Graph Theory Based Capacity Analysis for Vehicular Ad Hoc Networks

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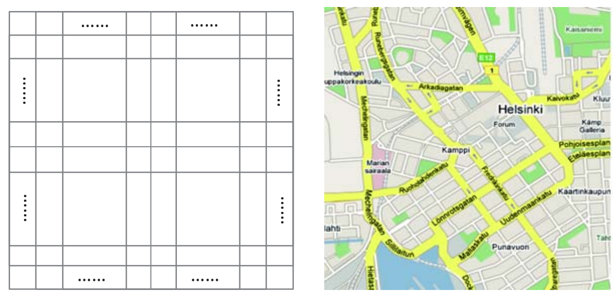
***Abstract*—Vehicular ad hoc networks (VANETs) which are deployed along roads make traffic systems safer and efficient. Existing theoretical results on capacity scaling laws provide insights and guidance for the design and deployment of VANETs. In this paper, we propose a novel fundamental framework RVWNM (Real Vehicular Wireless Network Model), which enables a more realistic capacity analysis in VANETs. We first introduce a Euclidean planar graph which can be constructed from any real map of urban area, and represents the practical geometry structure of the urban area. Then, an interference relationship graph is abstracted from the Euclidean planar graph which considers the transmission interference relations among the nodes in the network. Finally, we analyze theoretically the interference relationships in the interference relationship graph. As far as we know, we are the first to use a practical geometry structure to calculate the asymptotic capacity of VANETs. To verify the feasibility of RVWNM, we calculate the asymptotic capacity of urban area VANETs with the consideration of social-proximity based mobility of vehicles.**

I. Introduction

As the development of wireless networks [1-3], VANETs(Vehicular Ad hoc Networks) become to an important branch. VANETs can offer safety and traffic information without using a wired backbone. Capacity, as an important and fundamental property, is critical for VANETs theoretical analysis [1]. However, determining the capacity of distributed wireless networks is one of the most general but challenging problems. The existing techniques and inequalities from information theory cannot efficiently tackle this problem. Thus, some probability and statistical methods have been proposed to analyze and calculate the capacity of ad hoc networks [5, 6]. These technologies have brought a rapid progress in the study of capacity under various scenarios during the last decade.

However, three unique properties of VANET make the analysis by using probability and statistical technologies in VANETs difficult. Those properties can be summarized as follows.

- Since all vehicles can only move along roads, the layout of roads in an urban area directly affects the capacity of the VANET.



(a) Grid-like structure (b) Real-world map of urban area

Fig. 1. Grid-like structure and real map of an urban area

- Since the geometries of the roads in different urban areas are different, we cannot use a universal model for all urban areas.
- Since vehicles move neither randomly nor completely regularly, their mobility can only be characterized in a statistical mannner.

In 2007 [7], Pishro-Nik et al. proposed a grid-like construction to demonstrate all the roads in an urban area as shown Fig. 1 (a). Every line of the construction denotes a road, and the m vertical lines intersect with the m horizontal lines, which compose a grid-like construction. In 2012 [13], Lu et al. extended the work of Pishro-Nik et al. [7] and obtained an almost constant per-vehicle throughput in high-density scenario. We can see in Fig. 1 (b) that different areas of an urban have different road densities, and the shapes of the areas surrounded by the roads differ even more significantly. Therefore, if we use a normalized gird-like construction to model an urban area, we cannot obtain an accurate and suitable asymptotic capacity.

To solve the above problem, we propose a new fundamental framework RVWNM constructed by a Euclidean planar graph and an interference relationship graph. RVWNM is a real construction of vehicular network because it is abstracted from real-world map and has all the geometry property of such a real-world map. In addition, it is convenient to use methods in Graph Theory to analyze a RVWNM. The inter-

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ference relationship graph is obtained through abstracting the Euclidean planar graph which will be introduced in Section III. To schedule the interference in MAC layer of wireless transmission, we use the protocol interference model in [14] as our interference model.

Under the RVWNM, we calculate the throughput capacity limits of the urban social-proximity VANET to achieve $\Theta(1/n)$ in sparse area and constant capacity bounds in high density area, where n denotes the number of vehicles. The following is a summary of our contributions:

- We propose a new framework RVWNM constructed by a Euclidean planar graph and an interference relationship graph. The Euclidean planar graph can be abstracted from real map of any urban area. Different from the general and inaccurate result obtained under the general grid-like construction, which neglects the non-uniform of urban roads and the difference between different urban areas, the asymptotic capacity obtained under this framework is targeted and accurate.
- We abstract the interfenence relationship graph from Euclidean planar graph based on the interference between units. We introduce the independent set of graph theory to demonstrate the interference relationships between units. The independent set makes the analysis and calculation of concurrent transmission flows easier, which is important for the calculation of asymptotic capacity.
- To verify the feasibility of the proposed method, we calculate the asymptotic capacity of two-hop vehicular networks with the protocol interference model. We show that the constant capacity bounds could be achieved at high density area and the asymptotic capacity in sparse area is bounded by $\Theta(1/n)$.

