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RESEARCH ARTICLE

Performance analysis of cooperative spatial multiplexing networks with AF/DF relaying and linear receiver over Rayleigh fading channels

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ABSTRACT

Cooperative spatial multiplexing (CSM) system has played an important role in wireless networks by offering a substantial improvement in multiplexing gain compared with its cooperative diversity counterpart. However, there is a limited number of research works that consider the performance of CSM systems. As such, in this paper, we have derived exact performance of CSM with amplify-and-forward and decode-and-forward relays in terms of outage capacity and ergodic capacity. We have shown that CSM systems yield a unity diversity order regardless of the number of antennas at the destination and the number of relays in the networks, which is the direct result of diversity and multiplexing gain trade-off. Our analytical expressions are corroborated by Monte-Carlo simulations. Copyright © 2013 John Wiley & Sons, Ltd.

KEYWORDS
amplify-and-forward (AF); decode-and-forward (DF); relay networks; cooperative spatial multiplexing; outage probability; ergodic capacity

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1. INTRODUCTION

Cooperative communications have aroused much interest in recent years as practical transmission schemes for future wireless networks because of their substantial increase in spectral efficiency and reliability [1,2]. Most of the studies in cooperative communications have focused on the diversity gain aspect where relay nodes are deployed to assist direct communications in terms of the spatial diversity gain [3–6]. However, the transmission requires at least two hops for relay networks, which decreases the spectral efficiency compared with the direct communications.

To overcome this shortcoming, a series of seminars has investigated the spatial multiplexing gain of relay networks [7–11]. Specifically, by assuming that all decode-and-forward (DF) relays simultaneously transmit the source's message in the second hop, an improvement in capacity can be achieved by the cooperative spatial multiplexing (CSM) system compared to the cooperative diversity scheme [8]. The work in [8] then is extended to amplify-and-forward (AF) relays for dual-hop [9] and multihop [10]. However, these works, that is, [8–10], have evaluated the performance of CSM systems via simulations. Very recently, the performance analysis for CSM systems with AF relays has been reported in [11]. In particular, the symbol error probability (SEP) and the ergodic capacity, given in the forms of single integral, have been presented in [11]. Although these expressions enable us to evaluate the CSM system performance, their lack of tractability cannot reveal the insights on the impact of the networks parameters on the CSM performance. To the best of the authors' knowledge, there is no previous work deriving the exact closed-form expressions for the performance of CSM systems with either DF or AF relays.

Therefore, in this paper, we take a step further to analyze the performance of CSM systems by considering both DF and AF relays and deriving exact performance metrics of the system. In particular, assuming that a single-antenna source node communicates with a multiple-antenna destination through the assistance of multiple relays (each of which is equipped with a single antenna), the CSM system is formed to enhance the capacity of relay networks.

The relays operate in either DF or AF mode and simultaneously convey the source's signal to the destination. At the destination, a simple linear receiver, namely zero-forcing, is applied to the detect source's symbols. Our contributions are summarized as follows:

- The performance analysis for CSM systems is carried on for both DF and AF relays. The fading channel under consideration is independent but not necessarily identically distributed (i.n.i.d.) Rayleigh.
- We characterize statistical distribution of the end-to-end signal-to-noise ratio (SNR) by deriving the exact cumulative distribution function (CDF) and the probability density function (PDF).
- Utilizing the exact expressions for the statistics of SNR, we can derive the exact expressions for CSM systems in terms of outage probability, SEP, and ergodic capacity.
- We further derive the spatial diversity gain for CSM systems. It is interesting to obtain that the CSM system yields a unity diversity order for both DF and AF relays irrespective of the number of relays and the number of antennas at the destination.

The remainder of this paper is organized as follows: In Section 2, the system and the channel models for CSM with AF and DF relays are briefly described. In Section 3 and Section 4, the performance analysis for CSM has been derived for AF and DF relays, respectively. In Section 5, the numerical results are provided to illustrate the impact of CSM systems. Finally, Section 6 concludes our paper.

2. SYSTEM MODEL FOR COOPERATIVE SPATIAL MULTIPLEXING SYSTEMS WITH AMPLIFY-AND FORWARD AND DECODE-AND FORWARD RELAYS

Consider a CSM system with uplink communications consisting of a source node (equipped with a single antenna), R relay nodes (equipped with a single antenna), and a destination (equipped with n_{D} multiple antennas). Information is exchanged over dual-hop transmission. In the first hop, the source transmits R symbols $x_1, x_2, \ldots, x_R$ to the R relays with the average power per symbol $\mathcal{P}_s$. Then, the k-th relay for $k = 1, 2, \ldots, R$ will be active in the k-th duration to receive x_k. Depending on the relaying operation, the k-th relay can work either in DF or AF mode. In this work, we investigate the CSM system for both DF and AF relays. In the second hop, in contrast to cooperative diversity system where each relay node has to use orthogonal channel, the CSM system allows all relays to transmit simultaneously to the destination to increase the spectral efficiency. The destination can deploy the linear equalization to detect the multi-streams from the source, for example, zero-forcing or minimum mean square error.

