

Space-Time Coding for Wireless Communication: A Survey

Lei Poo

Stanford University

email: leipoo@stanford.edu

Abstract

Space-Time Codes were first introduced by Tarokh et al. from AT&T research labs [1] to provide transmit diversity in wireless fading channels using multiple transmit antennas. Two outstanding examples of space-time codes are space-time trellis codes [1] and space-time block codes [2], [3]. We survey the basic designs and diversity results of these two types of space-time codes, and summarize their relative performance.

I. INTRODUCTION TO SPACE-TIME CODES

Space-Time Codes (STC) were first introduced by Tarokh et al. from AT&T research labs [1] in 1998 as a novel means of providing transmit diversity for the multiple-antenna fading channel. Previously, multipath fading in multiple antenna wireless systems was mostly dealt with by other diversity techniques, such as temporal diversity, frequency diversity and receive antenna diversity, with receive antenna diversity being the most widely applied technique. However, it is hard to efficiently use receive antenna diversity at the remote units because of the need for them to remain relatively simple, inexpensive and small. Therefore, for commercial reasons, multiple antennas are preferred at the base stations, and transmit diversity schemes are growing increasingly popular as they promise high data rate transmission over wireless fading channels in both the uplink and downlink while putting the diversity burden on the base station.

The space-time coding scheme by Tarokh et al. [1], [2], is essentially a joint design of coding, modulation, transmit and receive diversity, and has been shown to be a generalization of other transmit diversity schemes, such as the bandwidth efficient transmit diversity scheme by Witneben [4] and the delay diversity scheme by Seshadri and Winters [5].

There are two main types of STCs, namely space-time block codes (STBC) and space-time trellis codes (STTC). Space-time block codes operate on a block of input symbols, producing a matrix output whose columns represent time and rows represent antennas. In contrast to single-antenna block codes for the AWGN channel, space-time block codes do not generally provide coding gain, unless concatenated with an outer code. Their main feature is the provision of full diversity with a very simple decoding scheme. On the other hand, space-time trellis codes operate on one input symbol at a time, producing a sequence of vector symbols whose length represents antennas. Like traditional TCM (trellis coded modulation) [6] for a single-antenna channel, space-time trellis codes provide coding gain. Since they also provide full diversity gain, their key advantage over space-time block codes is the provision of coding gain. Their disadvantage is that they are extremely hard to design and generally require high complexity encoders and decoders.

This project report aims to give a literature survey of the design and diversity results of STBCs and STTCs in Rayleigh fading. We discuss results primarily from [1], [2], [3], [7], and [8].

We start with a model of the multiple antenna system in §II. This is followed by a discussion of Alamouti's two-antenna transmit diversity scheme in relation to maximum ratio combining (MRC)

in §III, where we also outline the general construction of STBCs which generalized Alamouti's scheme. In §IV, we work through the analytical pairwise upper bounds for the performance of STTCs, and examine the design criteria for slow and fast Rayleigh fading channels with independent fading. An example of STTC using 4-PSK is given. In §V, we examine the performance comparison of STTC with concatenated STBC in [8], which was done keeping the transmit power, spectral efficiency and number of trellis states fixed. We conclude this survey with an outline of the recent developments of STC schemes.

II. SYSTEM MODEL

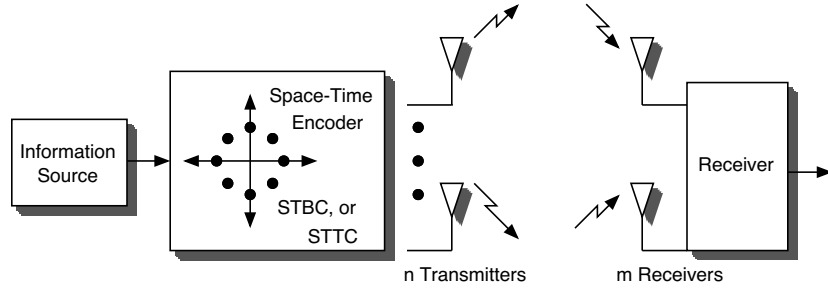


Fig. 1. System Block Diagram

Consider a mobile communications system where the base station is equipped with n antennas and the remote unit is equipped with m receive antennas (see Figure (1)). At each time slot t , signals c_t^i , $i = 1, 2, \dots, n$ are transmitted simultaneously from the n transmit antennas. The channel is flat-fading and the path gain from transmit antenna i to receive antenna j is denoted by $h_{i,j}$. The path gains are modeled as samples of independent complex Gaussian random variables with variance 0.5 per real dimension, i.e., $h_{i,j} \sim \mathcal{N}(0, 1)$, as we assume that signals received at different antennas experience independent fading. In this report, we will consider modelling the path gains in both slow and fast Rayleigh fading. For slow fading, it is assumed that the path gains are constant during a frame of length L and vary from one frame to another, i.e., channel is quasi-static. For fast fading, the path gains are constant within each symbol period and vary from symbol to symbol.

At time t , the signal r_t^j , received at antenna j is given by

$$r_t^j = \sum_{i=1}^n h_{i,j} c_t^i + \eta_t^j,$$

where the noise samples η_t^j are i.i.d. zero mean complex Gaussian with variance $\sigma^2 = 1/(2E_s/N_0) = 1/(2 \text{ SNR})$ per dimension. The average energy of the symbols transmitted from each antenna is normalized to one, so that the average power of the received signal at each receive antenna is n .

It is assumed that channel state information is only available at the receiver, who uses it to compute the decision metric

$$\sum_{t=1}^l \sum_{j=1}^m \left| r_t^j - \sum_{i=1}^n h_{i,j} c_t^i \right|^2$$

over all codewords $c_1^1 c_1^2 \dots c_1^n c_2^1 c_2^2 \dots c_2^n \dots c_l^1 c_l^2 \dots c_l^n$ and decide in favor of the code word that minimizes the sum.

Note that this transmission model can be rewritten in block matrix form: see Sandhu et al. [8].

The signals $\{c_i^i\}$, $i = 1, 2, \dots, n$, are encoded using either a STBC or an STTC. Both of these schemes are described in detail in the next two sections.

III. SPACE-TIME BLOCK CODES

A. Alamouti's Scheme

In 1998, Alamouti [7] proposed a simple transmit diversity scheme (see Figure (2)), which improves the signal quality at the receiver on one side of the link by simple processing across two transmit antennas at the opposite end.