To the best of our knowledge, this is the first framework that offers a targeted and accurate asymptotic capacity, which is different from the normalized grid-like construction. This framework also can be used to calculate asymptotic capacity under other scenarios with different interference models or different routing schemes.

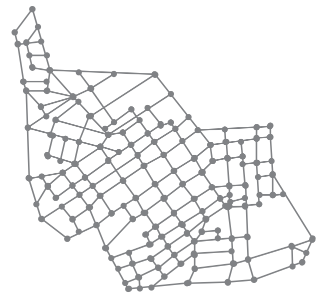
The rest of paper is organized as follows. Section II introduces the network model, definitions of capacity and known theorems used in the proof. Section III calculates the capacity of VANETs. Section IV discusses the drawbacks of this paper. Section V concludes the paper with future works.

II. System Model

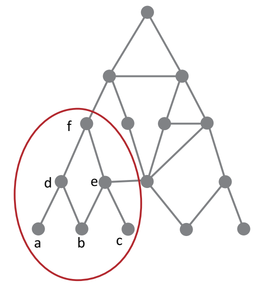
A. Network Model

The grid-like network geometry is convenient because of its partially normalized structure. To derive a more accurate theoretical capacity under a real scenario, we propose a novel network model, constructed by a Euclidean planar graph and an interference relationship graph. In this paper, we use the real map of an urban area to construct the network model. In Section III, we introduce the process of constructing the model from a real-world map in detail.

To obtain the Euclidean planar graph of RVWNM, we define the disk centered at the intersection in a real map with diameter



(a) A Euclidean planar graph



(b) A simple example of interference relationship graph

Fig. 2. Euclidean planar graph and interference relationship graph

r as a unit, where r is the transmission range. The transmission range of a wireless communication equipment is about 300 meters, so the road between two adjacent intersections that are 300 meters apart could be covered by units. Note that the length of most roads is less than 300 meters, and vehicles usually pass through the middle of the roads with high speed. Thus, we can neglect occasional cases when vehicles are not in coverage. The units in a real-world map are denoted by vertices on the Euclidean planar graph according to their coordinate in a real-world map. If there is a road between two units, we place an edge between the two corresponding vertices.

To an arbitrary real-world area, we can obtain an arbitrary RVWNM through abstract. For the general theoretical asymptotic capacity, we assume that the overall area is S and the perimeter is L. We consider units randomly distributed in the area and the connection between each two adjacent units is random. We abstract the above network area to derive a random RVWNM.

Given a real-world map (Fig. 1 (b)), we treat every intersection as a center and draw a circle with a diameter r. The road covered by a circle is a unit. All the units compose a unit set $U = \{u_1, u_2, \cdots, u_n\}$. The units in a real-world map are denoted by vertices on the Euclidean planar graph according to their coordinate in a real-world map. Place U on G_E according to the coordinate of units and place an edge between each two units that are connected in a real-world map. Thus, we derive an arbitrary Euclidean planar graph G_E.

To obtain the theoretical results, we randomly choose a region G_R with perimeter L. For simplicity, we use the derived Euclidean planar graph G_R as a random Euclidean planar graph as in Fig. 2 (a). In Fig. 2 (a), all the units denoted by vertexes compose the set $U_R = \{u_1, u_2, \cdots, u_{N_p}\}$, where N_p is the number of units in G_R.

B. Mobility Model

Since vehicles move in a localized region, the density of vehicles in the network is inhomogeneous. We use the probability density function to represent the inhomogeneous density of vehicles in the following contents.

In ad hoc networks, to simplify the calculation of asymptotic capacity, the $i.i.d.$ (independent and identically distributed) mobility model is widely used. However, the traffic of VANETs is not a random event, it has social-proximity properties. Therefore, we employ the restricted mobility model to represent the social-proximity traffic of VANETs. In the Euclidean planar graph G_R derived above, vehicles move between units. Since vehicles move in a localized region centered at a fixed home-point because of social activities, each vehicle uniformly chooses a unit in G_R as its home-point. We call the covered area by a home-point as a sub-area that does not overlap with each other.