The dual-hop fading is assumed as quasi-static flat channel, that is, the channel is fixed over the frame with duration of two hops being spanned over $R + 1$ symbols and changed over subsequent frames. The channel model under consideration is Rayleigh fading where both independent and identically distributed (i.i.d.) and i.n.i.d. fading can be taken into account. In particular, the channel vector from source to the R relays is defined as $[h_1, h_2, \ldots, h_R]$, where h_k is the channel coefficient from source to the k-th relay being modeled as a complex Gaussian random variable with zero mean and variance $\Omega_1 k$, $\mathcal{CN}\left(0, \Omega_{1_k}\right)$. The channel matrix from R relays to n_{D}-antenna destination is denoted as an $n_{\mathrm{D}} \times R$ matrix G. The (i, j)-th element of $\boldsymbol{G}$ for $i = 1, 2, \ldots, R$ and $j = 1, 2, \ldots, n_{\mathrm{D}}$ is the channel coefficient from the i-th relay to the j-th antenna of destination, which is also characterized as $\mathcal{CN}\left(0, \Omega_{2_k}\right)$.

For AF relaying mode, the post-processing SNR per symbol of the k-th sub-stream, $k = 1 \ldots R$, can be determined by [11]

$$\gamma_k^{AF} = \frac{\gamma_0 |h_k|^2}{1 + \left[(\boldsymbol{\Xi}^H \boldsymbol{\Xi})^{-1}\right]_{kk}} \tag{1}$$

where $[\boldsymbol{A}]_{ij}$ denotes the (i, j)-th entry of matrix $\boldsymbol{A}$, and $\gamma_0 = \frac{\mathcal{P}_s}{N_0}$ is the average transmitted SNR. Here, matrix Ξ is defined as

$$\boldsymbol{\Xi} = \begin{bmatrix} \alpha_1 G_{11} & \ldots & \alpha_R G_{1R} \\ \vdots & \ldots & \vdots \\ \alpha_1 G_{n_{\mathrm{D}}1} & \ldots & \alpha_R G_{n_{\mathrm{D}}R} \end{bmatrix} \tag{2}$$

and $\alpha_k = (\Omega_{1_k} + N_0/\mathcal{P}_s)^{-1}$, with N_0 being the noise variance.

For DF relays, the end-to-end SNR of a dual-hop transmission can be considered as the minimum among the two hops [12]. As such, the instantaneous SNR of the k-th symbol for CSM with DF relays can be given by

$$\begin{aligned} \gamma_k^{DF} &= \min(\gamma_{k1}, \gamma_{k2}) \\ &= \min\left(\gamma_0 |h_k|^2, \frac{\gamma_0}{[(\boldsymbol{G}_2^H \boldsymbol{G}_2)^{-1}]_{kk}}\right) \end{aligned} \tag{3}$$

3. PERFORMANCE ANALYSIS OF COOPERATIVE SPATIAL MULTIPLEXING SYSTEMS WITH AMPLIFY-AND-FORWARD RELAYS

3.1. Outage probability

From the definition of matrix $\boldsymbol{\Xi}$ in (2), we can observe that $\boldsymbol{\Xi}^H \boldsymbol{\Xi}$ is a complex Wishart matrix. Accordingly, the inverse of the k-th diagonal element of matrix $(\boldsymbol{\Xi}^H \boldsymbol{\Xi})^{-1}$, that is, $Y_k = 1/[(\boldsymbol{\Xi}^H \boldsymbol{\Xi})^{-1}]_{kk}$, is a weighted chi-square distributed random variable with $2(n_{\mathrm{D}} - R + 1)$ degrees of freedom [13], whose PDF can be expressed as

$$f_{Y_k}(y) = \frac{\exp(-y/\xi_k)}{\xi_k m!} \left(\frac{y}{\xi_k}\right)^m \tag{4}$$

where $\xi_k = \alpha_k^2 \Omega_{2_k}$ and $m = n_{\mathsf{D}} - R$. Furthermore, it is well-known that the PDF of random variable $X_k = 1/Y_k$ is given by

$$f_{X_k}(x) = \frac{1}{x^2} f_{Y_k}\left(\frac{1}{x}\right) = \frac{1}{\xi_k m! x^2} \exp\left(-\frac{1}{\xi_k x}\right)\left(\frac{1}{\xi_k x}\right)^m \quad (5)$$

where the second equality follows immediately from (4). We now can readily derive the CDF of the instantaneous SNR γ_k^{AF} of the k-th sub-stream as follows:

