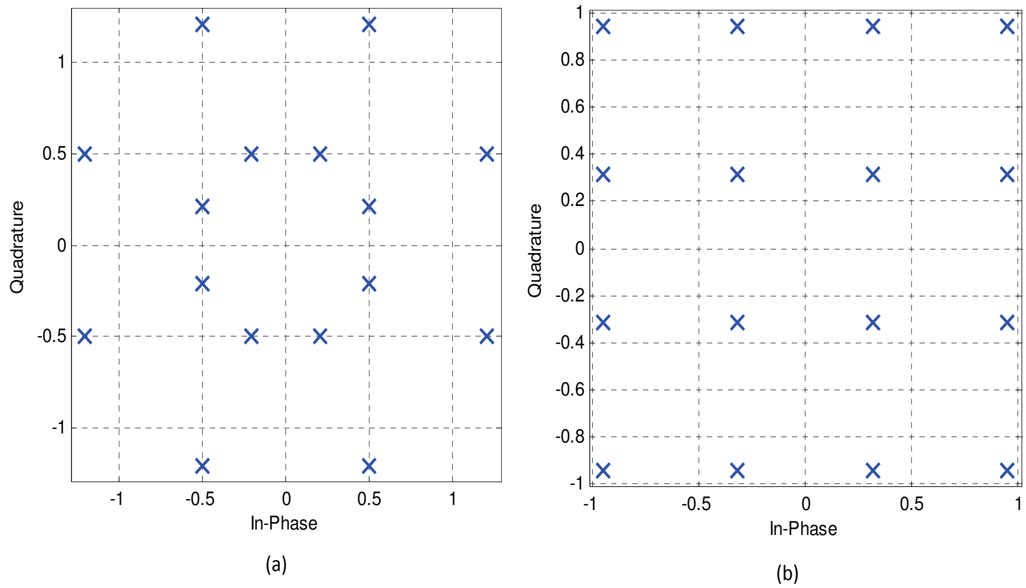

Fig. 3. We consider $K = 2$ sources and the transmit symbols of both sources are drawn from a 4–QAM alphabet. (a) Constellation for CFNC at R_u as proposed in [34], [11] (b) Constellation for SLNC after mapping the set of decoded source symbols to a square 16–QAM constellation. Both constellations have unit average energy.

$$\begin{aligned} P(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j) &\triangleq \mathcal{E}_{\boldsymbol{h}}\Big\{\sum_{i=0}^{G}\sum_{\mu=1}^{\binom{G}{i}}\sum_{q=1}^{K}\Big\{\prod_{p=1}^{i} P_R(\boldsymbol{c}_q,\, \hat{\boldsymbol{c}}_q|\boldsymbol{h}_{S_q R_{E_p^\mu}})\prod_{m=1}^{G-i}[1-P_R(\boldsymbol{c}_q,\, \hat{\boldsymbol{c}}_q|\boldsymbol{h}_{S_q R_{C_p^\mu}})]P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h},\boldsymbol{x}'_R)\Big\}\Big\} \\ &\leq \mathcal{E}_{\boldsymbol{h}}\Big\{\sum_{i=0}^{G}\sum_{\mu=1}^{\binom{G}{i}}\sum_{q=1}^{K}\Big\{\prod_{p=1}^{i} P_R(\boldsymbol{c}_q,\, \hat{\boldsymbol{c}}_q|\boldsymbol{h}_{S_q R_{E_p^\mu}})P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h},\boldsymbol{x}'_R)\Big\}\Big\} \\ &= \mathcal{E}_{\boldsymbol{h}}\Big\{P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h},\boldsymbol{x}_R)\Big\} + \sum_{i=1}^{G}\sum_{\mu=1}^{\binom{G}{i}}\sum_{q=1}^{K}\Theta(i,\mu,q) \end{aligned} \tag{6}$$

$$\begin{aligned} P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h},\boldsymbol{x}_R) = \Pr\Bigg\{&\sum_{k\in\mathcal{K}_j}\sum_{l=1}^{K}\Big(\gamma_{S_l D}[k]|X_l[k]-\tilde{X}_l[k]|^2 + \frac{2\Re\{\sqrt{P}H_{S_l D}[k](X_l[k]-\tilde{X}_l[k])N^*_{S_l D}[k]\}}{\sigma^2_{n_{S_l D}}[k]}\Big) \\ &+\sum_{u=1}^{G}\sum_{k\in\mathcal{K}_{j,u}}\Big(\gamma_{eq,u}[k]|X_{R_u}[k]-\tilde{X}_{R_u}[k]|^2 + \lambda_u[k]\frac{2\Re\{\sqrt{P}H_{R_u D}[k](X_{R_u}[k]-\tilde{X}_{R_u}[k])N^*_{R_u D}[k]\}}{\sigma^2_{n_{R_u D}}[k]}\Big) \leq 0\Bigg\} \end{aligned} \tag{8}$$

$\hat{\boldsymbol{x}}_{R_\mu} \triangleq \{\hat{X}_{R_{E_p^\mu}}[k]|k \in \mathcal{K}_j, E_p^\mu \in E^\mu\}$, $P_R(\boldsymbol{c}_q,\, \hat{\boldsymbol{c}}_q|\boldsymbol{h}_{S_q R_{E_p^\mu}})$ denotes the worst–case PEP at the relay if the signal of source S_q is received in error at $R_{E_p^\mu}$, and $P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h},\boldsymbol{x}'_R)$ denotes the PEP at the destination when the relays transmit $\boldsymbol{x}'_R$. Note that we can decompose $\boldsymbol{x}'_R$ in general as $\boldsymbol{x}'_R \triangleq \{\boldsymbol{x}_{R_c}, \hat{\boldsymbol{x}}_{R_e}\}$, where $\boldsymbol{x}_{R_c} \triangleq \{X_{R_u}[k]|k \in \mathcal{K}_{j,u},\ u \in C^\mu\}$ and $\hat{\boldsymbol{x}}_{R_e} \triangleq \{\hat{X}_{R_u}[k]|k \in \mathcal{K}_{j,u},\ u \in E^\mu\}$. The first term in (6), $\mathcal{E}_{\boldsymbol{h}}\Big\{P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h},\boldsymbol{x}_R)\Big\}$, corresponds to $i = 0$, i.e., when the decisions at all relays are correct, $\boldsymbol{x}_{R_c} = \boldsymbol{x}_R$ and $C^\mu = \{1,\ldots,G\}$. To derive the PEP and diversity gain, the following remark is useful which sheds some light on the asymptotic behavior of $P(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j)$ in (6).

Remark 1: We consider the case which yields the lowest possible diversity gain at high SNR, i.e., $\boldsymbol{x}$ and $\tilde{\boldsymbol{x}}$ differ in the minimum possible number of vector elements. From the theory of coding over fading channels [35], we know that the lowest achievable diversity gain is governed by the minimum Hamming distance between the transmitted and received signal sequences. As $\boldsymbol{c}_j$ and $\tilde{\boldsymbol{c}}_j$ differ in d_f bits which are mapped to $\boldsymbol{x}_j$ and $\tilde{\boldsymbol{x}}_j$, respectively, we have $\boldsymbol{x}_j \neq \tilde{\boldsymbol{x}}_j$. By inspection, we observe that $\boldsymbol{x}_j \neq \tilde{\boldsymbol{x}}_j$ leads to $\boldsymbol{x}'_R \neq \tilde{\boldsymbol{x}}_R$ if $\boldsymbol{x}_l = \tilde{\boldsymbol{x}}_l$, $l \in \{1,\ldots,K\}, l \neq j$. In that case, $\boldsymbol{x}$ and $\tilde{\boldsymbol{x}}$ differ in two vector elements. We study this case below in our diversity analysis. Note that the case where $\boldsymbol{x}_l \neq \tilde{\boldsymbol{x}}_l$ for more than one value of l causes $\boldsymbol{x}$ and $\tilde{\boldsymbol{x}}$ to differ in more than two vector elements, and thus will not be dominant at high SNR.

In the following, we calculate $\mathcal{E}_{\boldsymbol{h}}\Big\{P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h},\boldsymbol{x}_R)\Big\}$ and $\Theta(i,\mu,q)$.

No error at relay: When there is no decoding error at the relays, based on (5), the worst–case PEP of source S_j at the destination conditioned on $\boldsymbol{h}$ can be expressed as shown above in (8), where $\gamma_{S_l D}[k] \triangleq P|H_{S_l D}[k]|^2/\sigma^2_{n_{S_j D}}[k]$, $\forall l$, and we exploited the definition of $\lambda_u[k] \triangleq \gamma_{eq,u}[k]/\gamma_{R_u D}[k]$. Note that the BICM processing at the sources, i.e., the encoding and the interleaving (cf. Phase I in Section II), ensures that the d_f bits belonging to one error event are mapped onto d_f distinct sub–carriers. For the worst–case PEP, we only need to consider the bit metrics for a candidate set of $d_{\rm f}$ sub–carriers containing the $d_{\rm f}$ bits of the error event, as for the other bit positions, the codewords $\boldsymbol{c}_j$ and $\tilde{\boldsymbol{c}}_j$ are identical. Consequently, in (8), only the sub–carriers belonging to $\mathcal{K}_j$ are considered, where $\mathcal{K}_j$ denotes the set of $d_{\rm f}$ sub–carriers, see the first paragraph of this section.

For convenience, we define $d_l^2[k] \triangleq |X_l[k] - \tilde{X}_l[k]|^2$ and $d_{R_u}^2[k] \triangleq |X_{R_u}[k] - \tilde{X}_{R_u}[k]|^2$, $\forall l, u$. Using the Chernoff bound and exploiting the fact that $\gamma_{eq,u}[k] \le \gamma_{R_u D}[k]$, we obtain from (8)

$$P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \boldsymbol{x}_R) \le \frac{1}{2}\exp\Big(-\frac{1}{4}\big(\sum_{k\in\mathcal{K}_j}\sum_{l=1}^{K}\gamma_{S_l D}[k] d_l^2[k] + \sum_{u=1}^{G}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k] d_{R_u}^2[k]\big)\Big). \quad (9)$$

Assuming $d_l[k] = 0$ (cf. *Remark* 1), $\forall l,\ l \neq j$, we have $d_{R_u}^2[k] \ge d_{\min}^2/J$ ($d_{\min}$ minimum Euclidean distance of signal constellation $\mathcal{X}$), where $J > 1$ is a constant. For example if $K = 2$, both sources transmit 4-QAM symbols, and the network coded symbols are obtained by mapping the detected 4–QAM symbols to an 16-QAM constellation with unit average energy per symbol, we have $J = \sqrt{5}$. For other square constellations, J can be easily computed as $J \triangleq d_{\min}/d_{\min,R}$, where $d_{\min,R}$ denotes the minimum Euclidean distance of the constellation of size M^K used for the network coded symbols at the relay. From (9), we obtain

$$P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \boldsymbol{x}_R) \le \frac{1}{2}\exp\Big(-\xi\big(\gamma_{S_j D} + \frac{1}{J}\sum_{u=1}^{G}\gamma_{eq,u}\big)\Big), \quad (10)$$

where $\gamma_{S_j D} \triangleq \sum_{k\in\mathcal{K}_j}\gamma_{S_j D}[k]$, $\gamma_{eq,u} \triangleq \sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]$, $\xi \triangleq d_{\min}^2/4$, and we have replaced $d_j^2[k]$ and $d_{R_u}^2[k]$ by $d_{\min}^2$ and $d_{\min}^2/J$, respectively.

Now, the unconditional PEP is given by

$$\mathcal{E}_{\boldsymbol{h}}\Big\{P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \boldsymbol{x}_R)\Big\} \le \frac{1}{2}\mathcal{E}_{\boldsymbol{h}_{S_j D}}\Big\{\exp(-\xi\gamma_{S_j D})\Big\} \prod_{u=1}^{G}\mathcal{E}_{\boldsymbol{h}_{SR_u},\boldsymbol{h}_{R_u D}}\Big\{\exp(-\omega\gamma_{eq,u})\Big\}, \quad (11)$$

where $\omega \triangleq \xi/J$. The following proposition upper bounds $\mathcal{E}_{\boldsymbol{h}}\Big\{P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \boldsymbol{x}_R)\Big\}$.