A.1 Two Antenna Transmit Diversity Scheme

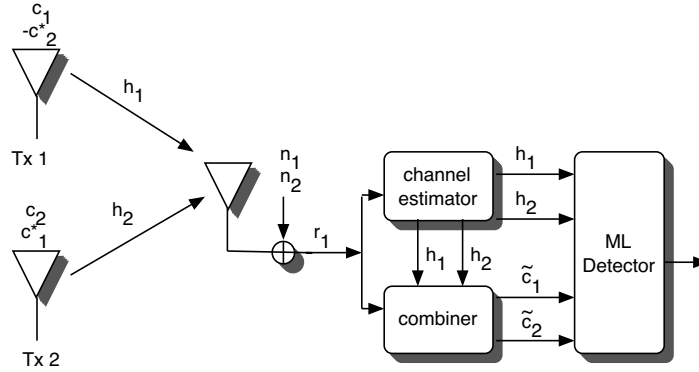


Fig. 2. Alamouti's Two Antenna Transmit Diversity Scheme

At a given symbol period, two signals are simultaneously transmitted from the two antennas, namely c_1 from the first antenna, Tx 1, and c_2 from the second antenna, Tx 2. In the next symbol period, signal $(-c_2^*)$ is transmitted from Tx 1 and signal c_1^* is transmitted from Tx 2, where $*$ denotes complex conjugation.

In accordance with our system model in §II, let $h_1(t)$ denote the path gain from Tx 1 to the receiver; similarly let $h_2(t)$ be that from Tx 2 to the receiver. If we assume that fading is constant across two consecutive symbols, we can write

$$\begin{aligned} h_1(t) &= h_1(t+T) = h_1 = \alpha_1 \exp j\theta_1 \\ h_2(t) &= h_2(t+T) = h_2 = \alpha_2 \exp j\theta_2 \end{aligned}$$

where T is the symbol period. The received signals are

$$\begin{aligned} r_1 &= r(t) = h_1 c_1 + h_2 c_2 + \eta_1 \\ r_2 &= r(t+T) = -h_1 c_2^* + h_2 c_1^* + \eta_2, \end{aligned}$$

where r_1 and r_2 are the received signals at time t and $t+T$.

The combiner combines the received signals as follows:

$$\begin{aligned} \tilde{c}_1 &= h_1^* r_1 + h_2 r_2^* = (\alpha_1^2 + \alpha_2^2) c_1 + h_1^* \eta_1 + h_2 \eta_2^* \\ \tilde{c}_2 &= h_2^* r_1 - h_1 r_2^* = (\alpha_1^2 + \alpha_2^2) c_2 - h_1 \eta_2^* + h_2^* \eta_1 \end{aligned}$$

and sends them to the maximum likelihood detector, which minimizes the following decision metric

$$|r_1 - h_1 c_1 - h_2 c_2|^2 + |r_2 + h_1 c_2^* - h_2 c_1^*|^2$$

over all possible values of c_1 and c_2 . Expanding this, and deleting terms that are independent of the codewords, the above minimization reduces to separately minimizing

$$|r_1 h_1^* + r_2^* h_2 - c_1|^2 + (\alpha_1^2 + \alpha_2^2 - 1) |c_1|^2$$

for detecting c_1 and

$$|r_1 h_2^* - r_2^* h_1 - c_2|^2 + (\alpha_1^2 + \alpha_2^2 - 1) |c_2|^2$$

for decoding c_2 . Equivalently, using the notation $d^2(x, y) = (x - y)(x^* - y^*) = |x - y|^2$, the decision rule for each combined signal $\tilde{c}_j$, $j = 1, 2$ becomes: Pick c_i if and only if (iff)

$$(\alpha_1^2 + \alpha_2^2 - 1) |c_i|^2 + d^2(\tilde{c}_j, c_i) \leq (\alpha_1^2 + \alpha_2^2 - 1) |c_k|^2 + d^2(\tilde{c}_j, c_k), \quad \forall i \neq k \quad (1)$$

For PSK signals (equal energy constellations), this simplifies to

$$d^2(\tilde{c}_j, c_i) \leq d^2(\tilde{c}_j, c_k), \quad \forall i \neq k \quad (2)$$

A.2 Maximum Ratio Combining

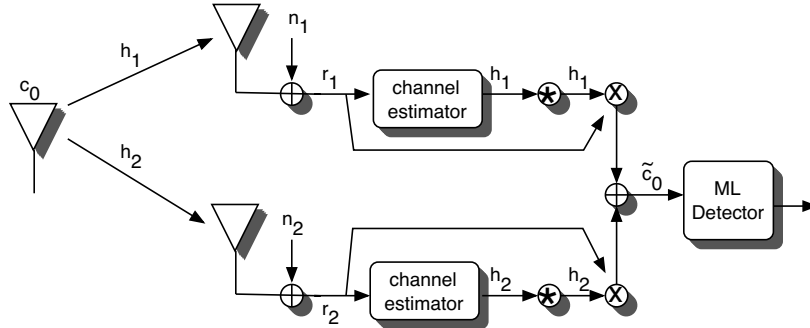


Fig. 3. Maximum Ratio Combining with 1 TX and 2 RX

In the case of maximum ratio combining (see Figure (3)), the resulting received signals are

$$\begin{aligned} r_1 &= h_1 c_0 + \eta_1 \\ r_2 &= -h_2 c_0 + \eta_2 \end{aligned}$$

and the combined signal is

$$\begin{aligned} \tilde{c}_0 &= h_1^* r_1 + h_2^* r_2 \\ &= (\alpha_1^2 + \alpha_2^2) c_0 + h_1^* \eta_1 + h_2^* \eta_2 \end{aligned}$$

The maximum likelihood detector decides signal c_i using exactly the same decision rule in (1), or in (2) for PSK signals.

Note that the MRC signal $\tilde{c}_0$ in (3) is equivalent to the resulting combined signals of the transmit diversity scheme in (1), except for a phase difference in the noise components which do not affect the effective SNR. This shows that the diversity order from Alamouti's two-antenna transmit diversity scheme is the same as that of the two-branch MRC.

A.3 Summary of Alamouti's scheme

Alamouti further extended this scheme to the case of 2 transmit antennas and m receive antennas, and showed that the scheme provided a diversity order of $2m$. Characteristics of this scheme include:

- no feedback from receiver to transmitter is required.
- no bandwidth expansion (as redundancy is applied in space across multiple antennas, not in time or frequency).
- low complexity decoders.
- identical performance as MRC if the total radiated power is doubled from that used in MRC, else if transmit power is kept constant, this scheme suffers a 3dB penalty in performance.
- added reliability as scheme includes a soft failure mode, where the transmitted signal may still be received with lower quality even if diversity gain was lost.
- no need for a complete redesign of existing systems to incorporate this diversity scheme.