$$\begin{aligned} F_{\gamma_k^{AF}}(\gamma) &= \Pr\left(|h_k|^2 < \frac{\gamma}{\gamma_0}(1+X_k)\right) \\ &= \mathbb{E}_{X_k}\left\{1 - \exp\left(-\frac{\gamma}{\gamma_0 \Omega_{1_k}}(1+X_k)\right)\right\} \\ &= 1 - \int_0^\infty \frac{\exp\left(-\frac{\gamma(1+x)}{\gamma_0\Omega_{1_k}}\right)\exp\left(-\frac{1}{\xi_k x}\right)\left(\frac{1}{\xi_k x}\right)^m}{\xi_k m! x^2} \\ &= 1 - \frac{2}{m!}\left(\frac{\gamma}{\gamma_0\Omega_{1_k}\xi_k}\right)^{(m+1)/2} \exp\left(-\frac{\gamma}{\gamma_0\Omega_{1_k}}\right)\mathcal{K}_{m+1}\left(2\sqrt{\frac{\gamma}{\gamma_0\Omega_{1_k}\xi_k}}\right) \end{aligned} \quad (6)$$

where $\mathbb{E}_x\{.\}$ is the expectation operator over the random variable X, and $\mathcal{K}_n(.)$ is the nth-order modified Bessel function of the second kind [14, Eq. (8.432.6)]. Having the CDF of γ_k^{AF} in hands allows us to evaluate the outage probability of the system under consideration. Denoting γ_{th} as the SNR threshold, below which the system is in outage, the outage probability can be obtained by replacing γ with γ_{th} in the CDF expression given previously as follows:

$$\begin{aligned} P_{\text{o}}^{AF} &= \frac{1}{R}\sum_{k=1}^{R} F_{\gamma_k^{AF}}(\gamma_{\mathsf{th}}) \\ &= \frac{1}{R}\sum_{k=1}^{R}\left[1 - \frac{2}{m!}\left(\frac{\gamma_{\mathsf{th}}}{\gamma_0\Omega_{1_k}\xi_k}\right)^{(m+1)/2}\exp\left(-\frac{\gamma_{\mathsf{th}}}{\gamma_0\Omega_{1_k}}\right)\mathcal{K}_{m+1}\left(2\sqrt{\frac{\gamma_{\mathsf{th}}}{\gamma_0\Omega_{1_k}\xi_k}}\right)\right] \end{aligned} \quad (7)$$

3.2. Symbol error probability

The PDF of γ_k^{AF} can be easily obtained by differentiating the CDF with respect to γ as follows:

$$\begin{aligned} f_{\gamma_k^{AF}}(\gamma) &= \frac{2}{m!}\left(\frac{1}{\gamma_0\Omega_{1_k}\xi_k}\right)^{\frac{m+2}{2}} \gamma^{\frac{m}{2}} \exp\left(-\frac{\gamma}{\gamma_0\Omega_{1_k}}\right)\mathcal{K}_m\left(2\sqrt{\frac{\gamma}{\gamma_0\Omega_{1_k}\xi_k}}\right) \\ &\quad + \frac{2}{m!}\left(\frac{1}{\gamma_0\Omega_{1_k}\xi_k}\right)^{\frac{m+1}{2}} \frac{\gamma^{\frac{m+1}{2}}}{\gamma_0\Omega_{1_k}} \exp\left(-\frac{\gamma}{\gamma_0\Omega_{1_k}}\right)\mathcal{K}_{m+1}\left(2\sqrt{\frac{\gamma}{\gamma_0\Omega_{1_k}\xi_k}}\right) \\ &= \sum_{\ell=0}^{1}\frac{2}{m!}\left(\frac{1}{\gamma_0\Omega_{1_k}\xi_k}\right)^{\frac{m-\ell+2}{2}}\left(\frac{1}{\gamma_0\Omega_{1_k}}\right)^{\ell}\gamma^{\frac{m+\ell}{2}}\exp\left(-\frac{\gamma}{\gamma_0\Omega_{1_k}}\right)\mathcal{K}_{m+\ell}\left(2\sqrt{\frac{\gamma}{\gamma_0\Omega_{1_k}\xi_k}}\right) \end{aligned} \quad (8)$$

The SEP of M-PSK can be given in exact form by [15]

$$P_e^{AF} = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \Phi_{\gamma_k^{AF}}\left(\frac{\mathsf{g}}{\sin^2\theta}\right) d\theta \quad (9)$$

where $\mathsf{g} = \sin^2\left(\frac{\pi}{M}\right)$ and $\Phi_{\gamma_k^{AF}}(s) = \mathbb{E}_{\gamma_k^{AF}}\{\exp(-s\gamma)\}$ is the moment generating function (MGF) of γ_k^{AF}. Applying the Laplace transform of $f_{\gamma_k^{AF}}(\gamma)$ yields the MGF of γ_k^{AF} as

$$\begin{aligned} \Phi_{\gamma_k^{AF}}(s) &= \frac{1}{1+\gamma_0\Omega_{1_k}s}\exp\left(\frac{1}{\xi_k(1+\gamma_0\Omega_{1_k}s)}\right) \\ &\quad \times \sum_{\ell=0}^{1}(m+1)^{\ell}\xi_k^{\ell-1}E_{m+\ell+1}\left(\frac{1}{\xi_k(1+\gamma_0\Omega_{1_k}s)}\right) \end{aligned} \quad (10)$$