Proposition 1: The expectation $\mathcal{E}_{\boldsymbol{h}}\Big\{P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \boldsymbol{x}_R)\Big\} \triangleq W_1$ can be upper bounded as

$$W_1 \le \frac{1}{2^{K d_{\rm f}+1}}\frac{1}{(\xi\bar{\gamma}_{S_j D})^{r_{S_j D}}\prod_{m=1}^{r_{S_j D}}\lambda_m(\boldsymbol{A}_{S_j D})} \times \prod_{u=1}^{G}\Bigg[\sum_{l=1}^{K}\frac{1}{(\omega\bar{\gamma}_{S_l R_u})^{r_{S_l R_u}}\prod_{m=1}^{r_{S_l R_u}}\lambda_m(\boldsymbol{A}_{S_l R_u})} + \frac{1}{(\omega\bar{\gamma}_{R_u D})^{r_{R_u D}}\prod_{m=1}^{r_{R_u D}}\lambda_m(\boldsymbol{A}_{R_u D})}\Bigg], \quad (12)$$

where $\boldsymbol{A}_{S_j D} = \sum_{k\in\mathcal{K}_j}\boldsymbol{w}_{S_j D}[k]\boldsymbol{w}_{S_j D}^H[k]$, and $\mathrm{rank}\{\boldsymbol{A}_{S_j D}\} = \min\{d_{\rm f}, L_{S_j D}\} \triangleq r_{S_j D}$. Similarly, $\boldsymbol{A}_Y = \sum_{k\in\mathcal{K}_{j,u}}\boldsymbol{w}_Y[k]\boldsymbol{w}_Y^H[k]$ and $\mathrm{rank}\{\boldsymbol{A}_Y\} = \min\{d_{{\rm f},u}, L_Y\} \triangleq r_Y$, $Y \in \{S_p R_u, R_u D\}$.

Proof: Please refer to Appendix A. ■

Next, we consider the case when at least one relay has made an error in decoding the codewords.

Error at relay: If the signal from source S_q is received in error at R_u, $u \in E^\mu$, then the conditional PEP at R_u can be upper bounded as

$$P_R(\boldsymbol{c}_q,\, \hat{\boldsymbol{c}}_q|\boldsymbol{h}_{S_q R_u}) \le \frac{1}{2}\exp\Big(-\frac{1}{4}\sum_{k\in\mathcal{K}_j}\big(\gamma_{S_q R_u}[k]\hat{d}_q^2[k]\big)\Big), \quad (13)$$

where $\hat{d}_q^2[k] \triangleq |X_q[k] - \hat{X}_q[k]|^2$. By replacing $d_q^2[k]$ by $d_{\min}^2$ in (13), we obtain for the conditional PEP at R_u

$$P_R(\boldsymbol{c}_q,\, \hat{\boldsymbol{c}}_q|\boldsymbol{h}_{S_q R_u}) \le \frac{1}{2}\exp\left(-\xi\gamma_{S_q R_u}\right), \quad (14)$$

where $\gamma_{S_q R_u} \triangleq \sum_{k\in\mathcal{K}_j}\gamma_{S_q R_u}[k]$. Furthermore, if there is a decoding error at R_u, $u \in E^\mu$, based on (5) the worst–case PEP for source S_j at D conditioned on $\boldsymbol{h}$, $P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \hat{\boldsymbol{x}}_{R_\mu})$, can be expressed as shown at the top of the next page in (15).

Employing the Chernoff bound and the fact that $\gamma_{eq,u}[k] \le \gamma_{R_u D}[k]$, we obtain $P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \hat{\boldsymbol{x}}_{R_\mu})$ as shown in (16). Now, we use $\hat{d}_{R_u,q}^2[k] = |\hat{X}_{R_u,q}[k] - \tilde{X}_{R_u}[k]|^2 - |\hat{X}_{R_u,q}[k] - X_{R_u}[k]|^2 \ge -\beta d_{\min}^2/J$ to obtain an upper bound, where $\beta \triangleq \alpha^2 - 1$, and $\alpha \triangleq d_{\max,R}/d_{\min,R} > 1$ denotes the ratio of the maximum and minimum Euclidean distances of the signal constellation (of size M^K) used for the network coded symbols at the relays. Assuming $d_l[k] = 0$, $\forall l,\ l \neq j$, we arrive at an expression for $P_D(\boldsymbol{c}_j,\, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \hat{\boldsymbol{x}}_{R_\mu})$ as shown in (17). We define $\gamma_{eq,u} \triangleq \sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]$, $\gamma_R^E \triangleq \sum_{u\in E^\mu}\gamma_{eq,u}$, and $\gamma_R^C \triangleq \sum_{u\in C^\mu}\gamma_{eq,u}$, and note that $\gamma_R \triangleq \sum_{u\in\{1,\cdots,G\}}\gamma_{eq,u} = \gamma_R^E + \gamma_R^C$ holds. Now, we obtain

$$\Theta(i,\mu,q) \le \frac{1}{4}\mathcal{E}_{\boldsymbol{h}}\left\{\exp\left(-\xi\sum_{p=1}^{i}\gamma_{S_q R_{E_p^\mu}}\right) \times \exp\left(-\xi\frac{(\gamma_{S_j D} + \frac{1}{J}\sum_{u\in C^\mu}\gamma_{R_u} - \frac{\beta}{J}\sum_{u\in E^\mu}\gamma_{R_u})^2}{\gamma_{S_j D} + \frac{1}{J}\sum_{u\in\mathcal{R}}\gamma_{R_u}}\right)\right\}. \quad (18)$$

The following proposition upper bounds $\Theta(i,\mu,q)$.

Proposition 2: The function $\Theta(i,\mu,q) \triangleq W_2$ can be upper bounded as

$$W_2 \le \frac{1}{2^{K d_{\rm f}+2}}\frac{\vartheta_{S_j D}}{(\xi\bar{\gamma}_{S_j D})^{r_{S_j D}}\prod_{m=1}^{r_{S_j D}}\lambda_m(\boldsymbol{A}_{S_j D})} \times \prod_{u=1}^{G}\Bigg[\sum_{l=1}^{K}\frac{\vartheta_{S_l R_u}}{(\omega\bar{\gamma}_{S_l R_u})^{r_{S_l R_u}}\prod_{m=1}^{r_{S_l R_u}}\lambda_m(\boldsymbol{A}_{S_l R_u})} + \frac{\vartheta_{R_u D}}{(\omega\bar{\gamma}_{R_u D})^{r_{R_u D}}\prod_{m=1}^{r_{R_u D}}\lambda_m(\boldsymbol{A}_{R_u D})}\Bigg], \quad (19)$$

where $\vartheta_Z \triangleq (1 + 1/\varphi^{r_Z})$, $Z \in \{S_j D, S_l R_u, R_u D\}$, $\forall j,l,u$, and φ is a modulation dependent parameter (cf. (47) in Appendix C) and independent of $\bar{\gamma}_Z$, $\forall Z$.

Proof: Please refer to Appendix C. ■

Next, we investigate the achievable diversity gain for the considered system based on the developed asymptotic upper bounds in (12) and (19). Note that using (12) and (19), we can

$$P_D(\boldsymbol{c}_j,\,\tilde{\boldsymbol{c}}_j|\boldsymbol{h},\hat{\boldsymbol{x}}_{R_\mu}) = \Pr\Bigg\{\sum_{k\in\mathcal{K}_j}\sum_{l=1}^{K}\gamma_{S_lD}[k]|X_l[k]-\tilde{X}_l[k]|^2+\sum_{u\in E^\mu}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k](|\hat{X}_{R_u,q}[k]-\tilde{X}_{R_u}[k]|^2-|\hat{X}_{R_u,q}[k]-X_{R_u}[k]|^2)$$
$$+\sum_{u\in C^\mu}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]|X_{R_u}[k]-\tilde{X}_{R_u}[k]|^2+\sum_{k\in\mathcal{K}_j}\sum_{l=1}^{K}\frac{2\Re\{\sqrt{P}H_{S_lD}[k](X_l[k]-\tilde{X}_l[k])N^*_{S_lD}[k]\}}{\sigma^2_{n_{S_lD}}}$$
$$+\sum_{u\in\mathcal{R}}\sum_{k\in\mathcal{K}_{j,u}}\lambda_u[k]\frac{2\Re\{\sqrt{P}H_{R_uD}[k](X_{R_u}[k]-\tilde{X}_{R_u}[k])N^*_{R_uD}[k]\}}{\sigma^2_{n_{R_uD}}}\leq 0\Bigg\} \quad (15)$$

$$P_D(\boldsymbol{c}_j,\,\tilde{\boldsymbol{c}}_j|\boldsymbol{h},\hat{\boldsymbol{x}}_{R_\mu})$$
$$\leq\frac{1}{2}\exp\left(-\frac{\big(\sum_{k\in\mathcal{K}_j}\sum_{l=1}^{K}\gamma_{S_lD}[k]d_l^2[k]+\sum_{u\in E^\mu}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]\hat{d}^2_{R_u}[k]+\sum_{u\in C^\mu}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]d^2_{R_u}[k]\big)^2}{4\big(\sum_{k\in\mathcal{K}_j}\sum_{l=1}^{K}\gamma_{S_lD}[k]d_l^2[k]+\sum_{u\in\mathcal{R}}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]d^2_{R_u}[k]\big)}\right) \quad (16)$$

$$P_D(\boldsymbol{c}_j,\,\tilde{\boldsymbol{c}}_j|\boldsymbol{h},\hat{\boldsymbol{x}}_{R_\mu})\leq\frac{1}{2}\exp\left(-\xi\frac{\big(\sum_{k\in\mathcal{K}_j}\gamma_{S_jD}[k]+\frac{1}{J}\sum_{u\in C^\mu}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]-\frac{\beta}{J}\sum_{u\in E^\mu}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]\big)^2}{\big(\sum_{k\in\mathcal{K}_j}\gamma_{S_jD}[k]+\frac{1}{J}\sum_{u\in\mathcal{R}}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]\big)}\right)$$
$$=\frac{1}{2}\exp\left(-\xi\frac{(\gamma_{S_jD}+\frac{1}{J}\sum_{u\in C^\mu}\gamma_{R_u}-\frac{\beta}{J}\sum_{u\in E^\mu}\gamma_{R_u})^2}{\gamma_{S_j}+\frac{1}{J}\sum_{u\in\mathcal{R}}\gamma_{R_u}}\right) \quad (17)$$

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also obtain an asymptotic upper bound on $P(\boldsymbol{c}_j,\,\tilde{\boldsymbol{c}}_j)$ in (6).

B. Diversity Gain

To get more insight into the system performance, we investigate the diversity gain. Let $\overline{\gamma}_{S_lD}=e_l\overline{\gamma}_t$, $\overline{\gamma}_{S_lR_u}=f_{l,u}\overline{\gamma}_t$, and $\overline{\gamma}_{R_uD}=g_u\overline{\gamma}_t$, $\forall l,u$, where e_l, $f_{l,u}$, and g_u are arbitrary positive constants. We define the diversity gain as the negative slope of the PEP as a function of $\overline{\gamma}_t$ on a double–logarithmic scale. Based on (12) and (19), the diversity gain for source S_j is obtained as G_d^j

$$=\min\{d_\mathrm{f},L_{S_jD}\}+\sum_{u=1}^{G}\min\{d_{\mathrm{f},u},\{\min_l L_{S_lR_u}\},L_{R_uD}\}. \quad (20)$$

Eq. (20) reveals that for source S_j, the maximum diversity gain of SLNC BICM–OFDM is limited by the free distance of the code, and the frequency diversity offered by the $S_j\to D$ link and all other relay links. We can extract the full frequency diversity offered by the channel by employing a code with sufficiently large free distance d_f. For channels that are rich in frequency diversity, i.e., $L_{S_jD}\geq d_\mathrm{f}$ and $\min\{\min_l\{L_{S_lR_u}\},L_{R_uD}\}\geq d_{\mathrm{f},u}$, we obtain $G_d^j=2d_\mathrm{f}$, as $\sum_{u=1}^{G}d_{\mathrm{f},u}=d_\mathrm{f}$, which is identical to the maximum diversity gain achievable in cooperative AF [36] and DF [30] BICM–OFDM systems. In a single relay setup, if the $S_j\to R$ and $R\to D$ channels are not rich in diversity and $\min\{L_{S_jR},\,L_{RD}\}<d_\mathrm{f}$, we can improve the diversity gain by adding a second relay. In doing so, we decrease $d_{\mathrm{f},1}$ and we may achieve the maximum diversity gain provided that the new $d_{\mathrm{f},1}$ and $d_{\mathrm{f},2}=d_\mathrm{f}-d_{\mathrm{f},1}$ do not exceed $\min\{L_{S_jR_1},\,L_{R_1D}\}$ and $\min\{L_{S_jR_2},\,L_{R_2D}\}$, respectively. Roughly speaking, by adding more relays we decrease $d_{\mathrm{f},u}$, $\forall u$, but increase the overall diversity by making up for the missing frequency diversity by adding more spatial diversity.