Simulations in Alamouti's paper [7] also demonstrated a significant improvement in the error performance of uncoded coherent BPSK for maximum ratio combining and the new transmit diversity scheme in Rayleigh fading (under the assumption of independent fading gains).

B. General Construction

Tarokh et al. [2], [3] later generalized Alamouti's transmit diversity scheme to an arbitrary number of transmit antennas, and presented more complex space-time block codes akin to Alamouti's. These codes require no channel state information at the transmitter, achieve maximum-likelihood decoding through linear processing at the receiver, and exhibit maximum diversity. For real signal constellations (such as PAM), they are known to provide the maximum possible transmission rate allowed by the theory of STC [1]. For complex constellations, STBCs can be constructed for any number of transmit antennas, and they provide full spatial diversity and half of the maximum possible rate allowed by the theory of STC.

B.1 Real Orthogonal Designs

An STBC is defined by a $p \times n$ transmission matrix G , whose entries are linear combinations of $x_1, \dots, x_k$ and their conjugates $x_1^*, \dots, x_k^*$, and whose columns are pairwise-orthogonal.

In the case when $p = n$ and $\{x_i\}$ are real, G is a *linear processing orthogonal design* which satisfies the condition that $G^T G = \mathcal{D}$, where $\mathcal{D}$ is a diagonal matrix with the (i, i) th diagonal element of the form $(l_1^i x_1^2 + l_2^i x_2^2 + \dots + l_n^i x_n^2)$, with the coefficients $l_1^i, l_2^i, \dots, l_n^i > 0$. Without loss of generality, the first row of G contains entries with positive signs; if not, one can always negate certain columns of G to arrive at a positive first row. Examples include the 2×2 and 4×4 design:

$$G_2 = \begin{pmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{pmatrix} \text{ and } G_4 = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{pmatrix}.$$

B.2 Encoding and Decoding

We assume that transmission at the baseband employs a signal constellation $\mathcal{A}$ with 2^b elements. At the first time slot, nb bits arrive at the encoder and select constellation signals $c_1, \dots, c_n$. Setting $x_i = c_i$ for $i = 1, \dots, n$ in G yields a matrix C whose entries are linear combinations of the c_i and their conjugates. While G contains the indeterminates $x_1, \dots, x_n$, C contains specific constellation symbols (or linear combinations of them), which are transmitted from the n antennas as follows:

At time t , the entries of row t of C are simultaneously transmitted from the n antennas, with the i th antenna sending the i th entry of the row. So each row of C gives the symbols sent at a certain time, while each column of C gives the symbols sent by a certain antenna.

The rate of transmission is clearly b bits/s/Hz, and the diversity order of the above coding scheme is shown in [2] to be nm . The maximum-likelihood decoding scheme is entirely analogous to that of Alamouti's (see (1)), except in more general form: choose c_i among all the constellation symbols c if

$$c_i = \arg \min_{s \in \mathcal{A}} \left| \sum_{t=1}^n \sum_{j=1}^m r_t^j h_{\epsilon_t(i),j}^* \Delta_k(i) - s \right|^2 + \left(-1 + \sum_{k,l} |h_{k,l}|^2 \right) |c|^2,$$

where $\Delta_k(i)$ denotes the sign of x_i in the k th row of G , ϵ_k denotes the permutation of the k -th row of G , and $\epsilon_k(p) = q$ means that x_p is up to a sign change the (k, q) th element of G .

In the case when $p < n$, G is a *generalized orthogonal design* (see [2]) in the variables $x_1, x_2, \dots, x_k$ if it satisfies

$$\mathcal{G}^T \mathcal{G} = (x_1^2 + x_2^2 + \dots + x_k^2) I.$$

Transmission using a generalized orthogonal design is similar to the case when $p = n$, except that now kb bits are sent during each p transmissions. Since p time slots are used to transmit k symbols, the rate R of this coding scheme is defined to be $kb/pb = k/p$. The diversity order remains nm .

For a given R , n , $A(R, n)$ is defined to be the minimum number p such that there exists a $p \times n$ generalized orthogonal design with rate at least R . If no such orthogonal design exists, then $A(R, n) \triangleq \infty$. A generalized orthogonal design attaining the value of $A(R, n)$ is called delay-optimal.

The value of $A(R, n)$ is the fundamental question of generalized orthogonal design theory. The most interesting part of this question is the computation of $A(1, n)$ since the generalized orthogonal designs of full rate are bandwidth-efficient.

B.3 Construction

- *Definition:* A set of $n \times n$ real matrices $\{B_1, B_2, \dots, B_k\}$ is called a size k Hurwitz-Radon family of matrices if

$$B_i^T B_i = I$$

$$B_i^T = -B_i, \quad i = 1, \dots, k$$

and

$$B_i B_j = -B_j B_i, \quad 1 \leq i < j \leq k$$

- *Radon's Theorem:* Let $n = 2^a b$, where b is odd and $a = 4c + d$ with $0 \leq d < 4$ and $0 \leq c$. Any Hurwitz-Radon family of $n \times n$ matrices contains strictly less than $\rho(n) = 8c + 2^d$ matrices. Furthermore, $\rho(n) \leq n$. A Hurwitz-Radon family containing $n - 1$ matrices exists iff $n = 2, 4$, or 8 .
- *Lemma:* For any n , there exists a Hurwitz-Radon family of matrices of size $\rho(n) - 1$ whose members contain entries in the set $\{-1, 0, 1\}$.
- *STBC Construction:* Let $X = (x_1, x_2, \dots, x_p)$ and $n \leq \rho(p)$. Using the Hurwitz-Radon family of matrices and the lemma defined above, we can explicitly construct a set of integer $p \times p$ matrices with $\rho(p) - 1$ members $\{A_1, A_2, \dots, A_{\rho(p)-1}\}$. Let $A_0 = I$ and consider the $p \times n$ matrix G whose j th column is $A_{j-1} X^T$ for $j = 1, 2, \dots, n$. The Hurwitz-Radon conditions imply that G is a generalized orthogonal design of full rate. The full-rate generalized orthogonal design has entries of the form $\pm x_1, \dots, \pm x_p$.

