SIMULTANEOUS FRACTAL IMAGE DENOISING AND INTERPOLATION

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ABSTRACT

In this paper, a simple and effective fractal-based simultaneous image denoising and interpolation scheme is proposed and implemented. The denoising is performed during the fractal encoding process while the interpolation is performed during the decoding process. The fractal-based image denoising involves predicting the fractal code of the original noiseless image from the statistics of the noisy observation. This fractal code can then be used to generate a fractally denoised estimate of the original image. The fractal interpolation can be easily achieved during the decoding process by iterating the predicted fractal code on a suitably sized blank intital image seed. The cycle spinning algorithm can also be incorporated in the proposed fractal joint denoising and resizing scheme in order to reduce some of the artifacts and enhance the visual quality of the fractally denoised and resized estimates.

1. INTRODUCTION

Image interpolation has wide applications in areas such as digital photography, the printing industry, HDTV and computer graphics [9]. In many applications, the image is transmitted over a noisy communication medium, such as an AWGN channel. Thus, there is a need to denoise the image before one can resize it. In this paper, we propose a fractal-based method that is capable of simultaneously denoising and resizing a noisy digital image.

Fractal image coding has received much interest over the past decade, mostly in the context of image compression. Over the years, a variety of highly competitive fractal fractal image compression methods have been [1, 2, 4, 7, 8]. Most of these schemes are based on exploiting the inherent local self-similarities in real world images by performing some form of block-based *collage coding* as originally introduced by Jacquin [7]. However, noisy structures have no resemblance or self-similarities and therefore cannot be represented well using fractal image coding schemes. Thus, one expects that some smoothness of the noise is achieved, when encoding a noisy image using a fractal coder. Exploiting these observations, we have previously developed a fractal-based denoising approach that predicts the fractal code of the original (noiseless) image from that of the noisy observation and yields a fractally denoised estimate of the original image. This fractal-based image denoising strategy can be performed in the spatial as well as the wavelet domains of the noisy image [5, 6].

In this work, we integrate a simple fractal-based interpolation strategy in our previously proposed fractal denoising scheme which allows us to modify the size of the image by a power of two factor, in each direction. This fractal interpolation method is performed entirely during the fast decoding process after the noisy image has been smoothed by the fractal denoising scheme. The resulting fractal scheme performs simultaneous image denoising and interpolation. The fractal code may be binary coded and stored and different denoised fractal interpolations may be generated from the same fractal code. The cycle spinning algorithm can also be incorporated within the proposed fractal scheme in order to reduce some of the artifacts and enhance the visual quality of the fractal denoised and resized estimate.

This paper is organized as follows: The basics of block-based fractal scheme are described in section 2 and its applications for the purpose of image denoising and interpolation are presented in section 3. Some experimental results are presented in section 4. Finally a brief discussion and concluding remarks are presented in section 5.

2. BLOCK-BASED FRACTAL IMAGE CODING

Originally, fractal-based methods sought to express a target set as a union of shrunken copies of itself. However, most real-world images are rarely so entirely self-similar. Instead, self-similarity may be exhibited only locally, in the sense that subregions of an image may be self-similar. This is the basis of the block-based fractal encoding scheme introduced by Jacquin [7]. Most fractal-based image coding methods are based on this scheme, which can be outlined as follows:

1. As illustrated in Fig. 1, the image is subdivided into two different non-overlapping partitions of sub-blocks:

- $M \times M$ domain (parent) blocks, D_i. For instance, when the image is square with a power of two size, we choose $M = 2^m$, for some integer $m \geq 1$.
- $N \times N$ range (child) blocks, R_k. Typically, we choose $N = 2 \times M$, so that the size of a parent block is four times that of a child block.

2. Each child block, R_k, $k = 1, 2, \ldots, N^2$, is then matched to its most "similar" parent block $D_{i^*(k)}$, for some $i^*(k) \in \{1, 2, \ldots, M^2\}$.

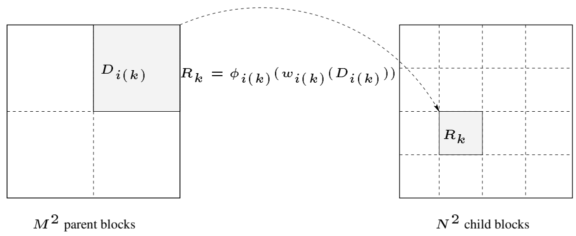

Fig. 1. Uniform image partitioning for fractal image coding.

