

Performance Analysis of K -Tier Cellular Networks with Time-Switching Energy Harvesting

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Abstract—Dense heterogeneous cellular networks (HCNs) with energy harvesting nodes are promising solutions to meet both the capacity and energy efficiency needs in the next generation cellular networks. This paper studies the system performance of both energy harvesting and information receiving in a K -tier cellular networks with time-switching simultaneous wireless information and power transfer (SWIPT). Based on stochastic geometry theory, we derive the closed-form formulas for the average harvested energy, the average transmission rate and the total power and information gains. Moreover, we propose a scheme that can optimally adjust the bias factors of tier selection while leveraging the system between energy harvesting and information receiving.

Index Terms—Poisson Point Process, K -tier heterogeneous cellular network, energy harvesting, SWIPT.

I. INTRODUCTION

With the rapid development of smart phones, tablets and machine-to-machine (M2M) devices, the number of global mobile-connected devices has grown to 7.9 billion in 2015 and will be over 11.6 billion by 2020 [1]. This brings significant challenges in the design of next generation cellular networks for enhanced capacity and energy efficiency. To meet the huge capacity demand, different types of base stations (BSs) are deployed with high density [2]. Meanwhile, energy harvesting is an effective solution to improve the energy efficiency of HCNs and to prolong the life-time of mobile devices [3]. Investigating HCNs with energy harvesting capability is thus critical to meet the growth of mobile devices.

Modeling the HCNs by multiple spatially and spectrally coexisting K -tiers has been advocated in recent works. The average ergodic rate of the typical user and the minimum average user throughput were derived in K -tier HCNs where load balancing is performed by biasing the user association among different layers of BSs [4]. Moreover, the downlink rate distribution of the K -tier HetNet was analyzed taking into account the shadowing effect and the optimal bias for maximizing the transmission rate was investigated in [5]. Meanwhile, some researches on K -tier energy harvesting HCNs or hybrid networks have been made recently. When user

terminals use dynamic power splitting technique for energy harvesting, the optimal ratio of power splitting was investigated in [6] for maximizing the average harvested energy of K -tier HCNs. The outage probability of uplink user terminals was studied in the hybrid networks in which random power beacons were applied to transfer power to mobiles [7].

According to the measurement results, the traffic of dense heterogeneous cellular networks exhibits high fluctuations both in time and over space [8]. Heterogeneous BS can provide the opportunity to utilize more power for energy harvesting when the traffic load of BS is low. Despite the previous works on K -tier energy harvesting HCNs, time-switching simultaneous wireless information and power transfer (SWIPT) in K -tier HCNs has been rarely addressed. To bridge the gap, this paper analyzes the system performances of both energy harvesting and information receiving for a general K -tier HCNs based on stochastic geometry. For typical user terminal in K -tier heterogeneous cellular network, the trade-off between energy harvesting and information receiving can be adjusted with the change of the bias of BSs in different tiers. Based on the derivation results, we further propose an optimization framework to efficiently leverage the system between information transmission and energy harvesting purposes by time switching.

Our contributions can be summarized as

- The average harvested energy of user terminals in K -tier HCNs are derived in closed-form expressions. The impacts of the threshold of energy harvesting and the bias of tier selection are revealed.
- A total gain scheme of time-switching based SWIPT is proposed to adjust the bias of tier selection. The impacts of the bias factor and the density of BSs on the information transmission and energy harvesting performance are analyzed.

The rest of this paper is organized as follows. In Section II, the system model based on the stochastic geometry is presented. The expression of average harvested energy are derived in Section III, where the impact of the tier selection bias is also studied. In Section IV, the expression of average information transmission is shown and the total gain scheme based on the information transmission and energy harvesting is proposed. In Section V, the impact of the bias of tier selection

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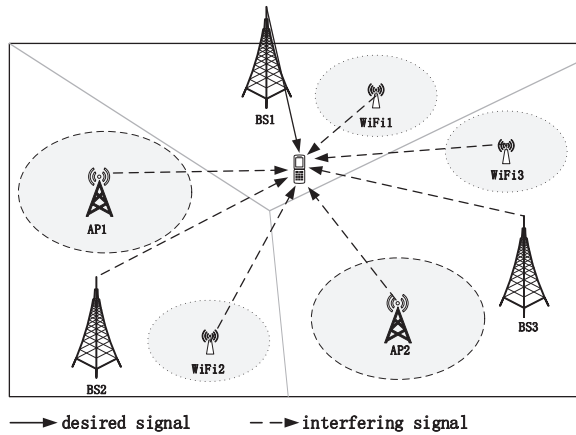