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Theorem 1. *The CSM with AF relays achieves the unity diversity gain, that is,*

$$d_{AF} \triangleq \lim_{\gamma_0 \to \infty} \frac{-\log P_e}{\log \gamma_0} = 1 \tag{11}$$

Proof. Because each sub-stream plays a similar role, we can rewrite (11) as follows:

$$\begin{aligned} d_{AF} &= \lim_{\gamma_0 \to \infty} \frac{-\log P_{ek}}{\log \gamma_0} \\ &= \lim_{\gamma_0 \to \infty} \frac{-\log \Phi_{\gamma_k^{AF}}(g)}{\log \gamma_0} \end{aligned} \tag{12}$$

From (10) and (12), we obtain

$$d_{AF} = \underbrace{\lim_{\gamma_0 \to \infty} \frac{\log(1 + \mathrm{g}\Omega_{1_k}\gamma_0)}{\log \gamma_0}}_{I_1} - I_2 \tag{13}$$

where I_2 is defined as

$$I_2 = \lim_{\gamma_0 \to \infty} \frac{\log\left[\sum_{\ell=0}^{1} \alpha_\ell \exp\left(\frac{1}{\xi_k(1+\gamma_0\Omega_{1_k}\mathrm{g})}\right)\right]}{\log \gamma_0} E_{m+\ell+1}\left(\frac{1}{\xi_k(1+\gamma_0\Omega_{1_k}\mathrm{g})}\right) \tag{14}$$

and $\alpha_\ell = (m+1)^\ell \xi_k^{\ell-1}$. It is easy to see that $I_1 = 1$. For the large SNR, that is, $\gamma_0 \gg 1$, I_2 can be expressed as

$$I_2 = \lim_{\gamma_0 \to \infty} \frac{\log\left[\sum_{\ell=0}^{1} \alpha_\ell \exp\left(\frac{\beta}{\gamma_0}\right) E_{m+\ell+1}\left(\frac{\beta}{\gamma_0}\right)\right]}{\log \gamma_0} \tag{15}$$

where $\beta = \frac{1}{\xi_k \Omega_{1_k} \mathrm{g}}$. The exponential integral function can be shown in the form of the incomplete gamma function

$$E_{m+\ell+1}\left(\frac{\beta}{\gamma_0}\right) = \left(\frac{\beta}{\gamma_0}\right)^{m+\ell} \Gamma\left(-m-\ell, \frac{\beta}{\gamma_0}\right) \tag{16}$$

where $\Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the incomplete gamma function. Furthermore, using the fact that $\Gamma(a,x) = e^{-x} x^a \sum_{i=0}^{\infty} \frac{L_i^a(x)}{i+1}$ [14, eq. (8.355)], where $L_i^a(x)$ is the Laguerre polynomial of order i and applying [14, eq. (8.970.1)], we obtain

$$E_{m+\ell+1}\left(\frac{\beta}{\gamma_0}\right) = \exp\left(-\frac{\beta}{\gamma_0}\right) \sum_{i=0}^{\infty} \sum_{j=0}^{i} \frac{(-1)^j}{(i+1)j!} \binom{i-m-\ell}{i-j} \frac{\beta^j}{\gamma_0^j}$$

From (15) and (17), I_2 can be written as

$$\begin{aligned} I_2 &= \lim_{\gamma_0 \to \infty} \frac{\log\left(\sum_{\ell=0}^{1} \sum_{i=0}^{\infty} \sum_{j=0}^{i} \frac{(-1)^j \alpha_\ell}{(i+1)j!} \binom{i-m-\ell}{i-j} \frac{\beta^j}{\gamma_0^j}\right)}{\log \gamma_0} \\ &= 0 \end{aligned} \tag{17}$$

which completes the proof. □

3.3. Ergodic capacity

The ergodic capacity of CSM with AF relays can be obtained by summing up the data rate of all sub-streams, given by (18) as shown

$$\begin{aligned} C_{AF} &= \frac{1}{R+1} \sum_{k=1}^{R} \mathbb{E}_{\gamma_k^{AF}} \{\log_2(1+\gamma_k)\} \\ &= \frac{2}{(R+1)m! \ln 2} \sum_{k=1}^{R} \sum_{\ell=0}^{1} \left(\frac{1}{\gamma_0 \Omega_{1_k} \xi_k}\right)^{\frac{m-\ell+2}{2}} \left(\frac{1}{\gamma_0 \Omega_{1_k}}\right)^{\ell} \int_0^\infty \gamma^{\frac{m+\ell}{2}} \exp\left(-\frac{\gamma}{\gamma_0 \Omega_{1_k}}\right) \\ &\quad \times \ln(1+\gamma) \mathcal{K}_{m+\ell}\left(2\sqrt{\frac{\gamma}{\gamma_0 \Omega_{1_k} \xi_k}}\right) d\gamma \end{aligned} \tag{18}$$