Remark 2: Adding more than d_f relays can increase the coding gain, but the diversity gain is still limited to $G_d^j\leq d_\mathrm{f}$. However, if more relays are available, we can perform sub–carrier based relay selection to exploit additional selection diversity. The relay selection problem will be discussed in Section IV.

Remark 3: The fact that the diversity gain in (20) is a function of d_f shows how critical BICM is to the system performance. In particular, if BICM is not used, i.e., coding and interleaving are not applied before the symbols are mapped to OFDM sub–carriers, $d_\mathrm{f}=1$ holds and the frequency diversity offered by the channel cannot be extracted.

Remark 4: The diversity gain for source S_j of a BICM–OFDM system employing GF(2) network coding (i.e., BLNC), K sources, and $G=1$ relay, is given by [31]

$$G_d^j=r_{S_jD}+\min\{\min_l\{r_{S_lR}\},\min_{p,\;p\neq j}\{r_{S_pD}\},r_{RD}\} \quad (21)$$
$$=\min\{d_\mathrm{f},L_{S_jD}\}+\min\{d_\mathrm{f},\min_l\{L_{S_lR}\},\min_{p,\;p\neq j}\{L_{S_pD}\},L_{RD}\}.$$

Note that for both BLNC and SLNC, different sources may enjoy different diversity gains depending on the quality of the channel of the different links. However, from (21), it is interesting to observe that for BLNC, the overall diversity gain of source S_j depends on the frequency diversity of the direct links of the other sources, which is different from SLNC.

Why do SLNC and BLNC result in different G_d^j?: The reason is that for SLNC, the symbol transmitted by the relay, $X'_R[k]$, is unique for different sets of decoded source symbols, whereas for BLNC, $X'_R[k]$ is not unique (due to the binary XOR operation at the bit level) and can be identical to $X_R[k]$ for erroneous transmission as well. In particular, for BLNC, $\boldsymbol{x}_l\neq\tilde{\boldsymbol{x}}_l$ (recall that $\boldsymbol{x}_l$ ($\tilde{\boldsymbol{x}}_l$) denote the transmitted (detected) symbols of S_l) for $k\in\mathcal{K}_j$, $l\in\{j,p\}$, $p\subseteq\{1,\ldots,K\}-j$, i.e., $\boldsymbol{x}$ and $\tilde{\boldsymbol{x}}$ differ in two vector elements, can cause $\boldsymbol{x}'_R=\tilde{\boldsymbol{x}}_R$, which is not the case in SLNC. Hence, the effective diversity gain of each source depends on the $S\to D$ links of the other sources for BLNC which can potentially limit the diversity gain if other $S\to D$ links are not rich in frequency diversity.

Moreover, we expect to observe a higher coding gain for SLNC because symbol combining is performed over the complex field and GF(2) rather than GF(2) alone as in BLNC.

IV. Relay Selection

In the analysis presented in Section III, uniform sub–carrier allocation among the relays was assumed. Hence, from (20), we observe that the diversity gain is limited by $d_{\mathrm{f},u}$, i.e., the number of erroneous bits transmitted by the relay R_u corresponding to the error event. However, in practice, the frequency diversity of the links associated with R_u may be larger than $d_{\mathrm{f},u}$. In this section, we consider two sub–carrier based relay selection schemes to further improve the diversity gain performance. In particular, we propose a) bulk selection and b) per–subcarrier selection based on the instantaneous PEP and derive the corresponding diversity gains[2]. As the high SNR performance is dominated by the worst–case error event, we only consider the set of sub–carriers, which contains the bits of the worst–case error event, for developing the selection rule and performance analysis.

As we are primarily interested in the diversity gain of the schemes, we use a general form of the instantaneous PEP in view of the analysis in Section III. Based on (10), (46) and (49) in Appendix C, the instantaneous PEP for both error free and erroneous detection at the relay can be expressed in general form as

$$P(\boldsymbol{c}_j, \widehat{\boldsymbol{c}}_j|\boldsymbol{h}) \leq \tau_0 \exp\Big(-\xi(\tau_S\gamma_{S_jD} + \tau_R\sum_{u=1}^{G}\gamma_{eq,u})\Big), \quad (22)$$

where τ_U, $U \in \{0, S, R\}$, are positive constants that absorb the scaling factors which do not affect the diversity gain. Hence, to derive the diversity gain for relay selection, we resort to (22) and do not analyze the error free and erroneous detection at the relay separately. Note that the worst–case sub–carrier set definitions for the two proposed selection schemes are different, as will be discussed below. We assume that the destination acquires the channel information and informs the selection outcome to the relays via low rate feedback link.

A. Bulk Selection

In this scheme, one relay is selected to transmit over all sub–carriers. For relay selection, we consider the link qualities of the relays for their corresponding worst–case sub–carrier sets. We explore all possible sets of d_{f} sub–carriers and determine the worst–case set for relay R_u as $\mathcal{K}^*_{j,u} = \arg\min_{\mathcal{K}_{j,u}\in\mathcal{K}}\sum_{k\in\mathcal{K}_{j,u}}\gamma_{eq,u}[k]$, where $\mathcal{K}$ is the ensemble set of all possible sets of d_{f} sub–carriers. Now, we define $\gamma_{\mathrm{worst},u} \triangleq \sum_{k\in\mathcal{K}^*_{j,u}}\gamma_{eq,u}[k]$, which is the sum of the instantaneous equivalent link SNRs of relay R_u corresponding to the worst–case sub–carrier set $\mathcal{K}^*_{j,u}$. Note that $\gamma_{eq,u}[k] = \min\{\min_j\gamma_{S_jR_u}[k], \gamma_{R_uD}[k]\}$. Here, $|\mathcal{K}_{j,u}| = d_{\mathrm{f}}$, as one relay is selected to transmit over the whole set of sub–carriers. Note that the worst–case sets for different relays are in general different as the relay links fade independently. As we have G relays, R_u, $u \in \{1, \ldots, G\}$, the best relay is selected as

$$u^* = \arg\max_u \gamma_{\mathrm{worst},u}. \quad (23)$$

To obtain the corresponding unconditional PEP, we have to calculate

$$P_{R_{u^*}}(\boldsymbol{c}_j, \widehat{\boldsymbol{c}}_j) \leq \mathcal{E}\{\exp(-\xi\tau_R\gamma_{\mathrm{worst},u^*})\}. \quad (24)$$

The cumulative distribution function (CDF) of $\gamma_{\mathrm{worst},u^*}$ is given by $\Pr(\gamma_{\mathrm{worst},u^*} < x)$

$$= \Pr\Big(\gamma_{\mathrm{worst},1} < x, \ldots, \gamma_{\mathrm{worst},G} < x\Big) = \prod_{u=1}^{G}\Pr(\gamma_{\mathrm{worst},u} < x). \quad (25)$$

Following (42) in Appendix A, $\Pr(\gamma_{\mathrm{worst},u} < x)$ can be asymptotically approximated as $\Pr(\gamma_{\mathrm{worst},u} < x)$

$$\doteq \frac{1}{2^{Kd_{\mathrm{f}}}}\Big(\sum_{j=1}^{K}\frac{1}{\Gamma(r_{S_jR_u}+1)\bar{\gamma}_{S_jR_u}^{r_{S_jR_u}}\prod_{l=1}^{r_{S_jR_u}}\lambda_l(\boldsymbol{A}_{S_jR_u})}x^{r_{S_jR_u}} + \frac{1}{\Gamma(r_{R_uD}+1)\bar{\gamma}_{R_uD}^{r_{R_uD}}\prod_{l=1}^{r_{R_uD}}\lambda_l(\boldsymbol{A}_{R_uD})}x^{r_{R_uD}}\Big). \quad (26)$$

Note that $d_{\mathrm{f},u}$ in (42) is replaced by d_{f} in (26) as only one relay is selected and forwards all the d_{f} erroneous bits. Consequently, we have $r_Z \triangleq \min\{d_{\mathrm{f}}, L_Z\}$, $Z \in \{S_jR_u, R_uD\}$. Hence, using (25), we obtain $\Pr(\gamma_{\mathrm{worst},u^*} < x)$

$$\doteq \frac{1}{2^{Kd_{\mathrm{f}}}}\prod_{u=1}^{G}\Bigg(\sum_{j=1}^{K}\frac{x^{r_{S_jR_u}}}{\Gamma(r_{S_jR_u}+1)\bar{\gamma}_{S_jR_u}^{r_{S_jR_u}}\prod_{l=1}^{r_{S_jR_u}}\lambda_l(\boldsymbol{A}_{S_jR_u})} + \frac{x^{r_{R_uD}}}{\Gamma(r_{R_uD}+1)\bar{\gamma}_{R_uD}^{r_{R_uD}}\prod_{l=1}^{r_{R_uD}}\lambda_l(\boldsymbol{A}_{R_uD})}\Bigg). \quad (27)$$

Now, we calculate the unconditional PEP as $P_{R_{u^*}}(\boldsymbol{c}_j, \widehat{\boldsymbol{c}}_j)$

$$\begin{aligned}&\leq \frac{1}{2^{Kd_{\mathrm{f}}}}\int_0^{\infty}\exp(-\xi\tau_R x)\frac{d}{dx}\Big[\Pr(\gamma_{\mathrm{worst},u^*} < x)\Big]dx\\&\leq \prod_{u=1}^{G}\Bigg(\sum_{j=1}^{K}\frac{\kappa_{S_jR_u}}{(\xi\tau_R\bar{\gamma}_{S_jR_u})^{r_{S_jR_u}}\prod_{l=1}^{r_{S_jR_u}}\lambda_l(\boldsymbol{A}_{S_jR_u})}\\&+\frac{\kappa_{R_uD}}{(\xi\tau_R\bar{\gamma}_{R_uD})^{r_{R_uD}}\prod_{l=1}^{r_{R_uD}}\lambda_l(\boldsymbol{A}_{R_uD})}\Bigg)+\text{higher order terms},\end{aligned} \quad (28)$$

where (28) is obtained by integration by parts and κ_Z, $\forall Z$, are positive constants which are independent of the average SNR. Hence, from (28), we get the diversity gain as

$$\begin{aligned}G_d^R &= \sum_{u=1}^{G}\min\{\min_j\ r_{S_jR_u}, r_{R_uD}\}\\&= \sum_{u=1}^{G}\min\{d_{\mathrm{f}}, \min_j\ L_{S_jR_u}, L_{R_uD}\} \leq Gd_{\mathrm{f}}.\end{aligned} \quad (29)$$

Now, by including direct transmission via the $S_j \to D$ link, the overall diversity gain is $G_d^j = \min\{d_{\mathrm{f}}, L_{S_jD}\} + G_d^R$.

[2]We note that in the considered system, the selected relays do not transmit over the same carrier. Each relay transmits SLNC symbols over a subset of the N data sub–carriers, and the sub–carrier sets assigned to the selected relays are disjoint, cf. Section II-B. On the other hand, if the CSI of the $R_u \to D$, $\forall u$ links are available to the relays, they can use transmit–side beamforming and all selected relays can jointly transmit the whole packet of SLNC symbols, mapped to the N data sub–carriers. However, for beamforming, the CSI of all $R_u \to D$ links has to be known at each relay [37] and perfect synchronisation is necessary, which is difficult to achieve in practice.