Fig. 1. A typical K -tier heterogeneous cellular networks model.

on the energy harvesting and the transmission rate is evaluated via numerical results. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a HCN consisting of K tiers of BSs. The BSs of different tiers are independent. The BSs in tier k is modeled by a homogeneous Poisson Point Process (PPP) Φ_k with intensity λ_k , where $k \in \{1, \dots, K\}$. Each BS in tier k operates at a fixed transmit power P_k . The bias factors are adopted to adjust the receiving power of user terminals to balance the load among K tiers BSs.

The transmission from the BS in tier k to the user terminal suffers from the multipath fading h_k and the shadowing effect s_k , where h_k and s_k are random variables. We consider Rayleigh fading, i.e., $h_k \sim \exp(1)$, and log-normal shadowing, i.e., $s_k \sim LN(\mu_k, \sigma_k^2)$, where μ_k and σ_k are the mean and standard deviation, respectively. As a result, the instantaneous biased-receiving-power (BRP) P_r at a typical user terminal from the K -th tier BSs is given as $P_r = P_k L_0 h_k s_k (R_k/r_0)^{-\alpha} B_k$, where L_0 is the path loss at the reference distance r_0 , typically $L_0 = -31.54$ dB when $r_0 = 1$ m. α is the path loss exponent, which is assumed to be the same for all tiers. B_k is the bias factor of the k -th tier BS. R_k is the distance between the typical user terminal and its connected BS in the k -th tier. Here, multipath fading does not affect the BS association since it changes over a much faster timescale compared with cell selection. The user terminals associate with the BS in tier k , from which the long-term average BRP $P_{r,k}$ is maximized.

$$P_{r,k} = P_k L_0 h_k s_k (R_k/r_0)^{-\alpha} B_k, \quad (1)$$

Due to the highly dense network deployment, the HCN downlink is interference-limited, in which the noise power is just negligible compared with the interference power caused by co-tier and inter-tier BSs. Denote the serving BS in the k -th tier by b_{k0} . The signal-to-interference ratio (SIR) at the typical user terminal is then given by

$$SIR_k = \frac{P_r}{I_k} = \frac{P_k h_k s_k R_k^{-\alpha}}{\sum_{j=1}^K \sum_{i \in \Phi_j \setminus b_{j0}} P_i h_i s_i R_i^{-\alpha}}, \quad (2)$$

where $I_k = \sum_{j=1}^K \sum_{i \in \Phi_j \setminus b_{j0}} P_i L_0 h_i s_i (R_i/r_0)^{-\alpha}$ denotes the inter-cell interference power.

However, the user terminals can harvest energy from interfering BSs of all tiers. We define A_k as the probability that a user terminal is associated with the k -th tier BSs. Consequently, the average harvested energy at the user terminal conditioned on A_k can be expressed as

$$P_{EH_k} = E \left[\sum_{j=1}^K \sum_{i \in \Phi_j} P_i L_0 s_i (R_i/r_0)^{-\alpha} | A_k \right], \quad (3)$$

where the expectation is taken with respect to the distribution of $\{s_k\}$. Based on the law of total probability, the average harvested energy of the whole network can be obtained as

$$P_{EH} = \sum_{k=1}^K P_{EH_k} \cdot A_k. \quad (4)$$

III. ANALYSIS OF ENERGY HARVESTING PERFORMANCE UNDER THRESHOLD CONSTRAINT

This section presents the average harvested energy expressions with the threshold constraint. When user terminals connect to the k -th tier serving BS based on the maximum long-term average BRP, the probability that a typical user terminal connects to a k -th tier BS described in Lemma 1.

Lemma 1 (Selection probability). The probability A_k , i.e., a typical user terminal connects to a k -th tier BS, is given by

$$A_k = \frac{\lambda_k E \left[s_k^{2/\alpha} \right] B_k^{2/\alpha} P_k^{2/\alpha}}{\sum_{j \in K} \lambda_j E \left[s_j^{2/\alpha} \right] B_j^{2/\alpha} P_j^{2/\alpha}}. \quad (5)$$

Proof: Due to page limits, please refer to [5, Lemma 2] for a similar proof. Based on Lemma 1, the selection probability can be affected by various system parameters, such as the transmission power, the shadowing effect, the path loss exponent and bias factor, because these parameters change the BRP of the typical user terminal in (1). ■

For a user terminal connected with the k -th tier serving BS, the distance between the user terminal and its serving BS is denoted by random variable X_k . Considering that the distribution of BSs in the k -th tier obeys the homogeneous PPP, the probability density function (pdf) of X_k , denoted by $f_{X_k}(x)$, can be analyzed by stochastic geometry theory.