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where the scalar $1/(R+1)$ accounts for the fact that the transmission incurs in $R+1$ symbol duration, and (18) follows immediately from (8). As can be observed from (18), we need to calculate the following integral to obtain the closed-form expression for the ergodic capacity

$$J = \int_0^{\infty} \gamma^{\frac{m+\ell}{2}} e^{-\frac{\gamma}{\gamma_0 \Omega_{1_k}}} \ln(1+\gamma) \mathcal{K}_{m+\ell}\left(2\sqrt{\frac{\gamma}{\gamma_0 \Omega_{1_k} \xi_k}}\right) d\gamma \tag{19}$$

To further simplify integral J, we express $\ln(1+\gamma)$ and $\mathcal{K}_{m+\ell}\left(2\sqrt{\frac{\gamma}{\gamma_0 \Omega_{1_k} \xi_k}}\right)$ in terms of the Meijer's G-function and Fox's H-function by making use of [16, eq. (8.4.6.5)], [16, eq. (8.4.23.1)], and [16, eq. (8.3.2.21)] as follows:

$$\begin{aligned}\ln(1+\gamma) &= G_{2,2}^{1,2}\left(\gamma \,\middle|\, \begin{matrix} 1,1 \\ 1,0 \end{matrix}\right) \\ &= H_{2,2}^{1,2}\left[\gamma \,\middle|\, \begin{matrix} (1,1),(1,1) \\ (1,1),(0,1) \end{matrix}\right]\end{aligned} \tag{20}$$

$$\begin{aligned}&\mathcal{K}_{m+\ell}\left(2\sqrt{\frac{\gamma}{\gamma_0 \Omega_{1_k} \xi_k}}\right) \\ &= \frac{1}{2} G_{2,0}^{0,2}\left(\frac{\gamma}{\gamma_0 \Omega_{1_k} \xi_k} \,\middle|\, \begin{matrix} \cdot & \cdot \\ \frac{m+\ell}{2}, & -\frac{m+\ell}{2} \end{matrix}\right) \\ &= \frac{1}{2} H_{2,0}^{0,2}\left[\frac{\gamma}{\gamma_0 \Omega_{1_k} \xi_k} \,\middle|\, \begin{matrix} \cdot & \cdot \\ (\frac{m+\ell}{2},1), & (-\frac{m+\ell}{2},1) \end{matrix}\right]\end{aligned} \tag{21}$$

where $G_{p,q}^{m,n}(\cdot)$ is the Meijer's G-function [16, eq. (8.2.1.1)], and $H_{p,q}^{m,n}[\cdot]$ is the Fox's H-function [16, eq. (8.3.1.1)]. By substituting (20) and (21) in (19) and using the help of [17, eq. (2.6.2)], we obtain

$$J = \frac{1}{2}\left(\frac{1}{\gamma_0 \Omega_{1_k}}\right)^{\frac{-m-\ell-2}{2}} H_{1,[2:2],0,[2:0]}^{1,2,2,1,0}\left[\begin{matrix} \gamma_0 \Omega_{1_k} \\ \\ \frac{1}{\xi_k} \end{matrix} \,\middle|\, \begin{matrix} (\frac{m+\ell+2}{2},1) \\ (1,1),(1,1) \\ (1,1),(0,1);(\frac{\overline{m+\ell}}{2},1),(-\frac{m+\ell}{2},1) \end{matrix}\right] \tag{22}$$

where $H_{E,[A:C],F,[B:D]}^{K,N,N',M,M'}[\cdot]$ is the generalized Fox's H-function [17, eq. (2.2.1)]. Finally, by combining (22) with (18), the ergodic capacity of CSM with AF relays can be given as

$$C_{AF} = \frac{1}{(R+1)m!\ln 2} \sum_{k=1}^{R} \sum_{\ell=0}^{1} \left(\frac{1}{\xi_k}\right)^{\frac{m-\ell+2}{2}} H_{1,[2:2],0,[2:0]}^{1,2,2,1,0}\left[\begin{matrix} \gamma_0 \Omega_{1_k} \\ \\ \frac{1}{\xi_k} \end{matrix} \,\middle|\, \begin{matrix} (\frac{m+\ell+2}{2},1) \\ (1,1),(1,1) \\ (1,1),(0,1);(\frac{\overline{m+\ell}}{2},1),(-\frac{m+\ell}{2},1) \end{matrix}\right] \tag{23}$$

4. PERFORMANCE ANALYSIS OF COOPERATIVE SPATIAL MULTIPLEXING SYSTEMS WITH FIXED DECODE-AND-FORWARD RELAYS

4.1. Outage probability

Defined as the probability that the minimum of its single-hop SNRs is below a given threshold SNR, the system outage probability of the CSM system with DF relays can be mathematically expressed as

$$P_o^{DF} = \frac{1}{R}\sum_{k=1}^{R} F_{\gamma_k^{DF}}(\gamma_{\text{th}}) \tag{24}$$

where γ_k^{DF} is given in (3), which leads to the fact that

$$F_{\gamma_k^{DF}}(\gamma) = 1 - \left[1 - F_{\gamma_{k1}}(\gamma)\right]\left[1 - F_{\gamma_{k2}}(\gamma)\right] \tag{25}$$