Let $X_i(t)$ denote the location of vehicle i at time t and $X_i^h(t)$ denote the location of the home-point of vehicle i at time t where t is an integer that denotes the slot sequence number. The Euclidean distance between vehicle i and its home-point at time t is defined by d_i, i.e. $d_i = \| X_i(t) - X_i^h(t) \|$. The spatial stationary distribution of node can be described by a generic, non-increasing function $\phi(d)$ in terms of the distance d from the home-point, and we assume that $\phi(d)$ decays as power law of exponent δ, i.e., $\phi(d) \sim d^{-\delta}$ with $\delta > 0$ as paper [10] did.

We denote a function $s(d) = \min(1, d^{-\delta})$ and normalize it to derive a probability density function over the network area $\phi(d) = \frac{s(d)}{\iint s(d)}$, where $\delta > 0$ denotes a uniform distribution over the space. The value of δ is analyzed in [11].

C. Communication and Interference Model

We assume that during a time slot t only one packet can be tansmitted. Due to the interference of wireless transmissions, one vehicle cannot transmit with two other vehicles at the same time slot. To schedule the transmission flows, we adopt the protocol interference model introduced in [12], which roughly represents the behavior of wireless MAC protocol. Protocol interference model schedule is defined as follows.

At each time slot, a transmission from vehicle i to vehicle j is successful only if: 1)

$$\|X_i(t) - X_j(t)\| \leq r$$

and 2) for any other vehicle l that transmits at t,

$$\|X_l(t) - X_j(t)\| \geq (1 + \Delta)r$$

where Δ is a guard factor defining a protection zone around the receivers.

D. Transmission Model and Relay Scheme

We assume each vehicle is the source of one transmission flow and the destination of another transmission flow. Thus, there are n transmission flows in the network concurrently.

Unicast flows transmit packets via two-hop relay scheme proposed in [13]. If the source vehicle and the destination vehicle of a transmission flow belong to the same home-point, the source vehicle will transmit packet to the destination vehicle directly. If they belong to different home-point, the source destination vehicle will relay the packets through one intermediate vehicle which has more contact opportunity with the destination vehicle.

E. Definitions of Capacity

In this paper, capacity denotes the feasible throughput. The capacity of VANETs is defined as follows.

Definition 1 (capacity of vehicle network) [14]: The average capacity of vehicular network is of order $\theta(g(n))$ [1] bits per second if there are deterministic constants $c > 0$ and $c < c' < +\infty$ such that:

$$\lim_{n\to\infty} \Pr(\lambda(n) = c(g(n)) \ is \ feasible) = 1$$

$$\lim_{n\to\infty} \inf \Pr(\lambda(n) = c(g(n)) \ is \ feasible) < 1$$

Definition 2 (throughput capacity) [15]: Let $G(T)$ denote the number of packets received by all the vehicles during time T. Capacity throughput $\lambda(n)$ is feasible if there is a scheduling scheme for which the following properties hold

$$\lim_{T\to\infty} \Pr\left(\frac{G(T)}{T} \geq \lambda\right) = 1.$$

F. Useful Known Results

In this paper, we will use the following existing results.

Lemma 1 (Groemer Inequality) [16]: Suppose that X is a compact convex set and U is a set of points with mutual distances at least one. Then

$$|U \cap X| \leq \frac{area(X)}{\sqrt{3}/2} + \frac{peri(X)}{2} + 1$$

where $area(X)$ and $peri(X)$ are the area and perimeter of X, respectively.

Lemma 2 (Borel's law of large numbers) [17]: Let $N(e)$ denote the number of times event e occurs in n trials and p is the probability that e occurs. For any positive integer ε, we have

$$\lim_{n\to\infty} Pr\left\{\left|\frac{N(e)}{n} - p\right| < \varepsilon\right\} = 1.$$

III. Capacity calculation of VANETs

A. Maximum Number of Concurrent Transmission Flows

We introduce the independent set and maximum independent number to analyze the wireless transmission interference under interference relationship graph G_P. The maximum independent set and maximum independent number are defined as follows.

Definition 3 (maximum independent set [18]*)*: In graph theory, an independent set of an interference relationship graph is a set of vertices that any two of them are non-adjacent. A maximum independent set is the largest independent set for a given graph.

Definition 4 (maximum independent number [18]*)*: The maximum independent number of a graph is the maximum size of a maximum independent set.