Lemma 2 (Distance distribution). The pdf of the distance X_k between the typical user terminal and its serving BS is given by

$$f_{X_k}(x) = \frac{2\pi \lambda_k E \left[s_k^{2/\alpha} \right]}{A_k} x \exp \left(-\pi \sum_{j=1}^K \lambda_j E \left[s_j^{2/\alpha} \right] B_j^{2/\alpha} x^2 \right). \quad (6)$$

Proof: The impact of shadowing in cell selection is investigated based on the displacement theorem in [9]. In particular, we first rewrite (1) as $P_{r,k} = P_k L_0 \left(\left\| s_k^{-1/\alpha} \left\| R_k / r_0 \right\|^{-\alpha} B_k \right\| \right)$, in which the large scale shadowing effect can be interpreted as a displacement of the locations of BSs in the k -th tier. Given that the BSs in the k -th tier obey the homogeneous PPP Φ_k with density λ_k , all BSs are independently displaced by the function $y = s_k^{-1/\alpha} x$, where y is the displaced location of BSs. According to the displace theorem, Φ'_k is defined as the new point process where the points are located at $\{y\}$. Φ'_k is also homogeneous PPP when $E \left[s_k^{-1/\alpha} \right] < \infty$.

For a lognormal shadowing s_k , the fractional moment $E \left[s_k^{2/\alpha} \right] = \exp \left(\frac{\ln 10}{5} \frac{\mu_k}{\alpha} + \frac{1}{2} \left(\frac{\ln 10}{5} \frac{\sigma_k}{\alpha} \right)^2 \right)$ is finite when both μ_k and σ_k are finite. The density of the new point process Φ'_k is given by $\lambda'_k = \lambda_k E \left[s_k^{2/\alpha} \right]$. Based on Lemma 3 in [4] and the fact that the density of Φ'_k is $\lambda'_k = \lambda_k E \left[s_k^{2/\alpha} \right]$, the pdf of X_k is readily available, which completes the proof. ■

When the typical user terminal harvests energy from the BSs, the receiving power should exceed a threshold that depends on the sensitivity of the power receiver [10]. We assumed that the threshold of the user terminals is P_{EH_min} . According to (1), we can obtain the maximum power transfer distance is obtained as $r_{EH_max} = r_0 \left(\frac{P_{EH_min}}{P_k L_0 E[s_k] B_k} \right)^{-\frac{1}{\alpha}}$ when the typical user terminal is associated with the BSs in the k -th tier. This means that the location of the BSs, from which the typical user terminal can harvest energy, is limited in a circle of radius r_{EH_max} . Based on Lemma 2, the average harvested energy P_{EH_k} from the BSs in the k -th tier can be obtained as the following lemma.

Lemma 3 (Average harvested energy). The average harvested energy P_{EH_k} in the k -th tier is given by

$$P_{EH_k} = \sum_{j=1}^K \lambda_j E[s_j] \pi r_0^2 (P_j L_0 E[s_j])^{\frac{2}{\alpha}} \times \left(P_{EH_min}^{1-\frac{2}{\alpha}} + \frac{\alpha}{\alpha-2} \left((E[p(a)])^{1-\frac{2}{\alpha}} - P_{EH_min}^{1-\frac{2}{\alpha}} \right) \right), \quad (7)$$

with

$$E[p(a)] = P_k L_0 E[s_k] B_k \left(\frac{\lambda_k E[s_k^{\frac{2}{\alpha}}]}{A_k \sum_{j=1}^K \lambda_j E[s_j^{\frac{2}{\alpha}}] B_j^{\frac{2}{\alpha}}} \right)^{-\alpha}. \quad (8)$$

Proof: Since multi-path fading can be averaged in the frequency selective channel, it does not impact the energy harvesting. The total power received at the typical user terminal from the BSs in the circle of radius b is given as $P = \sum_{x \in \Phi_k \cap B(0,b)} p(x)$, where $B(0,b)$ denotes the circle centered at the typical user terminal as having a radius of b , $p(x)$ is the receiving power at the typical user terminal

with distance x . P is a random variable, whose characteristic function is defined as $F_P(\omega) \triangleq E(e^{j\omega P})$.