Because $\gamma_{k1} = \gamma_0 |h_k|^2$ is an exponential random variable, we can observe that $F_{\gamma_{k1}}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\gamma_0 \Omega_{1_k}}\right)$. In addition, the CDF of $\gamma_{k2} = \frac{\gamma_0}{\left[(\mathbf{G}_2^H \mathbf{G}_2)^{-1}\right]_{kk}}$ can easily be obtained by differentiating (4), which enables us to rewrite (25) as follows:

$$F_{\gamma_k^{DF}}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\gamma_0 \Omega_{1_k}}\right) \frac{\Gamma(m+1, \gamma/(\gamma_0 \Omega_{2_k}))}{\Gamma(m+1)} \tag{26}$$

4.2. Symbol error probability

By making the derivative of (26) with respect to γ and using the fact that $\frac{\partial\Gamma(a+1,x)}{\partial x} = -\frac{x^a}{e^x}$, we obtain the PDF as follows:

$$f_{\gamma_k^{DF}}(\gamma) = \frac{\exp\left[-\left(\frac{1}{\gamma_0\Omega_{1_k}} + \frac{1}{\gamma_0\Omega_{2_k}}\right)\gamma\right]}{\gamma_0\Omega_{2_k}\Gamma(m+1)}\left(\frac{\gamma}{\gamma_0\Omega_{2_k}}\right)^m + \frac{\exp\left(-\frac{\gamma}{\gamma_0\Omega_{1_k}}\right)}{\gamma_0\Omega_{1_k}}\frac{\Gamma(m+1,\gamma/(\gamma_0\Omega_{2_k}))}{\Gamma(m+1)} \tag{27}$$

The MGF is then can be given by

$$\Phi_{\gamma_k^{DF}}(s) = \left(\frac{\Omega_{1_k}}{\Omega_{1_k}+\Omega_{2_k}+\gamma_0\Omega_{1_k}\Omega_{2_k}s}\right)^{m+1}\frac{\gamma_0\Omega_{1_k}s}{1+\gamma_0\Omega_{1_k}s} + \frac{1}{1+\gamma_0\Omega_{1_k}s} \tag{28}$$

Theorem 2. *The CSM with fixed DF relays achieves the first diversity gain, that is,*

$$d_{DF} \triangleq \lim_{\gamma_0\to\infty}\frac{-\log P_{\mathrm{e}}}{\log\gamma_0} = 1 \tag{29}$$

Proof. Similarly, as in the case of AF relays, the diversity order for fixed DF relays can be shown as

$$d_{DF} = \underbrace{\lim_{\gamma_0\to\infty}\frac{\log(1+\mathsf{g}\Omega_{1_k}\gamma_0)}{\log\gamma_0}}_{I_1} - \underbrace{\lim_{\gamma_0\to\infty}\frac{\log\left[1+\left(\frac{\Omega_{1_k}}{\Omega_{1_k}+\Omega_{2_k}+\gamma_0\Omega_{1_k}\Omega_{2_k}\mathsf{g}}\right)^{m+1}\gamma_0\Omega_{1_k}\mathsf{g}\right]}{\log\gamma_0}}_{I_3} \tag{30}$$

It is easy to observe that $I_3 = 0$, which finalizes the proof. □

4.3. Ergodic capacity

By expanding the incomplete gamma function in the summation of elementary functions in (8), the ergodic capacity for CSM with fixed DF relays can be given by

$$C_{DF} = \frac{1}{(R+1)\ln 2}\sum_{k=1}^{R}\int_0^\infty \ln(1+\gamma)\frac{\exp\left[-\left(\frac{1}{\gamma_0\Omega_{1_k}}+\frac{1}{\gamma_0\Omega_{2_k}}\right)\gamma\right]}{\gamma_0\Omega_{2_k}m!}\left(\frac{\gamma}{\gamma_0\Omega_{2_k}}\right)^m d\gamma$$
$$+ \frac{1}{(R+1)\ln 2}\sum_{k=1}^{R}\sum_{i=0}^{m}\int_0^\infty \ln(1+\gamma)\frac{\exp\left[-\left(\frac{1}{\gamma_0\Omega_{1_k}}+\frac{1}{\gamma_0\Omega_{2_k}}\right)\gamma\right]}{\gamma_0\Omega_{1_k}i!}\left(\frac{\gamma}{\gamma_0\Omega_{2_k}}\right)^i d\gamma \tag{31}$$

