Homework 1

Due: see website (Released: January 17)

Note: you can work in groups of up to three. List the IDs of all group members on the first page of your assignment.

0. Subscribe to the mailing list either by emailing sympa@ece.ubc.ca with "subscribe eece320@ece.ubc.ca" in the body of the message or by visiting https://oldlists.ece.ubc.ca/

1. Inference rules [8 points]. For each of the following, define proposition symbols for each simple proposition in the argument (for example: p = "It is a rainy day.") and then write out the logical form of the argument. If the argument form corresponds to an inference rule, state the rule. If not, prove using truth tables.

(a) This is an easy homework and I will get a good score on it. Therefore, I will get a good score on this homework.

(b) Yesterday was extremely hot or Paul did not drink enough water. Paul drank enough water. Therefore, yesterday was extremely hot.

(c) Vancouver will host the 2010 Winter Olympics. Therefore, Vancouver will host the 2010 Winter Olympics or there will be no snow in 2010.

(d) If I work hard, I can finish this homework. If I finish this homework, I will understand the material better. Therefore, if I work hard I will understand the material better.

2. Negations [2 points]. What is the negation of the statement "If I drive then I am sober."

3. Knights and knaves [20 points] Richard is either a knight or a knave. Knights always tell the truth, and only the truth; knaves always tell falsehoods, and only falsehoods. Someone asks Richard, "Are you a knight?" He replies, "If I am a knight then I'll eat my hat."

(a) Must Richard eat his hat? (2p)

(b) Set this up as problem in propositional logic. Introduce the following propositions: p = "Richard is a knight" and q = "Richard will eat his hat." Translate what we are given into propositional logic, i.e., re-write the premises in terms of these propositions. (5p)

(c) Prove that your answer from part (a) follows from the premises you wrote in part (b). (13p)

4. Numerical claims [5 points]. Prove or disprove that for all rational numbers a and b, $c = ab$ is also rational.

5. Your turn to grade [15 points]. Assign a grade of A (excellent) if the claim and proof are correct, F (failure) if the claim is incorrect, if the main idea in the proof is incorrect, or if most of the statements in it are incorrect. Assign a grade of C (partial credit) for a proof that is largely correct, but contains one or two incorrect statements or justifications. Whenever a proof is incorrect, explain your grade. Say what it is incorrect and why.

(a) Claim: For an integer m, if m^2 is odd then m is odd.

Proof. Assume m is odd. Then $m = 2k+1$ for some integer k. Therefore $m^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$, which is odd. Therefore if m^2 is odd, then m is odd.

(b) Claim: For an integer m, if m^2 is odd then m is odd.

Proof. Assume that m^2 is not odd. Then m^2 is even, and $m^2 = 2k$ for some integer k. Thus $2k$ is a perfect square; that is its square root *sqrt(2k)* is an integer. If *sqrt(2k)* is odd, then *sqrt(2k) = 2n+1* for some integer n, which means $m^2 = 2k = (2n+1)^2 = 4n^2+4n+1 = 2(2n^2+2n)+1$. Thus m^2 is odd contrary to our assumption. Therefore *sqrt(2k)* = m must be even. Thus if m^2 is not odd then m is not odd. Hence if m^2 is odd then m is odd.

(c) Claim: For some real number t, if t is irrational, then 5t is irrational.
Proof. Suppose $5t$ is rational. Then $5t = p/q$ where p and q are integers and $q \neq 0$. Therefore $t=p/5q$ where p and $5q$ are integers and $5q \neq 0$, so t is rational. Therefore if t is irrational, then $5t$ is irrational.

6. Prime numbers [20 points]. In class, we have sketched the proof that the set of prime numbers is infinite. Write the complete proof.

7. The principle of induction [12 points]

Let $P(k)$ be a proposition involving a natural number k. Suppose you only know that $\forall k, P(k) \rightarrow P(k+2)$. What can you say about the following statements? Definitely true? Definitely false? Possibly (not necessarily) true? Briefly explain your answers. You may assume that the universe for all quantifiers is the set of natural numbers.

(a) $\forall n, P(n)$
(b) $\forall n, \neg P(n)$
(c) $P(0) \rightarrow (\forall n, P(n+2))$
(d) $P(0) \wedge P(1) \rightarrow (\forall n, P(n))$
(e) $P(n) \rightarrow [\exists m > n, P(m)]$
(f) $(\forall n < 100, P(n)) \wedge (\forall n \geq 100, \neg P(n))$

8.) **Prove by induction.** [18 points]

a.) For n ≥ 1, $\sum_{k=1}^{n} \frac{1}{\sqrt{k}} \geq \sqrt{n}$.

b.) For any n ≥ 1, $1^3 + 2^3 + 3^3 + ... + n^3 = (1 + 2 + 3 + ... + n)^2$.