An example of an real orthogonal design that is delay-optimal with rate one is

$$G_3 = \begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \end{pmatrix}.$$

B.4 Complex Orthogonal Designs

Complex orthogonal designs of size n are completely analogous to their real counterparts, except that it now contains entries $\pm x_1, \pm x_2, \dots, \pm x_n$, their conjugates $\pm x_1^*, \pm x_2^*, \dots, \pm x_n^*$ or multiples of these indeterminants by $i = \sqrt{-1}$. It is shown by simple construction in [2] that a complex orthogonal design of size n determines a real orthogonal design of size $2n$, and a corollary of this is that complex orthogonal designs only exist for $n = 2$ or 4 , since their real counterparts only exist for $n = 2, 4$ or 8 .

A *complex linear processing orthogonal design*, $\mathcal{G}$, is a complex orthogonal design such $\mathcal{G}^T \mathcal{G} = \mathcal{D}$, where $\mathcal{D}$ is a diagonal matrix with all diagonal entries linear combinations of $|x_1|^2, \dots, |x_n|^2$ with all strictly positive coefficients. Note that complex linear processing orthogonal designs only exist when $n = 2$. Hence, the STBC proposed by Alamouti, which uses the following complex linear orthogonal design:

$$\mathcal{G}_2 = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}$$

is somewhat unique.

For *generalized complex orthogonal designs*, $\mathcal{G}_c$ is a $p \times n$ matrix whose entries are

$$0, \pm x_1, \pm x_1^*, \dots, \pm x_k, \pm x_k^*$$

or their product with i , which satisfies the condition that $\mathcal{G}_c^T \mathcal{G}_c = \mathcal{D}_c$, where $\mathcal{D}_c$ is a diagonal matrix with the (i, i) th diagonal entry of the form $(l_1^i |x_1|^2 + l_2^i |x_2|^2 + \dots + l_n^i |x_n|^2)$ with the coefficients $l_1^i, l_2^i, \dots, l_n^i > 0$.

As with generalized real orthogonal designs, the existence of such codes is not completely understood. Tarokh et al. [2] constructed examples of rate 1/2 codes and rate 3/4 codes, but it remains an open question whether there exist any other generalized designs in higher dimensions with rate greater than 1/2, other than those listed below.

Examples of rate 1/2 STBCs for three or four transmit antennas include:

$$\mathcal{G}_3 = \begin{pmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{pmatrix} \text{ and } \mathcal{G}_4 = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{pmatrix}.$$

Examples of rate 3/4 STBCs for three or four transmit antennas include:

$$\mathcal{H}_3 = \begin{pmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\ \frac{x_2^*}{\sqrt{2}} & \frac{x_1^*}{\sqrt{2}} & \frac{x_1 - x_1^* + x_2 - x_2^*}{2} \\ \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} & \frac{x_2 + x_2^* + x_1 - x_1^*}{2} \end{pmatrix} \text{ and } \mathcal{H}_4 = \begin{pmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\ \frac{x_2^*}{\sqrt{2}} & \frac{x_1^*}{\sqrt{2}} & \frac{-x_1 - x_1^* + x_2 - x_2^*}{2} & \frac{-x_2 - x_2^* + x_1 - x_1^*}{2} \\ \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} & \frac{x_2 + x_2^* + x_1 - x_1^*}{2} & -\frac{x_1 + x_1^* + x_2 - x_2^*}{2} \end{pmatrix}.$$

Simulations were done in [3] using codes $\mathcal{G}_2$, $\mathcal{G}_3$, $\mathcal{G}_4$, $\mathcal{H}_3$ and $\mathcal{H}_4$ over uncoded M -PSK and M -QAM constellations. Significant gains were seen in the various performance curves, namely:

- At 10^{-5} bit error rate (BER) and 3 bits/s/Hz spectral efficiency, the rate 3/4 16-QAM code $\mathcal{H}_4$ gave ~ 7 dB gain over the use of an 8-PSK $\mathcal{G}_2$ code.
- At 10^{-5} bit error rate (BER) and 2 bits/s/Hz spectral efficiency, the rate 1/2 16-QAM code $\mathcal{G}_4$ gave ~ 5 dB gain over the use of an 4-PSK $\mathcal{G}_2$ code.
- At 10^{-5} bit error rate (BER) and 1 bits/s/Hz spectral efficiency, the rate 1/2 4-PSK code $\mathcal{G}_4$ gave ~ 7.5 dB gain over the use of an BPSK $\mathcal{G}_2$ code. (Interestingly, if the number of receive antennas m is increased, the gain drops to ~ 3.5 dB, due to much of the diversity gain already having been obtained using two Tx and two Rx.)

IV. SPACE-TIME TRELLIS CODES

A. Pairwise Error Probability

The pairwise error probability (PEP) is the probability that the maximum likelihood decoder selects as its estimate a signal $\mathbf{e} = e_1^1 e_1^2 \dots e_1^n e_2^1 \dots e_2^n e_l^1 \dots e_l^n$ when in fact the signal $\mathbf{c} = c_1^1 c_1^2 \dots c_1^n c_2^1 \dots c_2^n c_l^1 \dots c_l^n$ was transmitted. This occurs if, summing over all symbols, antennas, and time periods,

$$\sum_{t=1}^l \sum_{j=1}^m \left| r_t^j - \sum_{i=1}^n h_t^{i,j} c_t^i \right|^2 \geq \sum_{t=1}^l \sum_{j=1}^m \left| r_t^j - \sum_{i=1}^n h_t^{i,j} e_t^i \right|^2$$

which can be rewritten as

$$\sum_{t=1}^l \sum_{j=1}^m 2 \operatorname{Re} \left\{ r_t^{j*} \sum_{i=1}^n h_t^{i,j} (e_t^i - c_t^i) \right\} \geq \sum_{t=1}^l \sum_{j=1}^m \left| \sum_{i=1}^n h_t^{i,j} (e_t^i - c_t^i) \right|^2 \quad (3)$$

where $\operatorname{Re}\{\cdot\}$ refers to the real part of the argument.