B. Per–Subcarrier Selection

In this scheme, we select the best relay on each sub–carrier as

$$u^*[k] = \arg\max_{u[k]\in\mathcal{R}} \gamma_{eq,u}[k], \quad \forall k, \tag{30}$$

i.e., relay R_{u^*} yields the best equivalent SNR on sub–carrier k. Now, we select the worst–case sub–carrier set for source S_j for the relaying phase as $\mathcal{K}^*_{j,R} = \arg\min_{\mathcal{K}_{j,R}\in\mathcal{K}} \sum_{k\in\mathcal{K}_{j,R}} \gamma_{eq,u^*}[k]$. Hence, the conditional PEP for the relaying phase is given by

$$P_R(\boldsymbol{c}_j, \widehat{\boldsymbol{c}}_j|\boldsymbol{h}_R) \le \exp\Big(-\xi\tau_R \sum_{k\in\mathcal{K}^*_{j,R}} \gamma_{eq,u^*}[k]\Big), \tag{31}$$

where $\boldsymbol{h}_R \in \{\boldsymbol{h}_{S_lR_u}, \boldsymbol{h}_{R_uD}\}$, $\forall l, u$, and $\gamma_{eq,u^*}[k] = \max_u \gamma_{eq,u}[k]$. Now, we need to derive the CDF of $\Upsilon \triangleq \sum_{k\in\mathcal{K}^*_{j,R}} \gamma_{eq,u^*}[k]$. We obtain

$$\Pr(\Upsilon < x) = \sum_{i\in\mathcal{D}} \Pr(\Theta_i < x), \quad \Theta_i \triangleq \sum_{u=1}^{G} \gamma^*_{eq,u,i}, \tag{32}$$

where $\mathcal{D}$ is the set of all possible distributions of sub–carriers among the relays for set $\mathcal{K}^*_{j,R}$, $\gamma^*_{eq,u,i} = \sum_{k\in\mathcal{K}^i_{j,u}} \gamma_{eq,u}[k]$ denotes the sum of equivalent SNRs for relay R_u if it is chosen to transmit over a subset of the worst–case sub–carrier set $\mathcal{K}^i_{j,u} \in \mathcal{K}^*_{j,R}$, $\forall i, u$. Note that $\gamma_{eq,u}[k] \ge \gamma_{eq,m}[k]$, $m \ne u$, $m \in \mathcal{R}$, $k \in \mathcal{K}^i_{j,u}$. The sets $\mathcal{K}^i_{j,u}$, $\forall u$, are disjoint and $|\mathcal{K}^i_{j,u}| = d^i_{\mathrm{f},u}$ holds. Here, $d^i_{\mathrm{f},u}$ denotes the number of worst–case error event bits contained in subset $\mathcal{K}^i_{j,u}$ and $\sum_u d^i_{\mathrm{f},u} = d_{\mathrm{f}}$, $\forall i$. Now, the CDF of $\gamma^*_{eq,u,i}$ is given by $\Pr(\gamma^*_{eq,u,i} < y)$

$$= \Pr(\gamma^u_{eq,1,i} < y, \ldots, \gamma^u_{eq,G,i} < y) = \prod_{m=1}^{G} \Pr(\gamma^u_{eq,m,i} < y), \tag{33}$$

where $\gamma^u_{eq,m,i} \triangleq \sum_{k\in\mathcal{K}^i_{j,u}} \gamma_{eq,m}[k]$ and the second equality in (33) holds due to the independence of $\gamma^u_{eq,m,i}$, $\forall m$. The probability density function (PDF) of $\gamma^u_{eq,m,i}$ decays as $\min\{d^i_{\mathrm{f},u}, \min_j L_{S_jR_m}, L_{R_mD}\}$ (cf. (42) in Appendix A) and, consequently, the PDF of $\gamma^*_{eq,u,i}$ decays as $\sum_{m=1}^{G} \min\{d^i_{\mathrm{f},u}, \min_j L_{S_jR_m}, L_{R_mD}\}$. The PDF of Θ_i in (32) can be obtained as

$$f_{\Theta_i}(x) = f_{\gamma^*_{eq,1,i}}(x) \circledast f_{\gamma^*_{eq,2,i}}(x) \circledast \ \ldots\ \circledast f_{\gamma^*_{eq,G,i}}(x), \tag{34}$$

where $\circledast$ denotes convolution. Following *Lemma* 1 in [38], it can be shown that the asymptotic PDF (i.e., when $\Theta_i \to 0$, cf. [39] for a detailed discussion on obtaining asymptotic PDF) $f_{\Theta_i}(x)$ decays with rate $\sum_{u=1}^{G}\sum_{m=1}^{G} \min\{d^i_{\mathrm{f},u}, \min_j L_{S_jR_m}, L_{R_mD}\}$. Now $\sum_{m=1}^{G} \min\{d^i_{\mathrm{f},u}, \min_j L_{S_jR_m}, L_{R_mD}\} \le Gd^i_{\mathrm{f},u}$, and $\sum_{u=1}^{G} Gd^i_{\mathrm{f},u} = Gd_{\mathrm{f}}$ holds independent of i and each term $\Pr(\Theta_i < x)$ inside the sum in (32) decays at the same rate. In view of (27)–(29), we can easily obtain that the diversity gain for the relaying phase is given by the decaying rate of $f_{\Theta_i}(x)$, i.e.,

$$G^R_d = \sum_{u=1}^{G}\sum_{m=1}^{G} \min\{d^i_{\mathrm{f},u}, \min_j L_{S_jR_m}, L_{R_mD}\} \le Gd_{\mathrm{f}}. \tag{35}$$

Hence, we observe that the maximum achievable diversity gain for both relay selection schemes is Gd_{f} and hence, identical.

V. Joint Source Subset and Relay Selection

In this section, we investigate the joint selection of a source subset and a relay for the considered K source and G relay SLNC BICM–OFDM network. In practice, the number of active sources K may be larger than the number of available orthogonal relay channels T. In that case, we select T out of K sources to transmit to the relay and destination over T time slots. Also, we may need to select one best relay to cooperate with the source subset. We note that joint source subset and relay selection incurs a larger feedback overhead compared to relay selection only because the selection decision has to be fed back not only to the relays but also to the sources. As we are primarily interested in the achievable diversity gain, we provide a sketch proof for the worst–case diversity gain. In fact, for multi–source multi–relay networks with direct link, it is very difficult to obtain a closed–form PEP expression for joint source and relay selection [40] due to correlation among the paths. There are some works on joint source and relay selection for flat–fading links [40], [41], but here we consider selecting a subset of sources which imposes further difficulty on the analysis. Hence, we focus on the diversity gain analysis, as the full PEP analysis would be even more involved for joint source subset and relay selection for a network–coded system.

A. Problem Formulation

Here, the objective is to choose a subset of sources and a relay jointly to minimize the worst–case instantaneous PEP. We assume K sources can form Q groups with T sources in each group. In our analysis, we assume the set of sources is partitioned into disjoint subsets. Partitioning into non–disjoint sets would induce correlation among the received SNRs, which leads to untractable analysis. Hence, we assume $Q \times T = K$, and out of the G relays, one best relay[3] is chosen jointly with the optimum source subset. The problem can be formulated as

$$\{i^*, j^*\} = \arg\max_{\forall i,j} \gamma_{\min,i,j}, \tag{36}$$

where $\gamma_{\min,i,j} \triangleq \min_{l\in\mathcal{G}_i} \gamma_{l,i,j}$, $\gamma_{l,i,j} = \tau_S\gamma_{S_lD,\mathcal{K}^*_{l,i,j}} + \tau_R \min\{\min_{m\in\mathcal{G}_i} \gamma_{S_mR_j,\mathcal{K}^*_{l,i,j}}, \gamma_{R_jD,\mathcal{K}^*_{l,i,j}}\}$ (cf. (22)), $l \in \mathcal{G}_i$, $\gamma_{Z,\mathcal{K}_a} = \sum_{k\in\mathcal{K}_a} \gamma_Z[k]$, $Z \in \{S_mR_u, S_mD, R_uD\}$, and $\mathcal{G}_i$ denotes the ith subset of sources where $|\mathcal{G}_i| = T$ holds. Here, $\mathcal{K}^*_{l,i,j}$ denotes the sub–carrier set containing the bits of the worst–case error event for source $S_l \in \mathcal{G}_i$ when relay R_j is chosen and it is obtained as $\mathcal{K}^*_{l,i,j} = \arg\min_{\mathcal{K}_{l,i,j}\in\mathcal{K}} \big(\tau_S\gamma_{S_lD,\mathcal{K}_{l,i,j}} + \tau_R \min\{\min_{m\in\mathcal{G}_i} \gamma_{S_mR_j,\mathcal{K}_{l,i,j}}, \gamma_{R_jD,\mathcal{K}_{l,i,j}}\}\big)$. When comparing different subsets of sources, the minimum of the received SNRs of all sources in that subset is considered. Basically the chosen subset for a particular relay has the maximum value of the minimum of the received SNRs among the sources in the subset. For analytical tractability, we assume that $\bar{\gamma}_{S_mR_j} = \bar{\gamma}_{S_mR}$, i.e., the links from a particular source to all

[3]As a single best relay is chosen, we adopt bulk allocation here, i.e., the chosen relay transmits on all sub–carriers.

the relays are i.i.d. Similar to relay selection, we assume that the destination informs the selection decision to the relays and sources via a low rate feedback link.

B. Analysis

We analyze the performance of S_l, $l \in \mathcal{G}_{i^*}$, when relay R_{j^*} is selected. Then, the conditional PEP of S_l can be expressed as (cf. (22))

$$P_D(\boldsymbol{c}_l, \tilde{\boldsymbol{c}}_l|\boldsymbol{h}) \leq \tau_0 \exp\big(-\xi\gamma_{l,i^*,j^*}\big) \leq \tau_0 \exp\big(-\xi\gamma_{\min,i^*,j^*}\big). \quad (37)$$

Next, we find the distribution of $\gamma_{\min,i^*,j^*}$. Note that the $\gamma_{\min,i^*,j}$ are independent for different j. Now, $\Pr(\gamma_{\min,i^*,j^*} < x)$

$$= \prod_{j=1}^{G} \Pr(\gamma_{\min,i^*,j} < x) = \prod_{j=1}^{G} \Pr(\max_i\{\gamma_{\min,i,j}\} < x). \quad (38)$$

Note that $\gamma_{\min,i,j}$ may not be independent for different subsets $\mathcal{G}_i$ as the same relay R_j can be used by different subsets, and depending on the overlap of the worst–case sub–carrier sets of different sources that belong to $\mathcal{G}_i$, $\forall i$, the minimum received SNR of different groups $\gamma_{\min,i,j}$, $\forall i$, can be correlated. Here, we study the worst case achievable diversity, i.e., we assume full overlap of the worst–case sub–carrier sets. In particular, we consider two cases: 1) $\bar{\gamma}_{R_jD} \gg \min_{m\in\mathcal{G}_i}\{\bar{\gamma}_{S_mR}\}$ and 2) $\bar{\gamma}_{R_jD} \ll \min_{m\in\mathcal{G}_i}\{\bar{\gamma}_{S_mR}\}$, and derive the corresponding diversity gains in the following propositions.

Proposition 3: For $\bar{\gamma}_{R_jD} \gg \min_{m\in\mathcal{G}_i}\{\bar{\gamma}_{S_mR}\}$, the achievable diversity gain is given by

$$G_d^1 = \sum_{i=1}^{Q} \min(d_{\mathrm{f}}, \min_{l\in\mathcal{G}_i} L_{S_lD}) + \sum_{i=1}^{Q}\sum_{j=1}^{G} \min(d_{\mathrm{f}}, \min_{l\in\mathcal{G}_i} L_{S_lR_j}). \quad (39)$$

Proof: Please refer to Appendix D. ■

The first term $\sum_{i=1}^{Q} \min(d_{\mathrm{f}}, \min_{l\in\mathcal{G}_i} L_{S_lD})$ corresponds to the contribution from the direct links of the sources in the subsets and the second term $\sum_{i=1}^{Q}\sum_{j=1}^{G} \min(d_{\mathrm{f}}, \min_{l\in\mathcal{G}_i} L_{S_lR_j})$ accounts for the contribution of the relay links, particularly of the $S_l \to R_j$ links as the event $\gamma_{R_jD} \gg \min_{m\in\mathcal{G}_i}\{\gamma_{S_mR_j}\}$ is dominant in this case. We observe that if $L_Y \geq d_{\mathrm{f}}$, $Y \in \{S_lD, S_lR_j\}$, the maximum achievable diversity gain in this case is $G^1_{d,max} = Q(1+G)d_{\mathrm{f}}$.

