

Homework 4 -- Due: see website (Released: March 19)

1. [20p] Show that Dijkstra's algorithm may not work if edges can have negative weight.
2. [20p] The *Mating Ritual* takes place over several days. The following events happen each day:

Morning: Each girl stands on her balcony. Each boy stands under the balcony of his favorite among the girls on his list, and he serenades her. If a boy has no girls left on his list, he stays home and does his EECE 320 homework.

Afternoon: Each girl who has one or more suitors serenading her, says to her favorite suitor, "We might get engaged. Come back tomorrow." To the others, she says, "No. I will never marry you! Take a hike!"

Evening: Any boy who is told by a girl to take a hike, crosses that girl off his list.

Termination condition: When every girl has at most one suitor, the ritual ends with each girl marrying her suitor, if she has one.

This ritual can be used to determine matchings in many other situations as well. A popular example is the matching of medical interns to hospitals. Use this algorithm as a basis to **find two stable assignments of students to companies (15 points)** ("the recruitment ritual") when the preference lists are as below (Tables 1 and 2).

In how many days is this algorithm guaranteed to terminate when there are n students and n companies (5 points)?

Student	Company
Arthur	IBM, Nokia, RIM, Microsoft
Galahad	RIM, Nokia, Microsoft, IBM
Guinevere	IBM, Microsoft, RIM, Nokia
Percival	Microsoft, RIM, Nokia, IBM

Table 1: Student preferences

Company	Student
RIM	Percival, Arthur, Guinevere, Galahad
Nokia	Guinevere, Galahad, Arthur, Percival
IBM	Percival, Guinevere, Arthur, Galahad
Microsoft	Galahad, Percival, Guinevere, Arthur

Table 2: Company preferences

3. [20p] Basic counting exercises. Show your work to receive full credit. For questions that involve a deck of cards, you may assume a standard deck of 52 cards.

- (a) (5 points) How many hands of 5 cards have exactly one pair and no three-of-a-kind or four-of-a-kind?
- (b) (5 points) How many hands of 5 cards have two or more Kings?
- (c) (5 points) How many n bit sequences contain exactly k zeros. (The remaining bits have to be 1s.)
- (d) (6 points) For fixed positive integer n , in how many ways can a class of $2*n$ students be divided into n pairs of students.

4. [20p] Fearing student animosity for long hours spent solving homework problems, Prof. Ripeanu decides to move to a bunker much like some former dictators. The entrance to the bunker is protected by an electronic lock that uses a ten digit numeric password. The password uses each of the digits 0;1; : : : ;9 exactly once. To keep the password secure, Prof. Ripeanu decides to avoid guessable sequence "320". Thus none of the potential passwords include this sequences at any position. How many ten digit passwords can Prof. Ripeanu choose from?

Practice problems – not graded: (problem number/page)
18, 15, 21, 22, 23/657; 35, 43, 44, 45, 47, 48/347