With the ideal channel state information at the receiver and for a given instance of channel path gains $\{h_{i,j}\}$, the term on the right of (3) is a constant equal to $d^2(\mathbf{e}, \mathbf{c})$, and the term on the left is a zero-mean Gaussian random variable with variance $4\sigma^2 d^2(\mathbf{e}, \mathbf{c})$. Hence, the PEP conditioned on knowing $\{h_{i,j}\}$ is given by

$$\begin{aligned} P(\mathbf{c} \rightarrow \mathbf{e} \mid h_{i,j}, i = 1, \dots, n, j = 1, \dots, m) &= Q\left(\frac{d(\mathbf{c}, \mathbf{e})}{2\sigma}\right) \\ &\leq \exp(-d^2(\mathbf{c}, \mathbf{e})E_s/4N_0) \end{aligned}$$

where $Q(x)$ is the complementary error function given by $Q(x) = \int_x^\infty e^{-t^2/2} dt$.

Now, $d^2(\mathbf{c}, \mathbf{e})$ can be rewritten as $\sum_{i=1}^n \sum_{i'=1}^n h_{i,j} \overline{h_{i',j}} (c_t^i - e_t^i) \overline{(c_t^{i'} - e_t^{i'})}$ where $\overline{x}$ is the complex conjugate of x . Thus

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{t=1}^l \sum_{j=1}^m \sum_{i=1}^n \sum_{i'=1}^n h_{i,j} \overline{h_{i',j}} (c_t^i - e_t^i) \overline{(c_t^{i'} - e_t^{i'})} \quad (4)$$

If we denote $\Omega_j = (h_{1,j}, h_{2,j}, \dots, h_{n,j})$, we can rewrite

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{j=1}^m \Omega_j A \Omega_j^\dagger, \quad (5)$$

where $\Omega_j^\dagger$ denotes the Hermitian transpose of Ω_j and $A = A(\mathbf{e}, \mathbf{c})$ is an $n \times n$ matrix independent of time and contains entries $A_{p,q} = \sum_{t=1}^l (c_t^p - e_t^p) \overline{(c_t^q - e_t^q)}$. So, the PEP becomes

$$P(\mathbf{c} \rightarrow \mathbf{e} \mid h_{i,j}, i = 1, \dots, n, j = 1, \dots, m) \leq \prod_{j=1}^m \exp\left(-\frac{\Omega_j A \Omega_j^\dagger E_s}{4N_0}\right) \quad (6)$$

Since A is Hermitian, there exists a unitary matrix V that satisfies $VV^\dagger = I$ and a real diagonal matrix D such that

$$VAV^\dagger = D.$$

The rows $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ of V are the eigenvectors of A , and form a complete orthonormal basis of an n -dimensional vector space. Furthermore, the diagonal elements of D are the eigenvalues λ_i , $i = 1, 2, \dots, n$ of A including multiplicities, and are nonnegative real numbers since A is Hermitian. By the construction of A , the *codeword difference matrix*, B , where

$$B(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdot & \cdot & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdot & \cdot & e_l^2 - c_l^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ e_1^n - c_1^n & e_2^n - c_2^n & \cdot & \cdot & e_l^n - c_l^n \end{pmatrix},$$

is a square root of A , i.e., $A = BB^\dagger$.

Next, we express $d^2(\mathbf{c}, \mathbf{e})$ in terms of the $\{\lambda_i\}$'s. Let $(\beta_{1,j}, \beta_{2,j}, \dots, \beta_{n,j}) = \Omega_j V^\dagger$, then

$$\Omega_j A \Omega_j^\dagger = \Omega_j V^\dagger D V \Omega_j^\dagger = \sum_{i=1}^n \lambda_i |\beta_{i,j}|^2. \quad (7)$$

Since $h_{i,j}$ are samples of a complex Gaussian random variable with mean $Eh_{i,j}$, let

$$K^j = (Eh_{1,j}, Eh_{2,j}, \dots, Eh_{n,j}).$$

Since V is unitary, this implies $\beta_{i,j}$ are independent complex Gaussian random variables with variance 0.5 per dimension and with mean $K^j \cdot v_i$.

B. Error Probability on Slow Fading Channels

In the case of flat Rayleigh fading, $Eh_{i,j} = 0$ for all i and j . To obtain an upper bound on the average probability of error, we average

$$\prod_{j=1}^m \exp\left(-\left(E_s/4N_0\right) \sum_{i=1}^n \lambda_i |\beta_{i,j}|^2\right) \quad (8)$$

with respect to the independent Rayleigh distributions of $|\beta_{i,j}|$ with probability density $p(|\beta_{i,j}|) = 2|\beta_{i,j}| \exp(-|\beta_{i,j}|^2)$. Letting $c = E_s/4N_0$, we get

$$E \prod_{j=1}^m e^{-c \sum_{i=1}^n \lambda_i |\beta_{i,j}|^2} = \prod_{j=1}^m \prod_{i=1}^n E(e^{-c \lambda_i |\beta_{i,j}|^2})$$

by independence, and

$$\begin{aligned}
 E(e^{-c\lambda_i|\beta_{i,j}|^2}) &= 2 \int_0^\infty e^{-c\lambda_i w^2} w e^{-w^2} dw \\
 &= 2 \int_0^\infty w e^{-w^2(1+c\lambda_i)} dw \\
 &= \frac{1}{1+c\lambda_i} \int_0^\infty u e^{-u} du \\
 &= \frac{1}{1+c\lambda_i},
 \end{aligned}$$

So,

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \left(\frac{1}{\prod_{i=1}^n (1 + \frac{E_s}{4N_0} \lambda_i)} \right)^m \quad (9)$$

Let r be the rank of A . Then the eigenvalue 0 has multiplicity $n - r$. Let the nonzero eigenvalues of A be $\lambda_1, \lambda_2, \dots, \lambda_r$. Then for high enough SNR, $(1 + \frac{E_s}{4N_0} \lambda_i) \approx \frac{E_s}{4N_0} \lambda_i$, and the PEP in (9) becomes

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq (\prod_{i=1}^r \lambda_i)^{-m} \left(\frac{E_s}{4N_0} \right)^{-rm} = \left(\prod_{i=1}^r \lambda_i^{1/r} \right)^{-rm} \left(\frac{E_s}{4N_0} \right)^{-rm} \quad (10)$$

Thus a *diversity advantage* of mr and a *coding advantage* of $(\prod_{i=1}^r \lambda_i)^{1/r}$ are obtained. Note that diversity advantage is defined as the power of the SNR in the denominator of the right hand expression of (10); and coding advantage is intuitively an estimate of the gain over an uncoded system with the same diversity advantage.

C. Design criteria for Slow Rayleigh Fading STTCs

- *The Rank Criterion:* To obtain maximal diversity, we need to maximize the minimum rank r of the matrix B over all pairs of distinct codewords. A diversity advantage of rm is achieved.
- *The Determinant Criterion:* Let rm be the target diversity advantage. Then the design goal is to maximize the minimum determinant $\prod_{i=1}^r \lambda_i$ of the matrix A along the pairs of distinct codewords with minimum rank.