Proposition 4: For $\bar{\gamma}_{R_jD} \ll \min_{m\in\mathcal{G}_i}\{\bar{\gamma}_{S_mR}\}$, the achievable diversity gain is given by

$$G_d^2 = \sum_{i=1}^{Q} \min\{d_{\mathrm{f}}, \min_{l\in\mathcal{G}_i} L_{S_lD}\} + \sum_{j=1}^{G} \min\{d_{\mathrm{f}}, L_{R_jD}\}. \quad (40)$$

Proof: Please refer to Appendix E. ■

In contrast to the previous case, here the second term reflects the contribution of the $R_j \to D$ links. This is because the event $\gamma_{R_jD} \ll \min_{m\in\mathcal{G}_i}\{\gamma_{S_mR_j}\}$ (i.e., $R_j \to D$ are the bottleneck links) is dominant in this scenario. The maximum achievable diversity gain is $G^2_{d,max} = (Q+G)d_{\mathrm{f}}$.

Remark 5: We note that Case 1 results in a higher diversity gain compared to Case 2. In Case 2, all effective SNRs for different subsets of sources sharing the same relay are strongly correlated as $R_j \to D$ is the bottleneck relay link. In Case 1, we can exploit the independent $S_l \to R_j$ links, $l \in \mathcal{G}_i$, $\forall i$. Hence, the effective SNRs of all pairs of relays and subsets are mutually independent in Case 1 and there are QG (cf. the second term in (39)) ways to establish a network coded connection consisting of a subset and a relay out of the Q subsets and the G relays, respectively.

Remark 6: The first term in (39) and (40) scales with the number of subsets, not with the number of sources per subset. This can be attributed to the fact that we consider the worst–case scenario for comparing the link strength from different subsets to the destination, i.e., the minimum end–to–end SNR of all sources in a subset is adopted as the basis for comparison. It is possible that in some instances, the end–to–end instantaneous SNR γ_{l,i^*,j^*} of source S_l, $l \in \mathcal{G}_{i^*}$, is larger than that of S_m, $m \in \mathcal{G}_{i^*}$, $m \neq l$. However, the performance at high SNR is dominated by the minimum end–to–end SNR event.

Example: If all links are flat–fading, i.e., $L_Z = 1$, $Z \in \{S_lD, S_lR_j, R_jD\}$, $\forall l, j$, and $G = Q = 2$, $G_d^1 = 6$ and $G_d^2 = 4$ results. The lower diversity gain in Case 2 results because the $R_j \to D$ links are the bottleneck links and the diversity offered by the $S_l \to R_j$, $l \in \mathcal{G}_i$, $\forall i$ links cannot be exploited.

Remark 7: Comparing (29) and (39)/(40) we observe that joint source subset and relay selection yields a larger diversity gain compared to relay selection only. However, joint source and relay selection may not always be desirable as it causes more feedback overhead compared to relay selection.

VI. Numerical Results

In this section, we present Monte–Carlo simulation results to investigate the impact of the various system and channel parameters on the performance of SLNC for BICM–OFDM systems. Throughout this section, we adopt the rate $1/2$ convolutional code with generator polynomials $(7,5)_8$ and free distance $d_{\mathrm{f}} = 5$, Gray labeling, and $N_t = 64$ sub–carriers of which $N = 60$ are data sub–carriers. We assume all network nodes employ the same channel code. The interleaver for BICM–OFDM is designed as outlined in [21]. The coefficients of the CIRs of all links are independent Rayleigh fading. We assume identical noise variances at R_u, $\forall u$, and D, i.e., $\sigma^2_{n_{S_jD}} = \sigma^2_{n_{S_jR_u}} = \sigma^2_{n_{RD}} = N_0$, $\forall j$. We assume $\sigma^2_{h_Z} = d_Z^{-\alpha}$, $Z \in \{S_jD, S_jR_u, R_uD\}$, with d_Z being the distance of link Z and path–loss exponent $\alpha = 2$. Unless otherwise mentioned, we assume the d_{S_jD} are normalized to 1, and all other link distances are 0.5.

A. Diversity Gain

First, we consider a system with $K = 2$ sources and $G = 1$ relay. We assume BPSK modulation, unless stated otherwise. Fig. 4 shows the bit error rate (BER) vs. transmit SNR (P/N_0) for source S_1 for different CIR lengths $\{L_{S_1D} = L_{S_2D} = L_{S_1R} = L_{S_2R} = L_{RD} = L\}$. We show results for $L = \{1, 2, 3, 5, 6\}$ (solid lines). We observe diversity gains of $G_d^1 = 2$, $G_d^1 = 4$, $G_d^1 = 6$, and $G_d^1 = 10$ for $L = 1$, $L = 2$, $L = 3$, and $L = 5$, respectively, as expected from (20). For $L = 6$, we do not observe any additional diversity gain compared to $L = 5$ but some additional coding gain. This is also confirmed by (20) and the maximum G_d^1 is limited by

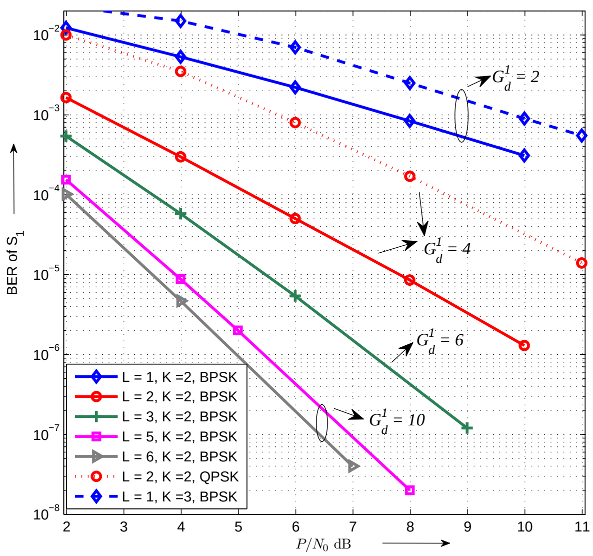

Fig. 4. BER vs. transmit SNR performance of single relay SLNC for BICM–OFDM systems. Equal SNRs for all links.

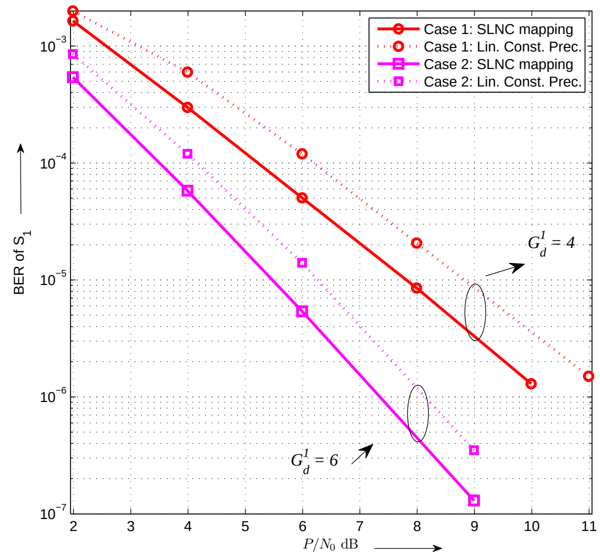

Fig. 6. BER vs. transmit SNR performance for linear constellation precoding and the mapping adopted for SLNC.

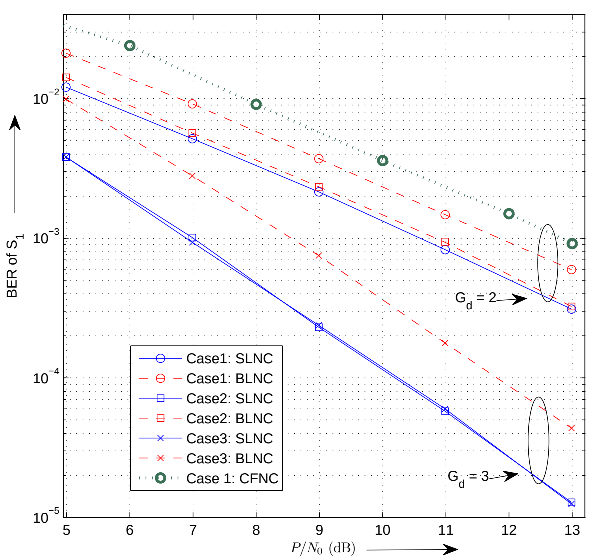

Fig. 5. BER vs. transmit SNR performance comparison of SLNC, BLNC [31], and CFNC [11] schemes in cooperative BICM–OFDM systems.

$2d_f = 10$ for the considered case. Furthermore, we observe that for $L = 2$, full diversity $G_d^1 = 4$ is also attained for QPSK modulation, and for $L = 1$ with $K = 3$, full diversity $G_d^1 = 2$ is obtained as well. This confirms that the diversity gain in (20) is valid for arbitrary modulation schemes and any number of sources. As K increases, some coding gain is sacrificed in exchange for an increase in throughput.

B. Performance Comparisons

In Fig. 5, we compare the performance of the considered SLNC scheme with other network coding protocols for BICM–OFDM systems. We consider a two source single relay network and adopt 8–PSK modulation for SLNC and BLNC. We present three cases: Case 1 ($L_{S_1D} = L_{S_2D} = L_{S_1R} = L_{S_2R} = L_{RD} = 1$), Case 2 ($L_{S_1D} = L_{S_2D} = 1$, $L_{S_1R} = L_{S_2R} = L_{RD} = 2$), and Case 3 ($L_{S_1D} = 1$, $L_{S_2D} = L_{S_1R} = L_{S_2R} = L_{RD} = 2$). For Cases 1 and 3, we observe that both SLNC and BLNC [31] achieve $G_d^1 = 2$ and $G_d^1 = 3$, respectively. However, for Case 2, SLNC results in $G_d^1 = 3$, whereas BLNC results in $G_d^1 = 2$, as expected from (20) and (21), respectively. This is because for BLNC, the diversity gain observed by S_1 depends on the $S_2 \to D$ link. In contrast, for SLNC we observe that Cases 2 and 3 result in identical performances as G_d^1 is independent of the frequency diversity of the $S_2 \to D$ link. Furthermore, we compare SLNC with CFNC in [11], where symbols from multiple sources are received simultaneously at the relays. As CFNC has a higher throughput (CFNC needs two channel uses compared to three required for SLNC to transmit the data of two sources), we assume QPSK transmission for CFNC to compare the performance for the same rate. In CFNC, multiuser detection is applied at the relay and destination which is more complex compared to the decoding procedure adopted for SLNC. We observe that both SLNC and BLNC outperform CFNC by some margin.

In Fig. 6, we compare linear constellation precoding [34] with the adopted multiuser mapping scheme at the relay for $K = 2$ and $G = 1$. We assume both sources transmit symbols from a 4–QAM alphabet. For constellation precoding, we adopt the choice $\theta_i = e^{j\pi(4n-1)(i-1)/2K}/\sqrt{K}$ for $K = 2^k$; and $\theta_i = e^{j\pi(6n-1)(i-1)/3K}/\sqrt{K}$ for $K = 3 \times 2^k$ and any $n \in \{1, \ldots, K\}$ [34]. Here, $K = 2$, and we choose $\theta_1 = 1/\sqrt{2}$ and $\theta_2 = e^{j3\pi/4}/\sqrt{2}$ for $n = 1$. We assume all links have identical frequency diversity L. We compare for $L \in \{2, 3\}$ and observe that both schemes achieve the same diversity gain for the considered cases. However, the mapping scheme performs approximately 1dB better than the constellation precoding scheme. This result can be attributed to the fact that when two sets of 4–QAM signals are mapped to a 16–QAM constellation, a larger $d_{\min}$ is achieved compared to that observed for constellation precoding (cf. Fig. 3).