The similarity between the parent block $D_{i^*(k)}$ and the child block R_k, is in the sense that the subimage on the parent block $D_{i^*(k)}$ can be transformed "closely" to the subimage on the child block R_k via a contractive mapping. This contractive transformation is a composition of a geometric mapping $w_{i(k)}$ followed by grey-level map, $\phi_{i(k)}$:

1. The geometric map, $w_{i(k)}$, is one of eight contractive affine mappings which can transform a square domain sub-block, D_i, into a smaller square block, $\tilde{D}_i$, of the same size as the range sub-block, R_k (four rotations, a horizontal flipping, a vertical flipping and two diagonal flippings).

2. A least-squares fit is then performed between the child block R_k and the geometrically transformed parent block $\tilde{D}_i$, using affine grey-level mapping

$$\phi(t) = \alpha t + \beta. \qquad (1)$$

The standard fractal encoding is briefly outlined next.

Fractal Encoding: For each range block R_k, $1 \leq k \leq N^2$, we seek the domain block $D_{i^*(k)}$, $1 \leq i^*(k) \leq M^2$, such that the sub-image $u(D_{i^*(k)})$ best approximates the sub-image $u(R_k)$ after a geometric transformation/decimation, $w_{i,k}^{(m)}$. In other words we find the indices $(i^*(k), m^*(k))$ that minimize the collage error

$$\Delta_{i,k}^2 = \| \alpha_{i(k)}^{m(k)} u(w_{i,k}^{(m(k))}(D_{i(k)})) + \beta_{i(k)}^{m(k)} - u(R_k) \|_2^2,$$

where $\alpha_{i(k)}^{(m)}$ and $\beta_{i(k)}^{(m)}$ denote the least-squares grey-level coefficients associated with the m^{th} geometric mapping $w_{i(k)}^{(m)}$. The fractal code of the target image is as follows:

$$\{M, N, i^*(k), w_{i^*(k)}^{(m^*)}, \alpha_{i(k)}^{(m^*)}, \beta_{i(k)}^{(m^*)}, k = 1, 2, \ldots, N^2\}. \qquad (2)$$

From this fractal code, a fractal approximation of the original image can be constructed as outlined next.

Fractal Decoding: The fractal decoding algorithm is a fast and recursive process which can be summarized as follows: Starting with any initial image (typically a blank image), each range sub-block, R_k, is predicted from its fractal code. This process is then repeated recursively until a desirable convergence is achieved. Convergence may be defined in terms of the difference between two consecutive iteration estimates.

Next, we outline the fractal-based joint image denoising and resizing scheme.

3. SIMULTANOUS FRACTAL IMAGE DENOISING AND INTERPOLATION

3.1. Fractal Image Denoising

In [6], a simple and effective fractal-based strategy for smoothing noisy images in the pixel as well as the wavelet domains of the noisy image was proposed. First, it was observed that straightforward fractal coding of a noisy image yields some degree of noise reduction. This may be explained by the fact that self-similar structures found in natural images are generally reconstructed rather well through fractal coding whereas the noisy components cannot be approximated well in this way. We have also shown that one can achieve better image denoising results by estimating the fractal code of the original noiseless image from that of the noisy observation. From this predicted fractal code, one can generate a fractally denoised estimate of the original image.

Due to space limitation, the details of this fractal-based image denoising scheme are not included here. The reader is referred to our previous work in [5, 6].

3.2. Fractal Image Interpolation

In order to increase the size of the image by a power of two (in each direction), a simple fractal strategy can be applied [8]. This method is performed entirely during the fast converging fractal decoding process. For instance, for the commonly used test image of "Lena" of size 512×512, and suppose we wish to double the size of this test image to obtain a fractally interpolated image of size 1024×1024 pixels.

- First, the original image is fractally encoded, using the standard fractal coding scheme, outlined above.

This must be done at a sufficiently high fractal resolution (i.e. small domain/range blocks) in order to achieve a sufficiently high quality fractal representation of the image. For instance for the standard fractal scheme, we use $(M, N) = (64, 128)$ which results in mapping a 8×8-pixel parent blocks $D_{i^*(k)}$ into an 4×4-pixel child blocks R_k.