[1] we use Knuth's notation: Given two functions and $g(n) \geq 0$, $f(n) = O(g(n))$ means $\limsup_{n\to\infty} f(n)/g(n) = c < \infty$; $f(n) = \Omega(g(n))$ is equivalent to $f(n) = O(g(n))$; $f(n) = \Theta(g(n))$ means $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

In the interference relationship graph G_p derived above, each vertex denotes a unique unit, and the edges represent the interference relationship between vertices. If there is an edge between vertices i and j, we say vertices i and j are adjacent and have interference. Vehicles in units i and j cannot transmit in the same time slot. According to the definition of maximum independent set, each two vertices in a maximum independent set Y are non-adjacent. Thus, the units denoted by the vertices in set Y cannot transmit in the same time slot. Therefore, the maximum independent number of a graph G_P is the maximum number of concurrent transmission flows M.

As shown in Fig. 2 (b), assume vertices a, b, c, d, e and f are the vertices of the interference relationship graph G_P. The edge in the graph means that the two vertices at the end of the edge cannot transmit packets at the same slot. According to definitions 4 and 5, vertices a, b, c, f compose an independent set A, vertices d and e compose an independent set B. Vertices a, b, c, f can transmit at the same time slot, vertices d and e as well. The elements of A cannot transmit when any element of B is transmitting. In the small interference relationship graph, we can easily find the maximum independent set A, and the maximum independent number is 4. Thus, in one time slot, at most 4 units can transmit packets without interference with other vertices.

For arbitrary network geometry and random network geometry, we introduce a different calculation method to get the maximum independent number, respectively. For the arbitrary interference relationship graph, it is easy to obtain a maximum independent number using the greedy algorithm. For a random interference relationship graph, we introduce the following Corollary from Lemma 1.

Corollary 1: In a square with area S and perimeter L, suppose that X is a compact convex set and U is a set of points with mutual distances at least $(1+\Delta)r$. Then

$$|U \cap X| \leq \frac{S}{\sqrt{3}/2[(1+\Delta)r]^2} + \frac{L}{2(1+\Delta)r} + 1$$

Proof 1: We scale down the Euclidean planar graph with the proportion $(1+\Delta)r : 1$. In the original Euclidean planar graph G_R, the distance between each two elements of independent sets is more than $(1+\Delta)r$. In the scaled down Euclidean planar graph G'_R, the distance between each two elements of independent set is more than 1. Simultaneously, the area and perimeter of Euclidean scale down as proportion. The scaled down area and perimeter are denoted by S' and L' respectively. In a square with area S' and perimeter L', i.e., $S' = S/[(1+\Delta)r]^2$ and $L' = L/(1+\Delta)r$. The scaled down Euclidean planar graph G'_R satisfies Lemma 1, the original Euclidean planar graph G_R satisfies Corollary 1.
Obviously, the maximum independent number is the maximum number of concurrent transmission flows. We can derive the following lemma according to Corollary 1.

Lemma 3: In the rectangular area with side length L, the number of concurrent transmission flows M satisfies

$$1 \leq M \leq \frac{S}{\sqrt{3}/2[(1+\Delta)r]^2} + \frac{L}{2(1+\Delta)r} + 1.$$

B. Bounds of Capacity

According to Lemma 3, we derive the upper bound of throughput capacity using the protocol interference model.

Theorem 1 : For the social-proximity vehicular networks, with the two-hop relay scheme, the average throughput $\lambda(n)$ cannot be better than

$$\lambda(n) \leq \frac{\frac{S}{\sqrt{3}/2[(1+\Delta)r]^2} + \frac{L}{2(1+\Delta)r} + 1}{n}.$$

Proof 2: $G_d(T)$ denotes the total number of packets transmitted through direct-transmission from source to destination during the time interval $[0, T]$, $G_r(T)$ denotes the total number of packets transmitted through relay-transmission during the time interval $[0, T]$. According to Definition 2, throughput $\lambda(n)$ satisfies:

$$\frac{G_d(T) + G_r(T)}{T} \geq n\lambda(n) - \varepsilon \tag{1}$$

where $\varepsilon > 0$ is an arbitrary and fixed number, $\varepsilon \to 0$ as $T \to \infty$. $K(T)$ denotes the total transmit opportunities during $[0, T]$. The total number of transmitted packets must be less than the total number of transmit opportunities during a long time. Since the relay-transmission needs the transmit opportunities twice to transmit one packet, we have

$$\frac{1}{T}K(T) \geq \frac{1}{T}G_d(T) + \frac{2}{T}G_r(T). \tag{2}$$

Substituting (1) into (2), we have

$$\lambda(n) \leq \frac{\frac{1}{T}K(T) + \frac{1}{T}G_d(T)}{2n}. \tag{3}$$

When $\varepsilon \to 0$ as $T \to \infty$.