To evaluate the integral in (31), we apply the following result from [18, Appendix B]

$$\int_0^\infty \ln(1+x)x^{n-1}\exp(-\alpha x) = (n-1)!\exp(\alpha)\sum_{j=1}^{n}\frac{\Gamma(-n+j,\alpha)}{\alpha^j} \tag{32}$$

where $n = 1, 2, \ldots$, $\alpha > 0$, which yields the desired expression as follows:

$$C_{DF} = \frac{1}{(R+1)\ln 2}\sum_{k=1}^{R}\exp\left(\frac{1}{\gamma_0\Omega_{1_k}}+\frac{1}{\gamma_0\Omega_{2_k}}\right)\left[\left(\frac{1}{\gamma_0\Omega_{2_k}}\right)^{m+1}\sum_{j=1}^{m+1}\frac{\Gamma\left(-m-1+j,\frac{1}{\gamma_0\Omega_{1_k}}+\frac{1}{\gamma_0\Omega_{2_k}}\right)}{\left(\frac{1}{\gamma_0\Omega_{1_k}}+\frac{1}{\gamma_0\Omega_{2_k}}\right)^j}\right.$$
$$\left.+\sum_{i=0}^{m}\frac{1}{\gamma_0\Omega_{1_k}}\left(\frac{1}{\gamma_0\Omega_{2_k}}\right)^i\sum_{j=1}^{i+1}\frac{\Gamma\left(-i-1+j,\frac{1}{\gamma_0\Omega_{1_k}}+\frac{1}{\gamma_0\Omega_{2_k}}\right)}{\left(\frac{1}{\gamma_0\Omega_{1_k}}+\frac{1}{\gamma_0\Omega_{2_k}}\right)^j}\right] \tag{33}$$

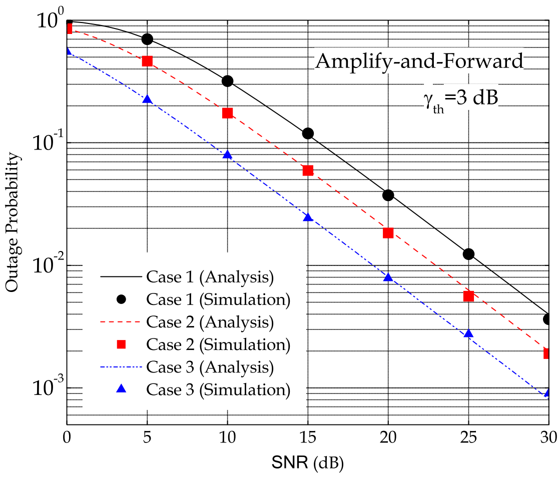

Figure 1. Outage probability of cooperative spatial multiplexing with AF relays versus SNR.

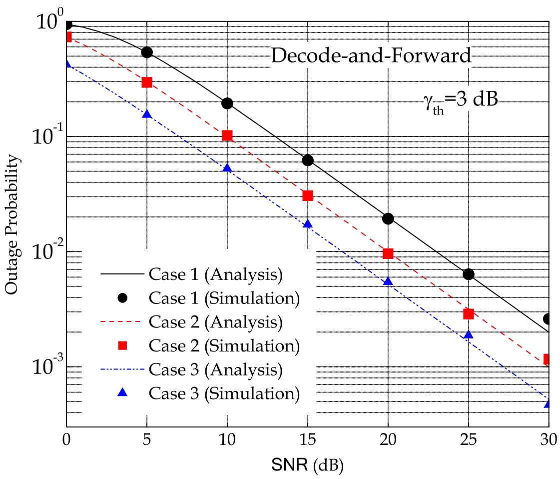

Figure 2. Outage probability of cooperative spatial multiplexing with DF relays versus SNR.

Then, applying some simplifications, we can obtain the ergodic capacity as in (34), where $\lambda_{1_k} = \frac{1}{\gamma_0 \Omega_{1_k}}$ and $\lambda_{2_k} = \frac{1}{\gamma_0 \Omega_{2_k}}$.

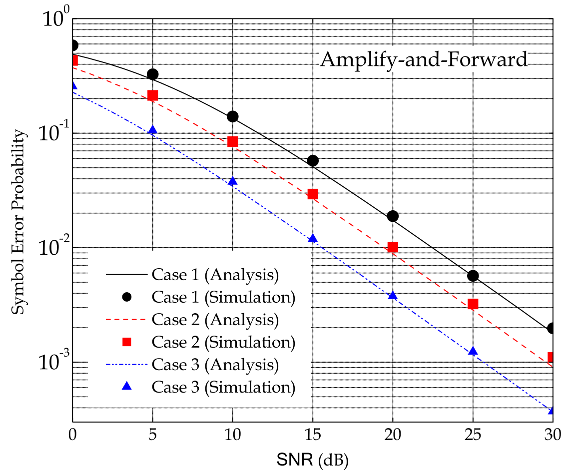

Figure 3. Symbol error probability of cooperative spatial multiplexing with AF relays versus SNR.