Assume that there are k BSs in the circle with radius b . Note that the k BSs are independently and identically distributed in the circle with pdf $f_R(r) = \frac{2r}{b^2}, 0 \leq r \leq b$. As a result, the characteristic function of P is given by the product of individual characteristic functions, i.e.,

$$F_P(\omega) = \left(\int_0^b \frac{2r}{b^2} e^{j\omega p(r)} dr \right)^k. \quad (9)$$

Moreover, according to the property of homogeneous PPP, the characteristic function of P can be obtained as

$$F_P(\omega) = \exp \left(\lambda'_k \pi b^2 \left(-1 + \int_0^b \frac{2r}{b^2} \exp(j\omega p(r)) dr \right) \right) = \exp \left(\lambda'_k \pi b^2 \left(-1 + \exp(j\omega P_{EH_min}) - \frac{1}{b^2} \int_0^b j\omega r^2 \exp(j\omega p(r)) d(p(r)) \right) \right), \quad (10)$$

The second equality in (10) follows the fact that when the BS is located on the edge of the circle $B(0,b)$, the receiving power at the typical user terminal satisfies $p(b) = P_{EH_min}$, λ'_k is the density of displaced point process taking account into the shadowing effect.

For the typical user terminal connected to the tier k , the maximum harvested energy depends on the distance to its serving BS, denoted by a . The distance distribution of a is given in Lemma 2. Let $p(r) = x$, the relationship between r and x can be obtained from (1) as $r = \left(\frac{x}{P_k L_0 E[s_k] r_0^\alpha B_k} \right)^{-\frac{1}{\alpha}}$. Substituting it into (10), we can get

$$F_P(\omega) = \exp \left(\lambda'_k \pi b^2 \left(-1 + \exp(j\omega P_{EH_min}) + \frac{j\omega (P_k L_0 E[s_k] r_0^\alpha)^{\frac{2}{\alpha}}}{b^2} \int_{p(b)}^{p(a)} x^{-\frac{2}{\alpha}} \exp(j\omega x) dx \right) \right). \quad (11)$$

The average harvested energy EH_k can be obtained using the relationship between the characteristic function and expectation. We have

$$E[P] = \lambda'_k \pi b^2 P_{EH_min} + \frac{\lambda'_k \pi (P_k L_0 r_0^\alpha)^{\frac{2}{\alpha}}}{1 - \frac{2}{\alpha}} \left((E[p(a)])^{1-\frac{2}{\alpha}} - P_{EH_min}^{1-\frac{2}{\alpha}} \right), \quad (12)$$

where a is the average distance between the serving BS and the typical user terminal, it can be gotten as $a = \frac{\frac{2}{\alpha} \lambda_k E[s_k^{\frac{2}{\alpha}}]}{A_k \sum_{j=1}^K \lambda_j E[s_j^{\frac{2}{\alpha}}] B_j^{\frac{2}{\alpha}}}$

easily from Lemma 2. $E[p(a)]$ is the average maximum harvested energy from the serving BS, it can be obtained as (8) by plugging the expression of a into (1). ■

Based on Lemmas 1 and 3, the expression of the average harvested energy in the total network, i.e., P_{EH} as defined in (4), can be obtained without difficulty.

IV. ANALYSIS OF INFORMATION RECEIVING PERFORMANCE AND TOTAL SYSTEM GAIN

A. Average Transmission Rate

For user terminals in the K -tier HCN, it can get both information and energy from K tiers of BSs. This section analyzes the average transmission rate of the K -tier HCN during information receiving. Moreover, we discuss how to leverage the system between energy harvesting and information receiving by defining the total system gain. For the downlink information transmission, let us investigate the user terminal accessing to the BSs in the k -th tier. The downlink rate of the typical user terminal is

$$R_k = E[\log_2(1 + SIR_k)], \quad (13)$$

Applying a similar argument as in [4, theorem 2], R_k can be obtained as

$$R_k = \frac{2\pi\lambda'_k}{A_k} \int_0^\infty \int_0^\infty x \exp\{-\pi \sum_{j=1}^K x^2 C_j(t)\} dt dx \quad (14)$$