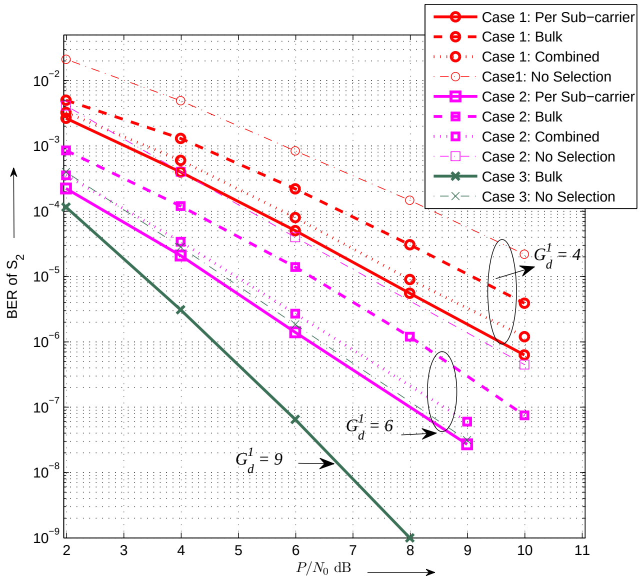

Fig. 7. BER vs. transmit SNR performance for different relay selection schemes.

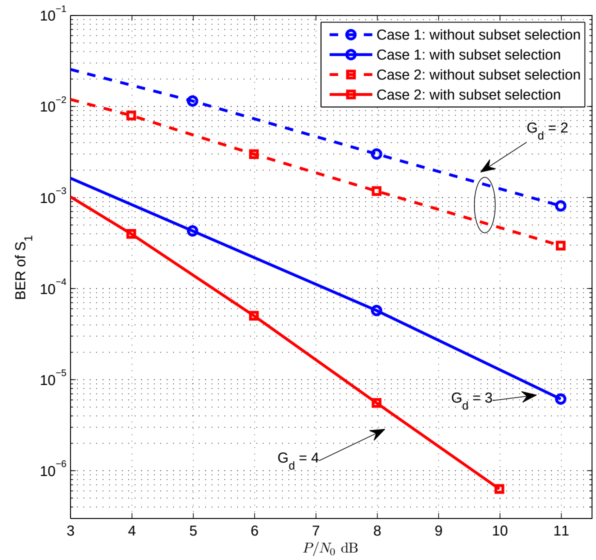

Fig. 8. BER vs. transmit SNR performance for selection of a subset of sources.

C. Relay Selection

In Fig. 7, we study the performance of different relay selection schemes for $K = 2$ and $G = 3$. We assume QPSK transmission by the sources. We assume $L_{S_jD} = L_{SD}$, $L_{S_jR_i} = L_{R_iD} = L_{R_i}$, $\forall i, j$, and consider three cases: Case 1 ($L_{SD} = L_{R_i} = 1$), Case 2 ($L_{SD} = 1$, $L_{R_1} = L_{R_2} = 2$, and $L_{R_3} = 1$), and Case 3 ($L_{SD} = 1$, $L_{R_1} = L_{R_2} = 3$, and $L_{R_3} = 2$). Along with bulk and per sub–carrier selection, here we also show the performance of combined selection, where a subset of all available relays are selected and a best relay is chosen for each sub–carrier among the subset[4]. For combined selection, here we assume each subset contains two relays. We observe that for Case 1 and Case 2, all three relay selection schemes have the same diversity gains of four and six, respectively (cf. Section IV). For Case 1, 'no relay selection' achieves also a diversity gain of four. However, for Case 2, it does not achieve full diversity gain. This is because for uniform sub–carrier allocation among the three relays, it is not guaranteed that the number of bits (i.e., $d_{\mathrm{f},u}$) transmitted by relay R_u corresponding to the worst–case error event will always match the frequency diversity of the $S \to R_u$ and $R_u \to D$ links (cf. (20)). For Case 3, we just show relay selection by bulk sub–carrier allocation and observe that indeed it achieves a diversity gain of nine, as predicted by the analysis in Section IV. However, no relay selection can at most achieve a diversity gain of six, as the maximum diversity for the relaying phase is limited to $d_{\mathrm{f}} = 5$ for uniform sub–carrier allocation among relays (cf. Section III). In conclusion, sub–carrier based relay selection schemes achieve a higher coding and diversity gain compared to 'no relay selection' as the frequency diversity of the involved links increases. It is obvious that per sub–carrier allocation is the most computationally complex of the schemes and bulk allocation is the most simple in terms of operation. In terms of complexity, combined selection stands in between the two schemes as per sub–carrier allocation is performed among a subset of relays.

[4]Following the analyses for bulk and per sub–carrier selection, we can easily show that combined selection also results in the same diversity gain. However, for brevity and space constraints, we do not show the analytical derivations rather validate the claim by simulation results.

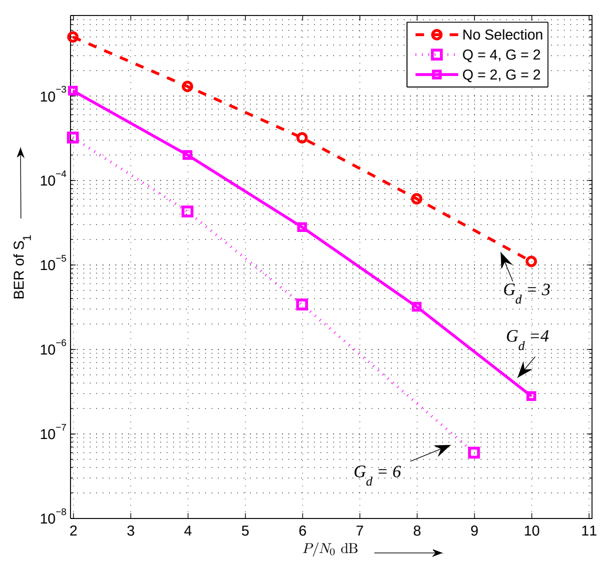

Fig. 9. BER vs. transmit SNR performance for the joint selection of a relay and a subset of sources.

D. Joint Source Subset and Relay Selection

In practice, source subset selection can be profitable when the number of available orthogonal multiple access channels is less than the number of sources. In Fig. 8, we study the performance of source subset selection compared to no selection. We assume $K = 4$, $G = 1$, and $T = 2$. There are $Q = 2$ subsets, each containing $T = 2$ sources: $\mathcal{S}_1 = \{S_1, S_2\}$ and $\mathcal{S}_2 = \{S_3, S_4\}$. We adopt BPSK and assume all links have an identical frequency diversity of one. We consider two cases; Case 1: $d_{RD} = 1$ and $d_{S_jR} = 0.25$ and Case 2: $d_{RD} = 0.5$

and $d_{S_jR} = 1$, $\forall j$. For Case 1, we observe that the system enjoys a diversity gain of three (cf. *Proposition* 4 in Section IV, Eq. (40)) and for Case 2, we observe a higher diversity gain of four as the i.i.d. $S_l \to R$ links experience a lower average SNR compared to the $R \to D$ link and are the bottleneck links (cf. *Proposition* 3 in Section IV, Eq. (39)). Note that no subset selection always achieves a diversity gain of two.

In Fig. 9, we consider joint source subset and relay selection when $Q = \{2, 4\}$, $T = 2$, and $G = 2$. Here, we assume BPSK transmission and all links are i.i.d. flat–fading. As all links are i.i.d. (i.e., same average SNR for all links), the probabilities of having a $S_k \to R_j$ or a $R_j \to D$ link as the bottleneck link are equal, however, the events when the $R_j \to D$ links are the bottleneck links will dominate performance, because the effective link from the source subsets to the destination via a particular relay will be correlated, which in turn limits the maximum achievable diversity gain, cf. *Proposition* 4 which describes the case when the $R_j \to D$ links are almost always the bottleneck links. We observe that for $Q = 2$ and $Q = 4$, diversity gains of four and six are obtained (cf. (40)), respectively, whereas no subset and relay selection results in only a diversity gain of three (cf. (20)). Hence, we conclude that a very high diversity gain can be realized by joint subset and relay selection.

VII. Conclusions

In this paper, we studied multi–source multi–relay SLNC for BICM–OFDM systems. We have shown that the presented SLNC scheme combined with C–MRC decoding achieves the maximum possible diversity gain of the considered system even if erroneous decisions at the relay are taken into account. The diversity gain of a particular source was shown to be independent of the direct links of other sources which is in contrast to BLNC based systems. Results revealed that significant coding and/or diversity gains can be observed with SLNC compared to BLNC and CFNC. Based on the instantaneous PEP, we have developed two sub–carrier based relay selection techniques and shown that they achieve identical diversity gain. Experimental results confirmed that sub–carrier based relay selection achieves higher coding and/or diversity gains compared to uniform sub–carrier allocation among all the relays. Finally, we considered the important problem of source subset selection. As it is difficult to obtain a general closed–form average PEP expression for source subset selection due to the correlation among the chosen paths, we analyzed two specific cases and provided diversity gain expressions for each of them for joint relay and source subset selection. Simulation results confirmed the high diversity gain achievable by the joint selection scheme.

Interesting topics for future work include power allocation among the transmitting nodes and performance analysis when outdated $S \to R$ link CSI is used for C–MRC decoding at the destination.

Appendix A

To evaluate $\mathcal{E}_{\boldsymbol{h}}\Big\{P_D(\boldsymbol{c}_j, \tilde{\boldsymbol{c}}_j|\boldsymbol{h}, \boldsymbol{x}_R)\Big\}$, we need to calculate the PDFs of γ_{S_jD} and $\gamma_{eq,u}$. The asymptotic PDF of γ_{S_jD} (i.e., $\gamma_{S_jD} \to 0$) can be obtained as [38]

$$f_{\gamma_{S_jD}}(x) \doteq \frac{1}{\Gamma(r_{S_jD})\bar{\gamma}_{S_jD}^{r_{S_jD}} \prod_{l=1}^{r_{S_jD}} \lambda_l(\boldsymbol{A}_{S_jD})} x^{r_{S_jD}-1}, \quad (41)$$

where $\gamma_{S_jD} = \bar{\gamma}_{S_jD} \sum_{k,d_{\rm f}} |\boldsymbol{w}_{S_jD}^H[k]\boldsymbol{h}_{S_jD}|^2 = \bar{\gamma}_{S_jD}\boldsymbol{h}_{S_jD}^H\boldsymbol{A}_{S_jD}\boldsymbol{h}_{S_jD}$, $\boldsymbol{A}_{S_jD} = \sum_{k,d_{\rm f}} \boldsymbol{w}_{S_jD}[k]\boldsymbol{w}_{S_jD}^H[k]$, and
$\text{rank}\{\boldsymbol{A}_{S_jD}\} = \min\{d_{\rm f}, L_{S_jD}\} = r_{S_jD}$. Now, we introduce the following *Lemma* to calculate the PDF of $\gamma_{eq,u}$.

Lemma 1: The asymptotic PDF of $\gamma_{eq,u} \triangleq \sum_{k\in\mathcal{K}_{j,u}} \gamma_{eq,u}[k]$, where $\gamma_{eq,u}[k] \triangleq \min\{\{\min_j \gamma_{S_jR_u}[k]\}, \gamma_{R_uD}[k]\}$, $\forall j$, is given by

$$\begin{aligned} f_{\gamma_{eq,u}}(x) \doteq& \frac{1}{2^{Kd_{\rm f},u}}\left(\sum_{p=1}^{K} f_{\gamma_{S_pR_u}}(x) + f_{\gamma_{R_uD}}(x)\right) \\ =& \frac{1}{2^{Kd_{\rm f},u}}\left(\sum_{p=1}^{K} \frac{x^{r_{S_pR_u}-1}}{\Gamma(r_{S_pR_u})\bar{\gamma}_{S_pR_u}^{r_{S_pR_u}} \prod_{l=1}^{r_{S_pR_u}} \lambda_l(\boldsymbol{A}_{S_pR_u})}\right. \\ &\left. + \frac{x^{r_{R_uD}-1}}{\Gamma(r_{R_uD})\bar{\gamma}_{R_uD}^{r_{R_uD}} \prod_{l=1}^{r_{R_uD}} \lambda_l(\boldsymbol{A}_{R_uD})}\right), \end{aligned} \quad (42)$$

where $r_{S_pR_u} \triangleq \text{rank}\{\boldsymbol{A}_{S_pR_u}\} = \min\{d_{{\rm f},u}, L_{\rm S_pR_u}\}$, and $r_{R_uD} \triangleq \text{rank}\{\boldsymbol{A}_{R_uD}\} = \min\{d_{{\rm f},u}, L_{\rm R_uD}\}$. Here, $\boldsymbol{A}_{S_pR_u}$ and $\boldsymbol{A}_{R_uD}$ are defined in a similar way as $\boldsymbol{A}_{S_jD}$ in (41) for set $\mathcal{K}_{j,u}$.