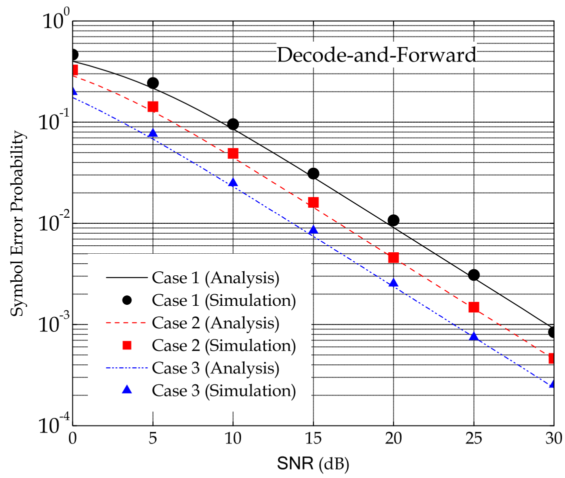

Figure 4. Symbol error probability of cooperative spatial multiplexing with DF relays versus SNR.

5. NUMERICAL RESULTS

In this section, numerical results are provided to validate our analysis. The number of relays is set as $R = 3$, and the number of antennas at the destination is selected as $n_D = 4$. We consider three different cases corresponding

$$C_{DF} = \frac{1}{(R+1)\ln 2}\exp(\lambda_{1_k}+\lambda_{2_k})\Bigg[(\lambda_{2_k})^{m+1}\sum_{j=1}^{m+1}\frac{\Gamma(-m-1-j,\lambda_{1_k}+\lambda_{2_k})}{(\lambda_{1_k}+\lambda_{2_k})^j} + \sum_{i=0}^{m}\lambda_{1_k}(\lambda_{2_k})^i\sum_{j=1}^{m+1}\frac{\Gamma(-i-1-j,\lambda_{1_k}+\lambda_{2_k})}{(\lambda_{1_k}+\lambda_{2_k})^j}\Bigg] \quad (34)$$

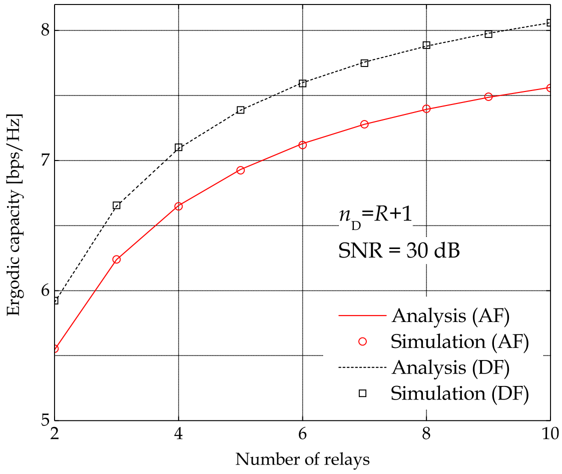

Figure 5. Ergodic capacity of cooperative spatial multiplexing versus number of relays.

i.i.d. and i.n.i.d. Rayleigh fading channels as follows: Case 1: $\{\Omega_1 k\}_{k=1}^3 = 1$, $\{\Omega_{2_k}\}_{k=1}^3 = 1$, Case 2: $\{\Omega_{1_k}\}_{k=1}^3 = 2$, $\{\Omega_{2_k}\}_{k=1}^3 = 2$, and Case 3: $\{\Omega_{1_k}\}_{k=1}^3 = \{3, 4, 5\}$, $\{\Omega_{2_k}\}_{k=1}^3 = \{6, 7, 8\}$.

Figures 1 and 2 show the outage probability for CSM with AF and DF relays, respectively, versus the average SNR with the outage threshold being chosen as $\gamma_{\text{th}} = 3$dB. In addition, Figures 3 and 4 display the SEP performance of Quadrature Phase Shift Keying modulation for AF and DF versus the average SNR, respectively. As can be seen from these four figures, the performance is improved with the increase of channel mean power of the fading channels, that is, Ω_{1_k} and Ω_{2_k}. It is observed that the best performance is seen with Case 3 because its channel mean powers are the highest among the three cases. It is also interesting to see that the curves are parallel in the high SNR regimes, which reveals the fact that the diversity gains are the same and do not depend on the channel mean powers. In addition, for the four figures, the analysis and the simulation curves match very well with each other, which validates the correctness of our analytical derivation.

Figure 5 displays the ergodic capacity of AF and DF relays, where the number of antennas at the destination is selected as $n_D = R + 1$. Here, we plot the capacity versus the number of relays R. As can be seen that the capacity performance is enhanced as the number of relays increases. More importantly, CSM with DF relays show a better performance than that of AF relays. This is straightforward because, in the latter case, the relays convey not only the source's information but also the relay's noise to the destination, which degrades the system performance.

6. CONCLUSIONS

In this work, we have analyzed the performance of CSM systems with AF and DF relays over i.n.i.d. Rayleigh fading channels. In particular, by assuming that each relaying terminal simultaneously transmits the source's signal to the destination, which deploys the linear receiver, the capacity of CSM systems can be significantly increased compared with cooperative diversity. We have derived exact analytical expressions for the outage probability, symbol error probability, and ergodic capacity. More importantly, we have shown that the capacity enhancement of CSM systems is achieved at an expense of the unity diversity gain for both AF and DF relays.

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