Due to the interference of wireless transmissions, the total transmissions must be less than concurrent transmissions during time $[0, T]$. According to the law of large numbers, we have

$$\lim_{x \to T} \frac{1}{T}K(T) \leq M. \tag{4}$$

Similarly, we have

$$\lim_{x \to T} \frac{1}{T}G_d(T) \leq M. \tag{5}$$

The two equalities hold when there exists always a transmission flow on each unit of a concurrent transmission group during each time slot. According to lemma 3, substituting (4) and (5) into (3), we have

$$\lambda(n) \leq \frac{M}{n}. \tag{6}$$

Substituting M into (6), we get

$$\lambda(n) \leq \frac{\frac{S}{\sqrt{3}/2[(1+\Delta)r]^2} + \frac{L}{2(1+\Delta)r} + 1}{n}.$$

Thus, the theorem then follows.
Remark : For a given urban region, the area S and perimeter L are constant. Thus, from Theorem 1 we can infer that the per-vehicle throughput $O(\frac{1}{n})$ is feasible. Due to the physical

size of vehicles and streets, the density of vehicles always bounded by a positive number. Therefore, the number of vehicles cannot increase endless. It will approach a constant number. Before the number of vehicles reaches the constant number, the capacity of VANET is scaled by the asymptotic upper bound $O(\frac{1}{n})$ till the number of vehicles cannot increase. We assume that an arbitrary urban has N_h home-point, and vehicles choose its home-point $i.i.d.$. In order to calculate the capacity lower bound, we will prove the following lemma before we go into Theorem 2.

$Lemma\ 4$: Let N_i denote the number of vehicles that belong to the same sub-area. N_i at most scales as $O(n)$ $w.h.p.$ (with high probability).

Proof 3: We use Borel's law of large numbers (Lemma 2) to prove the above lemma. It is easy to know that the probability that a vehicle belongs to a specified sub-area is $\frac{1}{N_h}$. According to Lemma 2, we have

$$\lim_{n\to\infty}\left\{\left|\frac{N_i}{n}-\frac{1}{N_h}\right|<c\right\}=1$$

where c is a positive integer. Thus,

$$\lim_{n\to\infty}\left\{N_i<n\left(\mathrm{c}+\frac{1}{N_h}\right)\right\}=1.$$

Since c and N_h are constant numbers, the lemma follows. According to the above Lemma, we know that at most $O(n)$ vehicles will belong to one specified sub-area. Thus, the transmission opportunities will be shared by at most $O(n)$ vehicles. Therefore, one vehicle can obtain at least $\Omega(\frac{1}{n})$ transmission opportunities in one time slot. Through the above analysis, we derive the following theorem.

$Theorem\ 2$: For the social-proximity vehicular networks, the throughput capacity $\lambda(n)$ can be bound by $\Theta(\frac{1}{n})$ $w.h.p.$.

$Remark$: The lower bound will be also a constant when the number of vehicles cannot increase due to the physical size. For the time being, the capacity of VANET is constant.

IV. Discussion

The delay of VANETs is not considered in this paper. It may be large. However, in a realistic scenario, the delay is one of the most important properties. Therefore, the delay and delay-capacity trade-off are needed to be discussed in our planned future work. In addition, traffic is never a random event. Although the restricted mobility model used in this paper is more accurate than the random mobility model, it also needs improvement. The capacity calculation in this paper is derived with a pure ad hoc scenario. The infrastructure in a city may improve the performance of VANETs remarkably. Thus, the hybrid scenario of VANET is worth to be studied. Since the interference model of this paper is simple, the use of more complex interference model, such as a Gaussian Channel Model, can make the results more accurate.

V. Conclusion

This paper analyzes the asymptotic capacity for social-proximity urban vehicular networks. We proposed a new framework abstracted from real world. The proposed framework established with an Euclidean planar graph and an interference relationship graph. The Euclidean planar graph shows the distribution of units, and the interference relationship graph shows the interference relationship between each two units. The independent set in graph theory is used to analyze the interference relationship in interference relationship graph. We showed that the constant capacity bounds could be achieved at high density area and the asymptotic capacity in sparse area is bounded by $\Theta(1/n)$. Our results indicate that the VANETs are scalable to be deployed in urban environments.

Acknowledgment

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