D. Case of Fast Rayleigh Fading

The analysis for slow fading channels in the previous sections can be directly applied to fast fading channels. At each time t , we define a *space-time symbol difference vector*, $F(\mathbf{c}, \mathbf{e})$ as

$$F(\mathbf{c}, \mathbf{e}) = [c_t^1 - e_t^1, c_t^2 - e_t^2, \dots, c_t^n - e_t^n]^T \quad (11)$$

Following the derivation in [1], we consider an $n \times n$ matrix $C = C(\mathbf{c}, \mathbf{e})$, defined as $C = F(\mathbf{c}, \mathbf{e})F^\dagger(\mathbf{c}, \mathbf{e})$.

It is clear that C is Hermitian, so there exists a unitary matrix V_t and a real diagonal matrix D_t such that

$$V_t C V_t^\dagger = D_t.$$

The diagonal entries of D_t , $\{D_t^i, i = 1, \dots, n\}$ are the eigenvalues of C ; the rows of V_t , $\{v_t^i, i = 1, \dots, n\}$ are the eigenvectors of C , which form a complete orthonormal basis of an n -dimensional vector space.

Note that C is a rank 1 matrix if $\mathbf{c} \neq \mathbf{e}$, and is rank 0 otherwise. It follows that $n-1$ elements in the list $\{D_t^i, i = 1, \dots, n\}$ are zeros, and WLOG, we can let the single nonzero element in this list be D_t^1 , with corresponding eigenvector v_t^1 . Since expressions for $d^2(\mathbf{c}, \mathbf{e})$ in (4), (5) are still valid here, we can directly substitute D_t^1 into equation (9), and obtain the PEP in the case of fast Rayleigh fading:

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{i=1}^n \left(1 + D_t^i \frac{E_s}{4N_0}\right)^{-m} \quad (12)$$

Let $\mathcal{V}(\mathbf{c}, \mathbf{e})$ denote the set of time instances $1 \leq t \leq l$ such that $D_t^1 \neq 0$, and let $|\mathcal{V}(\mathbf{c}, \mathbf{e})|$ denote the number of elements of $\mathcal{V}(\mathbf{c}, \mathbf{e})$. Then it follows from (12) that

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{t \in \mathcal{V}(\mathbf{c}, \mathbf{e})} \left(D_t^1 \frac{E_s}{4N_0}\right)^{-m} \quad (13)$$

It follows that a diversity of $m|\mathcal{V}(\mathbf{c}, \mathbf{e})|$ is achieved. Examining the coefficient of $(E_s/4N_0)^{-m|\mathcal{V}|}$ leads to the following design criterion.

E. Design criteria for Fast Rayleigh Fading STTCs

- *The Distance Criterion:* To obtain a diversity of rm , we require for any two codewords $\mathbf{c}$ and $\mathbf{e}$, that the Hamming distance between $\mathbf{c}$ and $\mathbf{e}$ be at least r .
- *The Product Criterion:* Let $\mathcal{V}(\mathbf{c}, \mathbf{e})$ be the set of time instances such that $c_1^t c_2^t \dots c_t^n \neq e_1^t e_2^t \dots e_t^n$ and let $D_t^1 = \sum_{i=1}^n |c_t^i - e_t^i|^2$. To obtain maximal coding advantage in a fast fading environment, we maximize the minimum of the products

$$\prod_{t \in \mathcal{V}(\mathbf{c}, \mathbf{e})} |D_t^1|^2$$

over distinct codewords $\mathbf{c}$ and $\mathbf{e}$.

F. Encoding/Decoding of STTCs for Quasi-Static Flat Fading Channels

The encoding for STTCs are similar to TCM, except that at the beginning and the end of each frame, the encoder is required to be in the zero state. At each time t , depending on the state of the encoder and the input bits, a transition branch is selected. If the label of the transition branch is $c_t^1, c_t^2, \dots, c_t^n$, then transmit antenna i is used to send the constellation symbols c_t^i , $i = 1, 2, \dots, n$ and all these transmissions are in parallel. An example of a STTC encoder is shown in Figure (IV-F) for 4-PSK. The encoder coefficient set, denoted by

$$\mathbf{g}^i = [(g_{0,1}^i, g_{0,2}^i, \dots, g_{0,n}^i), (g_{1,1}^i, g_{1,2}^i, \dots, g_{1,n}^i), \dots, (g_{\nu_i,1}^i, g_{\nu_i,2}^i, \dots, g_{\nu_i,n}^i)] \quad (14)$$

is usually found next to the trellis diagram of the trellis code. Each $g_{l,k}^i$ is an element of the 4-PSK constellation set $\{0, 1, 2, 3\}$, and ν_i is the memory order of the i th shift register. Multiplier outputs are added modulo 4.

Assuming that r_t^j is the received signal at antenna j at time t , the branch metric is given by

$$\sum_{j=1}^m \left| r_t^j - \sum_{i=1}^n h_{i,j} q_t^i \right|^2.$$

The Viterbi algorithm is used to compute the path with the lowest metric. In the absence of ideal channel state information, an analysis in [9] gives the appropriate branch metrics for decoding.

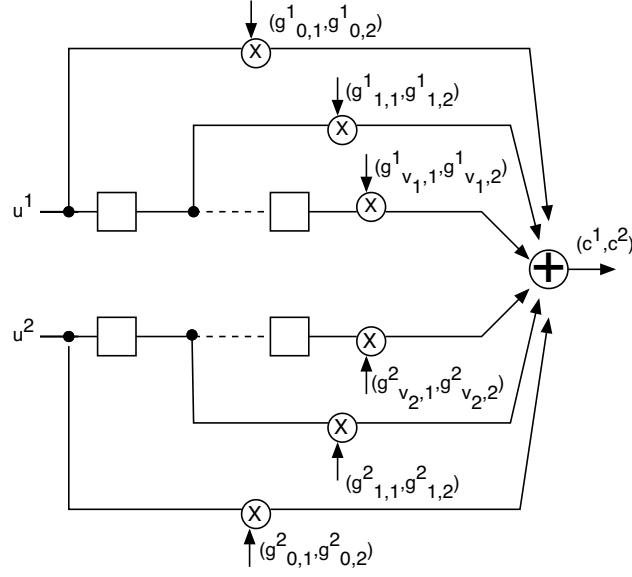


Fig. 4. Space-time trellis code encoder for 4-PSK

G. Code construction for Quasi-Static Flat Fading Channels

Note that:

- If a space-time trellis code guarantees a diversity advantage of r for the quasistatic flat fading channel model (given one receive antenna), then it is an r -STTC.
- The constraint length of an r -STTC is at least $r - 1$.
- If the diversity advantage is nm then the transmission rate is at most b bits/s/Hz with a 2^b signal constellation. Thus 4-PSK, 8-PSK, and 16 QAM will be upper bounded by 2, 3, 4 bits/s/Hz respectively.
- If b is the transmission rate, the trellis complexity is at least $2^{b(r-1)}$.
- A STTC is shown to be geometrically uniform (see Forney [10]) and its performance is independent of the transmitted codeword.