Proof: Please refer to Appendix B.

Now, we calculate the expectations $\mathcal{E}_{\boldsymbol{h}_{S_jD}}\Big\{\exp(-\xi\gamma_{S_jD})\Big\}$ and $\mathcal{E}_{\boldsymbol{h}_{SR_u},\boldsymbol{h}_{R_uD}}\Big\{\exp(-\omega\gamma_{eq,u})\Big\}$ in (11) using $f_{\gamma_{S_jD}}(x)$ in (41) and $f_{\gamma_{eq,u}}(x)$ in (42), respectively, and after some simple manipulations, we arrive at (12).

Appendix B

Here, we prove *Lemma* 1. By the law of total probability, we obtain the expression for the PDF of $\gamma_{eq,u}$ at the top of next page, where $C = (K+1)^{d_{{\rm f},u}} - K - 1$, $\gamma_{S_pR_u} \triangleq \sum_{k\in\mathcal{K}_{j,u}} \gamma_{S_pR_u}[k]$, $\gamma_{R_uD} \triangleq \sum_{k\in\mathcal{K}_{j,u}} \gamma_{R_uD}[k]$, $\gamma_{S_pR_u}^l \triangleq \sum_{k\in\mathcal{K}_{j,u,S_p}^l} \gamma_{S_pR_u}[k]$, $\gamma_{R_uD}^l \triangleq \sum_{k\in\mathcal{K}_{j,u,R_u}^l} \gamma_{R_uD}[k]$, $\cup_{B\in\{S_1,\ldots,S_K,R_u\}}\mathcal{K}_{j,u,B}^l = \mathcal{K}_{j,u}$, and $\cap_{B\in\{S_1,\ldots,S_K,R_u\}}\mathcal{K}_{j,u,B}^l = \varnothing$. In view of *Lemma* 1 in [38], we observe that for $\bar{\gamma}_Z \to \infty$, $Z \in \{S_lR_u, R_uD\}, \forall l, u$, the third term in (43), which corresponds to the mixed event, i.e., when $S_p \to R_u$ is the bottleneck link on some worst–case sub–carriers $k \in \mathcal{K}_{j,u,S_p}^l$ and the $R_u \to D$ link is the bottleneck link on some other worst–case sub–carriers $k \in \mathcal{K}_{j,u,R_u}^l$, decays much faster than the first and second terms and

$$\begin{aligned} &\prod_{k\in\mathcal{K}_{j,u}} \Pr(\gamma_{S_pR_u}[k] < \gamma_Y[k]|Y \in \{S_iR_u, R_uD\}, i \neq p, i \in \mathcal{S}) \\ &\doteq \prod_{k\in\mathcal{K}_{j,u}} \Pr(\gamma_{R_uD}[k] \leq \gamma_Z[k]|Z \in \{S_1R_u, \ldots, S_KR_u\}) \doteq \frac{1}{2^{Kd_{{\rm f},u}}} \end{aligned} \quad (44)$$

holds. Hence, using (cf. (41)), the asymptotic PDF of $\gamma_{eq,u}$ can be obtained as shown in (42). Note that this *Lemma* can be considered as an extension to the results shown in *Lemma* 1 in [38], where a PDF of $\gamma_{eq,u}$ was derived for a single relaying path only, i.e., for one source and one destination.

$$f_{\gamma_{eq,u}}(x) = \underbrace{\sum_{p=1}^{K} f_{\gamma_{S_pR_u}}(x) \prod_{k\in\mathcal{K}_{j,u}} \Pr(\gamma_{S_pR_u}[k] < \gamma_Y[k] | Y \in \{S_iR_u, R_uD\}, i \neq p, i \in \mathcal{S})}_{\text{When } S_p \to R_u \text{ is the bottleneck link}}$$
$$+ \underbrace{f_{\gamma_{R_uD}}(x) \prod_{k\in\mathcal{K}_{j,u}} \Pr(\gamma_{R_uD}[k] \leq \gamma_Z[k] | Z \in \{S_1R_u, \ldots, S_KR_u\})}_{\text{When } R_u \to D \text{ is the bottleneck link}}$$
$$+ \sum_{l=1}^{C} f_{\gamma^l_{S_1R_u}}(x) \circledast \ldots \circledast f_{\gamma^l_{S_KR_u}}(x) \circledast f_{\gamma^l_{R_uD}}(x)$$
$$\times \prod_{k\in\mathcal{K}^l_{j,u,S_1}} \cdots \prod_{k\in\mathcal{K}^l_{j,u,S_K}} \prod_{k\in\mathcal{K}^l_{j,u,R_u}} \Pr(\gamma_{S_1R_u}[k] < \gamma_Y[k] | k \in \mathcal{K}^l_{j,u,S_1}, Y \in \{S_2D, \ldots, S_KD, R_uD\}) \ldots$$
$$\underbrace{\times \Pr(\gamma_{S_KR_u}[k] < \gamma_Y[k] | k \in \mathcal{K}^l_{j,u,S_K}, Y \in \{S_1D, \ldots, S_{K-1}D, R_uD\}) \Pr(\gamma_{R_uD}[k] \leq \gamma_Z[k] | k \in \mathcal{K}^l_{j,u,R_u})}_{\text{Mixed event}} \quad (43)$$

Appendix C

Here, we derive an expression for $\Theta(\mu, q)$ (dropping index i). From (18), we have $\sum_{p=1}^{i} \gamma_{S_qR_{E^\mu_p}} = \sum_{u\in E^\mu} \gamma_{S_qR_u} \geq \sum_{u\in E^\mu} \gamma_{R_u} = \gamma^E_R$. Then $\Theta(\mu, q)$ can be further upper bounded as

$$\Theta(\mu, q) \leq \frac{1}{4}\mathcal{E}_{\boldsymbol{h}}\left\{\exp\left(-\xi\left(\gamma^E_R + \frac{(\gamma_{S_jD} + \frac{1}{J}\gamma^C_R - \frac{\beta}{J}\gamma^E_R)^2}{\gamma_{S_jD} + \frac{1}{J}(\gamma^C_R + \gamma^E_R)}\right)\right)\right\}. \quad (45)$$

Let $\gamma_{S_jD} + \frac{1}{J}\gamma^C_R \triangleq \gamma_m$. Then, we obtain

$$\Theta(\mu, q) \leq \frac{1}{4}\mathcal{E}_{\boldsymbol{h}}\left\{\exp\left(-\xi\left(\gamma^E_R + \frac{(\gamma_m - \frac{\beta}{J}\gamma^E_R)^2}{\gamma_m + \frac{1}{J}\gamma^E_R}\right)\right)\right\}. \quad (46)$$

We calculate an upper bound on $\Theta(\mu, q)$ as it is difficult to obtain a closed–form result for the expression in (46). For this purpose, first we obtain a lower bound on $\gamma^E_R + \frac{(\gamma_m - \frac{\beta}{J}\gamma^E_R)^2}{\gamma_m + \frac{1}{J}\gamma^E_R}$. Following (17) – (24) in [30] where a similar function is lower bounded, we can show that

$$\gamma^E_R + \frac{(\gamma_m - \frac{\beta}{J}\gamma^E_R)^2}{\gamma_m + \frac{1}{J}\gamma^E_R} \geq \begin{cases} \gamma_m + \frac{1}{J}\gamma^E_R, & \gamma^E_R > \frac{2\beta+1}{J\beta^2}\gamma_m \\ \varphi\big(\gamma_m + \frac{1}{J}\gamma^E_R\big), & \gamma^E_R \leq \frac{2\beta+1}{J\beta^2}\gamma_m \end{cases}, \quad (47)$$

where $\varphi \triangleq \rho J\beta^2/(J\beta^2 + 2\beta + 1) > 0$ is a modulation dependent parameter and ρ is a function of β and J, and defined as

$$\rho = -\frac{J^2 + 2J\beta + 2\beta^2}{J^2} + \frac{J^2 + 2J\beta + 2\beta^2}{J^2}\sqrt{4J^2 + \frac{8J\beta^3 + 4\beta^4}{J^2 + 2J\beta + 2\beta^2}} > 0. \quad (48)$$

Exploiting $\gamma_m \triangleq \gamma_{S_jD} + \frac{1}{J}\gamma^C_R$ and $\gamma_R \triangleq \gamma^E_R + \gamma^C_R$, we simplify (47) as

$$\gamma^E_R + \frac{(\gamma_m - \frac{\beta}{J}\gamma^E_R)^2}{\gamma_m + \frac{1}{J}\gamma^E_R} \geq \begin{cases} \gamma_{S_jD} + \frac{1}{J}\gamma_R, & \gamma^E_R > c\gamma_m \\ \varphi\big(\gamma_{S_jD} + \frac{1}{J}\gamma_R\big), & \gamma^E_R \leq c\gamma_m \end{cases}, \quad (49)$$

where $c \triangleq \frac{2\beta+1}{J\beta^2}$. Note that $\gamma_R = \sum_u \gamma_{eq,u} = \sum_u \sum_{k\in\mathcal{K}_j} \gamma_{eq,u}[k]$. We divide the calculation of $\Theta(\mu, q)$ in (46) into two parts corresponding to the cases $\gamma^E_R > c\gamma_m$ and $\gamma^E_R \leq c\gamma_m$, and each part has a similar form as the right hand side of (11). Following similar steps shown in Appendix A to arrive at (12), we can obtain (19).

Appendix D

When $\gamma_{R_jD} \gg \min_{m\in\mathcal{G}_i}\{\gamma_{S_mR_j}\}$, which corresponds to the case when $\bar{\gamma}_{R_jD} \gg \min_{m\in\mathcal{G}_i}\{\bar{\gamma}_{S_mR}\}$, $\gamma_{i,l,j}$ simplifies to $\gamma_{i,l,j} = \gamma_{S_lD} + \min_{k\in\mathcal{G}_i}\{\gamma_{S_kR_j}\}$, $l \in \mathcal{G}_i$. Then the minimum received SNR $\gamma_{\min,i,j}$ of group $\mathcal{G}_i$ can be expressed as $\gamma_{\min,i,j}$

$$= \min_{l\in\mathcal{G}_i}\big\{\gamma_{S_lD} + \min_{m\in\mathcal{G}_i}\{\gamma_{S_mR_j}\}\big\} = \min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\} + \min_{l\in\mathcal{G}_i}\{\gamma_{S_lR_j}\}. \quad (50)$$

Hence, $\gamma_{\min,i^*,j^*}$ can be expressed as

$$\gamma_{\min,i^*,j^*} = \max_{i,j} \gamma_{\min,i,j} = \max_{i,j}\big\{\min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\} + \min_{l\in\mathcal{G}_i}\{\gamma_{S_lR_j}\}\big\}$$
$$= \max_{i}\big\{\min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\} + \max_{j}\min_{l\in\mathcal{G}_i}\{\gamma_{S_lR_j}\}\big\}. \quad (51)$$

For the considered case, we have $\gamma_{\min,i,j^*} = \min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\} + \max_j \min_{l\in\mathcal{G}_i}\{\gamma_{S_lR_j}\}$. Note that γ_{S_lD} and $\gamma_{S_lR_j}$ are independent for different $l \in \mathcal{G}_i$, and consequently, $\min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\}$ and $\min_{l\in\mathcal{G}_i}\{\gamma_{S_lR_j}\}$ are independent. We define $\gamma_{SD,i} \triangleq \min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\}$ and $\gamma_{SR_j,i} \triangleq \min_{l\in\mathcal{G}_i}\{\gamma_{S_lR_j}\}$. To obtain the asymptotic PDFs of $\gamma_{SD,i}$ and $\gamma_{SR_j,i}$, we exploit the following Lemma.