with

$$\begin{aligned} C_j(t) &= \lambda_j (\hat{P}_j)^{2/\alpha} \left((\hat{B}_j)^{2/\alpha} + Z(2^t - 1, \alpha_j, \hat{B}_j) \right), \\ Z(2^t - 1, \alpha_j, \hat{B}_j) &= \frac{2(2^t - 1)(\hat{B}_j)^{\frac{2}{\alpha_j} - 1}}{\alpha_j - 2} {}_2F_1\left[1, 1 - \frac{2}{\alpha_j}; 2 - \frac{2}{\alpha_j}; -\frac{1-2^t}{\hat{B}_j}\right] \end{aligned} \quad (15)$$

where $\hat{P}_j = P_j/P_k$, $\hat{B}_j = B_j/B_k$ and ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ is a Gauss hypergeometric function.

Similarly as (4), the average transmission rate of the whole network can be obtained as

$$R = \sum_{k=1}^K R_k \cdot A_k. \quad (16)$$

Moreover, R can be readily obtained by plugging (5) into (16).

B. Trade-Offs between Information & Energy Receiving

The user terminals in K -tier HCN can both receive information and harvest energy by applying policies such as time switch, power switch and receiver (antenna) switch. There exist obvious trade-offs between the information gain and the power gain [11]. To study the information-power trade-off in the K -tier HCN, we consider a fixed time switch SWIPT policy and define the total system gain as

$$G = \tau R + (T - \tau) \cdot P_{EH}, \quad (17)$$

where τ is the duration of information receiving, $T - \tau$ is the duration of energy harvesting at the user terminals.

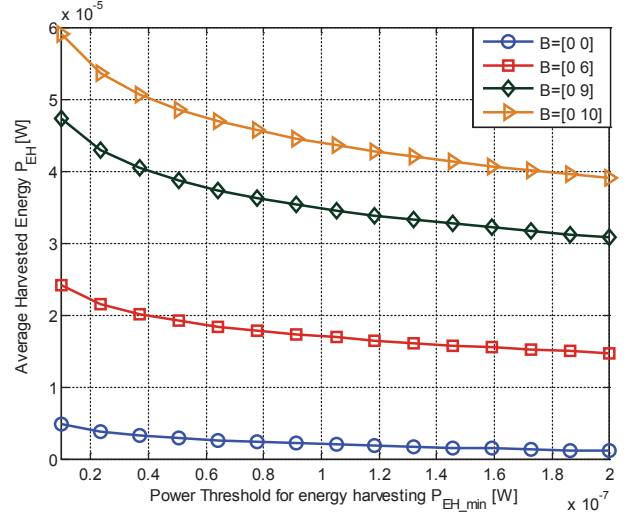


Fig. 2. Average harvested energy of the user terminal under different threshold of energy harvesting.

According to the analyse in Sections III and IV, the bias factor B_k impacts on the cell selection and the distance of serving BS. The total system gain is also affected by the bias factor. When G is optimized over the bias factors, the system gain is automatically leveraged between information and power receiving.

V. NUMERICAL SIMULATIONS

In this section, we evaluate the results in a two-tier HCN, e.g., macro and pico cells. In particular, the average harvested energy, the average transmission rate and the total system gain are investigated based on numerical results. We assume that the shadowing distribution for each tier is log-normal with mean $\mu = [0\ 0]$ dB and standard deviation $\sigma = [4\ 4]$ dB. For the first and the second tiers, we set the BS density as $\lambda_{BS} = 4 \times 10^{-5}/\text{m}^2$ and $\lambda_{BS} = 28 \times 10^{-5}/\text{m}^2$ and the total power as $P_1 = 41\text{dBm}$ and $P_2 = 33\text{dBm}$, respectively.

A. Effect of Energy Harvesting Threshold

Suppose that the path loss exponent is the same for both tiers with $\alpha_1 = \alpha_2 = 2.1$. The average harvested energy of the user terminals is evaluated for different second-tier selection bias. The results are shown in Fig. 2. It can be observed that the average harvested energy decreases with an increasing energy harvesting threshold; meanwhile, the average harvested energy of the user terminal is higher with a bigger second-tier selection bias. This result can be understood intuitively: if the threshold of energy harvesting increases, it is more difficult for the user to capture the energy.