G.1 Example using 4-PSK

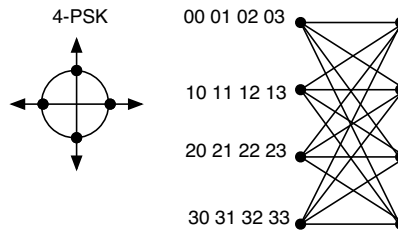


Fig. 5. 2-space-time trellis code, 4-PSK, 4 states, 2bits/s/Hz

For 4-PSK, signal points are labeled in $\mathbb{Z}_4$. The edge label is denoted by c_1c_2 , where c_1 is transmitted from antenna 1 and c_2 is transmitted from antenna 2. With (b_k, a_k) as the input sequence

of binary bits, the output signal pair $c_1^k c_2^k$ at time k is given by

$$\begin{aligned} (c_1^k, c_2^k) &= b_{k-1}(2, 0) + a_{k-1}(1, 0) + b_k(0, 2) + a_k(0, 1) \mod 4 \\ &= ((2b_{k-1} + a_{k-1}) \mod 4, (2b_k + a_k) \mod 4) \end{aligned}$$

For the diversity advantage to be 2 (to qualify as a 2-space-time code), the rank of $B(\mathbf{c}, \mathbf{e})$ has to be 2. This can be seen from Figure (5) or the algebraic formulation above, since if the paths corresponding to codewords $\mathbf{c}$ and $\mathbf{e}$ diverge at time t_1 and remerge at time t_2 , then the vectors $(e_1^{t_1} - c_1^{t_1}, e_2^{t_1} - c_2^{t_1})$ and $(e_1^{t_2} - c_1^{t_2}, e_2^{t_2} - c_2^{t_2})$ are linearly dependent (with $(e_1^{t_1} - c_1^{t_1} = e_2^{t_1} - c_2^{t_1}) = 0$, $e_1^{t_2} - c_1^{t_2} \neq 0$ and $e_2^{t_2} - c_2^{t_2} \neq 0$).

To compute the coding advantage, we need to find codewords $\mathbf{c}$ and $\mathbf{e}$ such that the determinant

$$\det \left(\sum_{k=1}^l (e_k^1 - c_k^1, e_k^2 - c_k^2)^\dagger (e_k^1 - c_k^1, e_k^2 - c_k^2) \right) \quad (15)$$

is minimized. Using the theorem that the code is geometrically uniform, we can assume WLOG that $\mathbf{c}$ is the all-zeros codeword.

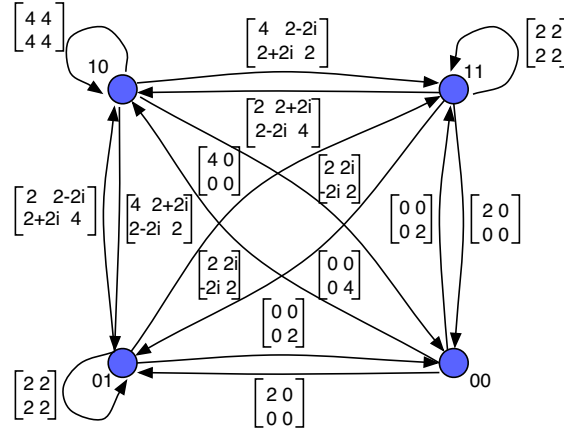


Fig. 6. State diagram for 4-PSK example

We can attack (15) by replacing the edge label (c_1, c_2) by the complex matrix

$$\begin{pmatrix} (\mathbf{j}^{-c_1} - 1)(\mathbf{j}^{c_1} - 1) & (\mathbf{j}^{c_1} - 1)(\mathbf{j}^{-c_2} - 1) \\ (\mathbf{j}^{-c_1} - 1)(\mathbf{j}^{c_2} - 1) & (\mathbf{j}^{c_2} - 1)(\mathbf{j}^{-c_2} - 1) \end{pmatrix}.$$

See Figure (6) for the corresponding labeling. Diverging from the zero state contributes a matrix of the form

$$\begin{pmatrix} 0 & 0 \\ 0 & t \end{pmatrix},$$

and remerging to the zero state contributes a matrix of the form

$$\begin{pmatrix} s & 0 \\ 0 & 0 \end{pmatrix}$$

with $s, t \geq 2$. Thus, (15) can be written as

$$\det \left(\begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix} + \begin{pmatrix} a & b \\ \bar{b} & d \end{pmatrix} \right)$$

with $a, d \geq 0, |b|^2 \leq ad$. So the minimum determinant is 4.

The *design rules* that guarantee diversity for the 4-PSK and 8-PSK code are:

- Transitions departing from the same state differ in the second symbol.
- Transitions arriving at the same state differ in the first symbol.

Simulations were done in [1] for the 4-PSK codes at spectral efficiency 2 bits/s/Hz and for the 8-PSK code at spectral efficiency 3 bits/s/Hz, both assuming two transmit antennas and one receiver antenna. Results showed a diversity advantage of two for both codes, and that they performed within 2.5dB of the outage capacity given by Foschini et al. [11]. Results also implied that the coding advantage obtained by increasing the number of state increases with the number of receive antennas. Interestingly, the coding advantage over the 4-state code failed to be as large as that predicted by the determinant criterion. This is clearly an avenue of research, as we outline in §VI.

V. STBC VERSUS STTC IN PERFORMANCE

From the previous sections, we have learnt that space-time block codes and space-time trellis codes are two very different transmit diversity schemes. Space-time block codes are constructed from known orthogonal designs, achieves full diversity, are easily decodable by maximum likelihood decoding via linear processing at the receiver, but suffers from a lack of coding gain. On the other hand, space-time trellis codes possess both diversity and coding gain, yet is complex to decode (since maximum likelihood sequence estimation is required), and arduous to design. In both cases, however, code design for a large number of transmit antennas remains an open question.