- Then, in order to double the size of the fractally decoded image, during the decoding process one simply starts with an initial blank image seed of the same size as the desired interpolated image (i.e. 1024×1024 pixels). Hence, during the fractal decoding process, 16×16-pixel parent blocks $D_{i^*(k)}$ are now mapped into 8×8-pixel child blocks R_k, using the fractal code obtained from the original image, of size 512×512.

This simple fractal-based process results in a fractally interpolated image of size 1024×1024 pixels. Of course, modifying (increase or decreasing) the size of the image by any power of 2 in each direction can be performed in a similar manner, by simply applying the fractal code on an image blank image seed of a suitable size.

3.3. Additional Enhancement via Cycle Spinning

The idea of using the cycle spinning algorithm was originally proposed for the purpose of reducing the pseudo-Gibbs disturbing artifacts that are often present in wavelet thresholding denoised estimates [3]. In our case, this cycle spinning idea can be incorporated in the simultaneous fractal image denoising and resizing scheme for the purpose of enhancing the results. This can be summarized as follows:

$$\hat{\mathbf{x}}_K = \frac{1}{K} \sum_{h=0}^{K-1} D_{-h}((FDI((D_h(\mathbf{y}))), \qquad (3)$$

where the noisy image ($\mathbf{y}$) is first shifted, using a diagonal shifting operator, D_h. The factal denoising and interpolation (FDI) scheme is then applied and the resulting image is then unshifted, by the same amount. This process is repeated for each of K shifts and the respective results are then averaged to obtain one enhanced denoised and resized estimate of the image.

4. EXPERIMENTAL RESULTS

Figs. 2(a) and (b) illustrate the original as well as the noisy version of the standard *Lena* image (512×512 pixel, 8 bits per pixel) as corrupted by an AWGN noise with intensity $\sigma_w = 25$. Fig. 3(a) illustrates the results of performing simultaneous fractal denoising and resizing of the noisy image. Fig. 3(b) illustrates the results of incorporating the cycle spinning idea described above. Clearly, the use of the cycle spinning algorithm has resulted in a significant enhancement of the visual quality of the fractally denoised and resized estimate of the image. Experimentally, it was observed that the quality of the denoised estimate improves significantly after a set of initial shifts and then becomes stable with little or no further gains are achieved through additional shifts. In this case, a total of $K = 40$ diagonal shifts were used. Clearly the cycle spinning algorithm can be computationally expensive. Indeed, when incorporating this algorithm with K shifts for any denoising method, the computational complexity is multiplied by a factor of K.

(a) Original image: 512×512 pixels

(b) Noisy test image: $\sigma_w = 25$

Fig. 2. (a) the original image, and (b) its noisy observation as corrupted by an AWGN noise with intensity, $\sigma_w = 25$.

5. DISCUSSION AND CONCLUSIONS

In this paper, we presented a simple and effective fractal-based image denoising and interpolation scheme. The fractal denoising is performed during the encoding process while the fractal interpolation is performed during the fractal decoding process.

(a) Simultaneous fractal denoising and interpolation
1024×1024 pixels

(b) Incorporating the cycle spinning: $K = 40$ shifts.
1024×1024 pixels

Fig. 3. Experimental results: (a) simultaneous fractal denoising and resizing of the noisy image, (b) enhancement of the denoised estimates using the cycle spinning algorithm with $K = 40$ shifts.

The main advantage of the proposed fractal-based method is that unlike most standard interpolation methods, the proposed method performs simultaneous image denoising and interpolation. Furthermore, since fractal transforms were originally developed for the purpose of image coding, the proposed fractal scheme may also be optimized to perform simultaneous image denoising, interpolation and compression. Since the interpolation is performed during the decoding process, no extra bits are required to encode the interpolation information.

In conclusion, this work broadens the scope of fractal-based methods which have been used mainly for image compression. Recently, we have investigated the potentials of developing and applying fractal-based methods for the purpose of image denoising [5, 6]. In this work, we explored another potential application of fractal-based methods, namely image interpolation. The main contribution lies in developing a simple fractal image denoising and resizing scheme. This proposed fractal-based image denoising and resizing strategy can be extended to most fractal-based schemes, as performed in the pixel or the wavelet domain of the image.

6. REFERENCES

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