B. Effect of the Density of BSs

We assume that $\alpha_1 = \alpha_2 = 2.1$ and evaluate the average harvested energy of the user terminals with the selection bias set as $B = \{[0\ 0], [0\ 4], [0\ 9], [0\ 10]\}$ dB. As shown in Fig. 3, the average harvested energy increases with an increasing

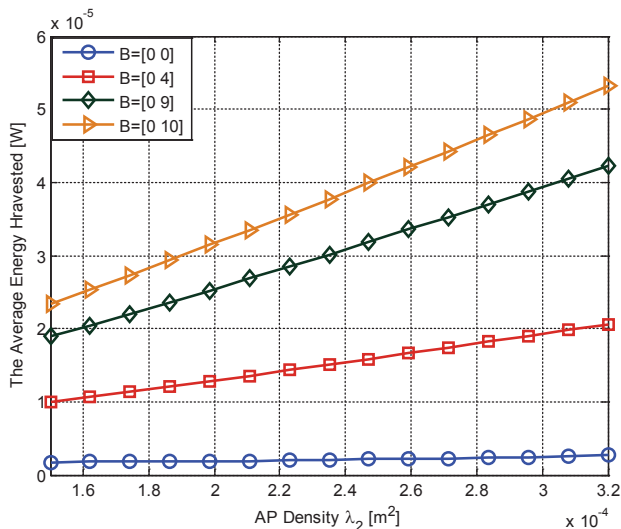


Fig. 3. Average harvested energy of the user terminal under different density of BSs

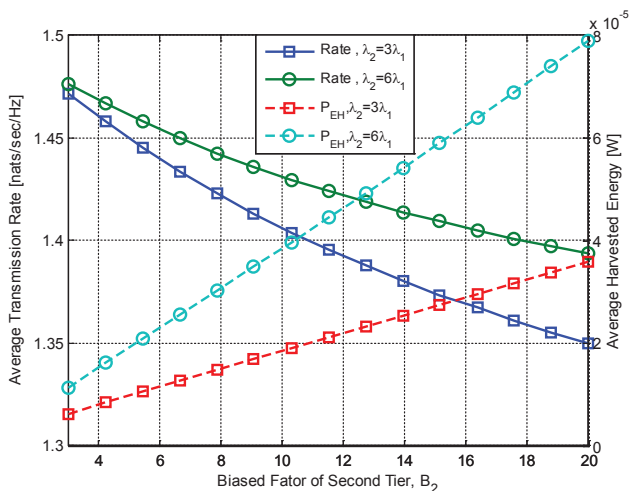


Fig. 4. Average transmission rate and average harvested energy in two tier HCNs for varying second biased factor B_2 .

BS density as well as a larger selection bias. When the bias factor of the second tier increases, more user terminals will connect to the second tier and in the meantime only user terminals with shorter access distance remain connected the first tier. As a result, the average access distance decreases as the bias factor of the second increases, which further results in an increase in the average harvested energy. On the other hand, when the density of BSs increases, the BSs are also closer to the user terminals. Consequently, the average harvested energy increases due to the shorter power transfer distances.

C. Trade-Offs between Information and Power Gains

In Fig. 4, the system gains including the average transmission rate and average harvested energy are evaluated as the bias factor B increases for different BS densities. It can be

seen that, with an increasing of the bias factor, the average transmission rate decreases, while the average harvested energy increases. This is because, when the bias factor increases, the BSs in the second tier serve more user terminals. Since the power of BSs in the second tier is lower than that in the first tier, the average transmission rate of user terminals decreases. When the density of BSs increases, however, both the average transmission rate and the average harvested energy increase. This result can be explained by the fact that the distance between BSs and user terminals decreases with an increasing density of BSs. As a result, the receiving power of user terminals increases, which further leads to the increase in both average transmission rate and average harvested energy.

VI. CONCLUSIONS

In this paper, the average harvested energy and the average transmission rate of user terminals are analyzed based on close-form derivation for K -tier SWIPT cellular networks, where the BSs are distributed according to homogeneous PPPs. The impact of various system parameters, such as the bias factor, the density of BSs, the transmission power of serving BS, the shadowing effect and the path loss exponent, on the energy harvesting and the information transmission performances are revealed. We illustrate the trade-off between the power and the information gains when the bias factor of tier selection is adjusted. These results provide new insights into the dynamical operation and control of K -tier SWIPT cellular networks.

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