As for performance comparison between the two schemes, it is only fair to use concatenated STBC since STBC inherently lacks coding gain. Concatenated codes that have been used so far include AWGN Trellis codes [8] or turbo codes [12]. An attempt to provide such a comparison is found in [13], but the comparison was unfair in terms of the data rate and computational complexity. A fair comparison was given in [14], but only for the fast fading channel. Bauch concatenated turbo codes with STBC in [12], but did not provide a performance comparison with STTC. Sumeet et al. [8] later attempted a fair comparison of the performance of STBC with STTC over the flat fading quasi-static channel, presenting results in terms of the frame error rate (FER), while keeping the transmit power and spectral efficiency constant. It is her results that this section discusses.

A. Performance Analysis

In general, any STC can be analysed in the same way as STTC using diversity advantage and coding advantage. The diversity advantage and coding advantage affect the performance curve differently. Diversity advantage causes the slope of the FER versus SNR graph to change in the following way: the larger the diversity, the more negative the slope. Coding advantage shifts the graph horizontally: the greater the coding advantage, the larger is the left shift. In Sumeet's paper [8], full diversity codes were used, and so the slopes of their FER graphs were identical.

To see the contribution of the coding advantage in the high SNR regimes of interest (typically 4 to 18 dB), we first take the logarithm of the PEP expression in (10) for the k th code. This yields $P_k = \log(\text{PEP}) \approx -mns_k - mnc_k$, where mn is the full diversity advantage, $s_k = \log(\frac{E_s}{4N_0})$ is the SNR term, and $c_k = \log(\prod_{i=1}^n \lambda_i^{1/n})$ is the coding advantage term. If we let $\delta_P = P_k - P_l$,

$\delta_c = c_k - c_l$, and $\delta_s = s_k - s_l$ for the k th and l th code, then

$$\delta_P = -mn\delta_s - mn\delta_c \quad (16)$$

If k is a better code, then $\delta_c > 0$. At a given SNR, $\delta_s = 0$, and the PEP for k is less than that for l by $\delta_P \approx mn\delta_c$. Clearly, this difference increases with m , the number of receive antennas. Thus, the coding advantage improves further when more receive antennas were used.

B. Simulation Results

Using full diversity codes with a fixed number of trellis states (either 4 or 8) with two Transmit Antenna and at spectral efficiency of 2 bits/s/Hz, the following results are observed:

- With the same number of trellis states, concatenated STBCs outperforms STTCs at SNRs of interest, namely 4 to 12 dB for the case of two receivers, and 10 to 18dB for the case of one receiver.
- In the case of one receive antenna, it was noted that STBC by itself had a performance almost on par with all the STTCs used (notably the STTC provided by Tarokh et al. [1]) despite having no coding gain. This is attributed to the multidimensional structure of STBCs, since each codeword spans two time symbols and averages over noise.
- With increasing number of antennas and trellis states, STTC begins to outperform concatenated STBC.

Future work includes looking at block codes that preserve capacity [15], which may improve the performance of concatenated STBCs. It would also be particularly useful to have a more unified framework for concatenated space-time systems.

VI. RECENT DEVELOPMENTS OF SPACE-TIME CODES

A. Newer STTCs

Since the advent of space-time coding schemes, there has been rapid progress in the field, targeted at finding better codes with full diversity advantage and with greater coding advantage than those provided in [1]. Baro et al. [16] reported improved STTCs that were found through exhaustive computer search over a feedforward convolutional coding (FFC) generator; Ionescu et al. [17] reported improved 8 and 16 state STCs for 4-PSK in the case of two transmitters in flat Rayleigh fading via a modified determinant criterion. Similarly, Yuan et al. [18] derived a more accurate code design criteria by using a tighter bound for the $Q(\cdot)$ function in the PEP approximation in (??), and yielded new STTCs with better performance than the original Tarokh codes. This helped to explain why simulations for STTCs over a 4-state 4-PSK code in [1] produced a coding advantage that was lower than what the determinant criterion predicted. Also, a more structured method of code construction that ensures full diversity is provided in [19], along with a number of new code designs, such as codes that yield the best distance spectrum properties among all codes with a given coding advantage [20].

B. Correlated Fading Channels

The issue of receive and transmit correlation in physically motivating multiple-input-multiple-output (MIMO) systems on error performance of STCs was raised by Böleskei et al. [21]. They showed that the resulting maximum diversity order was given by the ranks of the receive and transmit correlation matrices. Further work has been undertaken to study the performance of STBCs and STTCs and to develop robust codes for correlated fading channels [22].

C. Multilevel Designs

There has also been much interest in multilevel designs of STCs, since they appear to be an efficient means of constructing and decoding codes when the number of antennas is large. Some recent work in this area includes investigating systematic designs of space-time codes employing multiple trellis coded modulation [23].

D. Channel State Information

There are several interesting questions raised when we remove the assumption of CSI at the receiver in the space-time coding schemes introduced by Alamouti [7], and Tarokh et al. [1], [2], [3]. For instance, what is the transmitter diversity gain when the receiver has no CSI or has no knowledge of the number of transmitters? Much motivation of this work stems from the desire to create STC systems for channels that are known neither to the transmitter nor to the receiver. Recently, a step in this direction was undertaken by Hochwald et al. [24], who proposed a systematic method for designing unitary space-time constellations for MIMO systems. Unitary space-time signals are orthonormal in time across the antennas, and have been shown to be well-suited to a Rayleigh fading channel where neither the transmitter nor the receiver knows the fading channel gains.

E. Applications in OFDM

Another active area is the incorporation of space-time coding schemes into orthogonal frequency division multiplexing (OFDM). OFDM is commonly used for high data rate wireless applications due to its inherent ability to combat intersymbol interference (ISI). To achieve optimum performance with such systems, some form of diversity (time, frequency, channel coding) is often used. Recent work has shown that the performance of OFDM systems using STBCs yielded significant gains (~ 5 dB) [25] in the absence of any time-interleaving, and hence would be particularly attractive for delay-sensitive applications [26].

F. Concatenation of STBCs

For STBCs, coding gain is only achieved if it is concatenated with an outer code, such as a TCM code [8] or a Turbo-TCM Code [12]. This was mentioned in §V and is an ongoing, exciting area of research.

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