Lemma 2: Consider L non–negative random variables $\{X_i\}$[5], $i \in \{1, \cdots, L\}$, and define $W \triangleq \min(X_1, \cdots, X_L)$. Then when $w \to 0^+$, the PDF $f_W(w)$ of W is given by

$$f_W(w) = f_{X_1}(w) + f_{X_2}(w) + \cdots + f_{X_L}(w). \quad (52)$$

Proof: Please refer to *Lemma* 2 in [42]. ■

Now using (41) and *Lemma* 2, we obtain the asymptotic PDFs

[5]Here, the random variables $\{X_i\}$, $\forall i$, do not need to be independent.

$$f_{\gamma_{SD,i}}(x) = \sum_{l\in\mathcal{G}_i} f_{\gamma_{S_lD}}(x) = \sum_{l\in\mathcal{G}_i} \frac{1}{\Gamma(r_{S_lD})\bar{\gamma}_{S_lD}^{r_{S_lD}}\prod_{m=1}^{r_{S_lD}}\lambda_m(\boldsymbol{A}_{S_lD})} x^{r_{S_lD}-1} \tag{53}$$

and $f_{\gamma_{SR_j,i}}(x)$

$$= \sum_{l\in\mathcal{G}_i} \frac{1}{\Gamma(r_{S_lR_j})\bar{\gamma}_{S_lR_j}^{r_{S_lR_j}}\prod_{m=1}^{r_{S_lR_j}}\lambda_m(\boldsymbol{A}_{S_lR_j})} x^{r_{S_lR_j}-1}. \tag{54}$$

From (53), $f_{\gamma_{SD,i}}(x)$ can be approximated as $f_{\gamma_{SD,i}}(x) \approx \Omega(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i})x^{r_{SD,i}-1}$, where $r_{SD,i} \triangleq \min_{l\in\mathcal{G}_i}\{r_{S_lD}\}$, and $\Omega(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i})$ is defined as

$$\Omega(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i}) = \begin{cases} \sum_{l\in\mathcal{G}_i}\mu_l, & r_{s_1D} = \cdots = r_{s_QD}; \\ \mu_l, & r_{S_lD} < r_{S_mD}, \quad m\in\mathcal{G}_i, m\neq l; \\ \sum_{l\in\mathcal{G}_{i,1}}\mu_l, & r_{\mathcal{G}_{i,1}} < r_{S_mD}, m\in\mathcal{G}_{i,2}, \\ & r_{\mathcal{G}_{i,1}} = \{r_{S_lD}\}_{l\in\mathcal{G}_{i,1}}, \mathcal{G}_{i,1}\cap\mathcal{G}_{i,2} = \emptyset, \end{cases} \tag{55}$$

and

$$\mu_l \triangleq \frac{1}{\Gamma(r_{S_lD})\bar{\gamma}_{S_lD}^{r_{S_lD}}\prod_{m=1}^{r_{S_lD}}\lambda_m(\boldsymbol{A}_{S_lD})}. \tag{56}$$

Similarly from (54), $f_{\gamma_{SR_j,i}}(x)$ can be approximated as $f_{\gamma_{SR_j,i}}(x) \approx \Lambda(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i})x^{r_{SR_j,i}-1}$, where $r_{SR_j,i} \triangleq \min_{l\in\mathcal{G}_i}\{r_{S_lR_j}\}$, $\Lambda(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i})$ is defined in a similar fashion as $\Omega(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i})$ above, and ν_l is defined as

$$\nu_l \triangleq \frac{1}{\Gamma(r_{S_lR_j})\bar{\gamma}_{S_lDR_j}^{r_{S_lR_j}}\prod_{m=1}^{r_{S_lR_j}}\lambda_m(\boldsymbol{A}_{S_lR_j})}. \tag{57}$$

Now, we define $\gamma_{SR_{j^*},i} \triangleq \max_j \gamma_{SR_j,i}$. The asymptotic CDF of $\gamma_{SR_j,i}$ can be obtained as $\Pr(\gamma_{SR_j,i} < x) = \Theta(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i})x^{r_{SR_j,i}}$, where $\Theta(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i}) = \Lambda(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i})/r_{SR_j,i}$. Now, the CDF of $\gamma_{SR_{j^*},i}$ can be expressed as

$$\Pr(\gamma_{SR_{j^*},i} < x) = \prod_{j=1}^{G}\Theta(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i})x^{r_{SR_j,i}} = \left(\prod_{j=1}^{G}\Theta(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i})\right)x^{\sum_{j=1}^{G}r_{SR_j,i}}. \tag{58}$$

Let us define $r_{SR,i} \triangleq \sum_{j=1}^{G} r_{SR_j,i}$. Now, the PDF of $\gamma_{SR_{j^*},i}$ is given by

$$f_{\gamma_{SR_{j^*},i}}(x) = r_{SR,i}\left(\prod_{j=1}^{G}\Theta(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i})\right)x^{r_{SR,i}-1} = \Sigma(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i}, \forall j)x^{r_{SR,i}-1}, \tag{59}$$

where $\Sigma(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i}, \forall j)$

$$= r_{SR,i}\left(\prod_{j=1}^{G}\Theta(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i})\right). \tag{60}$$

Now, we find the PDF of $\gamma_{\min,i,j^*} = \gamma_{SD,i} + \gamma_{SR_{j^*},i}$ by exploiting the following Lemma.

Lemma 3: Consider two non–negative independent random variables Z_1 and Z_2. The PDF of Z_i is approximated as $f_{Z_i}(z) \approx \zeta_i z^{\kappa_i}$ (assuming that $z \to 0$). Let $V = Z_1 + Z_2$. Then the PDF of V can be expressed as

$$f_V(v) \approx \frac{\zeta_1\zeta_2\Gamma(\kappa_1+1)\Gamma(\kappa_2+1)}{\Gamma(\kappa_1+\kappa_2+2)}v^{\kappa_1+\kappa_2+1}. \tag{61}$$

Proof: Please refer to *Proposition* 3 in [43]. ■

Following *Lemma* 3, we obtain the PDF of $\gamma_{\min,i,j^*}$ as

$$f_{\gamma_{\min,i,j^*}}(x) = \Omega(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i})\Sigma(\{\nu_l\}_{l\in\mathcal{G}_i}, r_{SR_j,i}, \forall j) \times \frac{\Gamma(r_{SD,i})\Gamma(r_{SR,i})}{\Gamma(r_{SD,i}+r_{SR,i})}x^{r_{SD,i}+r_{SR,i}-1}. \tag{62}$$

As $\gamma_{\min,i,j^*}$ is independent over i and provides a diversity gain of $r_{SD,i}+r_{SR,i}$, following Proposition 4 in [39], we obtain the diversity gain for $\gamma_{\min,i^*,j^*} = \max_i \gamma_{\min,i,j^*}$ (cf. (51)) as

$$G_d^1 = \sum_{i=1}^{Q}(r_{SD,i}+r_{SR,i}) = \sum_{i=1}^{Q}\min_{l\in\mathcal{G}_i} r_{S_lD} + \sum_{i=1}^{Q}\sum_{j=1}^{G}\min_{l\in\mathcal{G}_i} r_{S_lR_j} = \sum_{i=1}^{Q}\min(d_{\mathrm{f}}, \min_{l\in\mathcal{G}_i} L_{S_lD}) + \sum_{i=1}^{Q}\sum_{j=1}^{G}\min(d_{\mathrm{f}}, \min_{l\in\mathcal{G}_i} L_{S_lR_j}) \tag{63}$$

This concludes the proof.

Appendix E

When $\gamma_{R_jD} \ll \min_{m\in\mathcal{G}_i}\{\gamma_{S_mR_j}\}$, which corresponds to the case when $\bar{\gamma}_{R_jD} \ll \min_{m\in\mathcal{G}_i}\{\bar{\gamma}_{S_mR}\}$, $\gamma_{i,l,j}$ simplifies to $\gamma_{i,l,j} = \gamma_{S_lD} + \gamma_{R_jD}$, $l \in \mathcal{G}_i$. Then, the minimum received SNR $\gamma_{\min,i,j}$ of group $\mathcal{G}_i$ can be expressed as $\gamma_{\min,i,j}$

$$= \min_{l\in\mathcal{G}_i}\big\{\gamma_{S_lD} + \gamma_{R_jD}\big\}\big\} = \min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\} + \gamma_{R_jD}. \tag{64}$$

Hence, $\gamma_{\min,i^*,j^*}$ can be expressed as

$$\gamma_{\min,i^*,j^*} = \max_{i,j}\gamma_{\min,i,j} = \max_{i,j}\big\{\min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\} + \gamma_{R_jD}\big\} = \max_i\big\{\min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\}\big\} + \max_j\gamma_{R_jD}. \tag{65}$$

Let $\gamma_{SD,i^*} = \max_i\big\{\min_{l\in\mathcal{G}_i}\{\gamma_{S_lD}\}\big\} = \max_i\big\{\gamma_{SD,i}\big\}$. The asymptotic CDF of $\gamma_{SD,i}$ can be obtained following (53) and (55) as $\Pr(\gamma_{SD,i} < x) = \Upsilon(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i})x^{r_{SD,i}}$, where $\Upsilon(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i}) = \Omega(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i})/r_{SD,i}$. Now, the PDF of γ_{SD,i^*} can be obtained following (58) and (59) as $f_{\gamma_{SD,i^*}}(x) = \Xi(\{\mu_l\}_{l\in\mathcal{G}_i,\{r_{SD,i}\},\forall i})x^{r_{SD}-1}$, where $r_{SD} = \sum_{i=1}^{Q}r_{SD,i}$ and $\Xi(\{\mu_l\}_{l\in\mathcal{G}_i,\{r_{SD,i}\},\forall i}) = r_{SD}\big(\prod_{i=1}^{Q}\Upsilon(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i})\big)$. Similarly, we can obtain the PDF of $\gamma_{R_{j^*}D} = \max_j\gamma_{R_jD}$ as $f_{\gamma_{R_{j^*}D}}(x) = \Psi(\tau_j, r_{R_jD}, \forall j)x^{r_{RD}-1}$, where $r_{RD} = \sum_{j=1}^{G}r_{R_jD}$ and $\Psi(\tau_j, r_{R_jD}, \forall j) = r_{RD}\prod_{j=1}^{G}\tau_j$, and

$$\tau_j \triangleq \frac{1}{\Gamma(r_{R_jD})\bar{\gamma}_{R_jD}^{r_{R_jD}}\prod_{m=1}^{r_{R_jD}}\lambda_m(\boldsymbol{A}_{R_jD})}. \tag{66}$$

Note that $\tau_j x^{r_{R_jD}-1}$ is the PDF of γ_{R_jD} (cf. (41)). Now, following *Lemma* 3, we obtain the PDF of $\gamma_{\min,i^*,j^*}$ as

$$f_{\gamma_{\min,i^*,j^*}}(x) = \Xi(\{\mu_l\}_{l\in\mathcal{G}_i}, r_{SD,i}, \forall i)\Psi(\tau_j, r_{R_jD}, \forall j) \times \frac{\Gamma(r_{SD})\Gamma(r_{RD})}{\Gamma(r_{SD}+r_{RD})}x^{r_{SD}+r_{RD}-1}. \tag{67}$$

Finally, following [39], we can show that for the considered case, the diversity gain provided by $\gamma_{\min,i^*,j^*}$ is given by the decaying rate of $f_{\gamma_{\min,i^*,j^*}}(x)$ as

$$G_d^2 = r_{SD} + r_{RD} = \sum_{i=1}^{Q} r_{SD,i} + \sum_{j=1}^{G} r_{R_j D}$$
$$= \sum_{i=1}^{Q} \min\{d_{\mathrm{f}}, \min_{l\in\mathcal{G}_i} L_{S_l D}\} + \sum_{j=1}^{G} \min\{d_{\mathrm{f}}, L_{R_j D}\}. \quad (68)$$

This concludes the proof.

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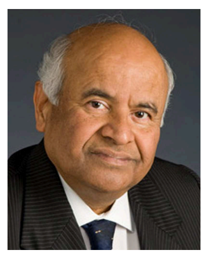

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