A Differential Mass Component for Signal Processing Using MEMS

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Abstract—**A complete topological equivalence between network representations of mechanical and electrical systems is hindered by the inertial mass element, represented as an equivalent capacitor with a pin always connected to ground (inertial reference system). This limits the class of synthesizable mechanical filters from equivalent electrical prototypes by excluding the schematics that have floating capacitors. This restriction motivates the introduction of a differential mass component. It is shown that this new component would make it possible to implement a simple third order Chebyshev bandpass filter which can not be implemented using standard masses, springs and dampers.**

Index Terms—**MEMS, filter synthesis, Kirchhoff network, across-through representation, dynamic response, frequency domain analysis, impedance, modeling.**

I. INTRODUCTION

The field of micro-electro-mechanical systems (MEMS) has reached a maturity level where any advance in performance relies on the presence of structured design methods for the control of complex system interactions. There is a need to expand the well-developed methods of analysis and synthesis of electrical networks to the field of electro-mechanical systems to enable more advanced signal processing tasks for mechanical signals. For instance, miniaturized, highly selective filters are of strong interest in mobile telecommunication equipment. An increase in filter selectivity can simplify the signal processing chain, and ultimately reduce size and power consumption which are important goals in this market. MEMS based filters have become increasingly popular because mechanically oscillating structures can achieve much higher quality factors than their electrical LC counterparts. This is mainly due to the present limitations in implementing inductors using modern CMOS technology. Typically, high-order MEMS filters are built using simple coupled-resonators architectures, resulting in "all-pole" transfer functions [1].

The design of advanced inertial microsensors creates a need for more complex signal filtering schemes. Although the input stimuli is mechanical, electrically-tunable filters for mechanical signal processing enable innovative functions such as real-time spectrum analysis of mechanical vibrations [2].

A unified representation of electro-mechanical systems is required for a structured design methodology to take advantage of the large theoretical and practical framework made available by electrical network theory. The main unifying concept is the energetic coupling between subsystems that suggests three categories of primitive components in the case of lumped-parameter systems:

- Components defined strictly in the electrical domain: resistors (R), inductors (L), capacitors (C), etc.
- Components defined strictly in the mechanical domain: inertial masses (M), springs (K), dampers (B), etc.
- Electro-mechanical coupling elements which interface between mechanical and electrical energy domains such as a micro-mechanical comb drive.

The power/energy coupling formalism allows two equivalent (at least for planar Kirchhoff networks) correspondences between the across and through variables in the mechanical and electrical domains. Depending on how the force balance and velocity compatibility relations are mapped to Kirchhoff current and voltage laws for an electrical circuit, forces can be represented as across or through variables. The "force as voltage" association has been widely used in the context of bond-graph theory (effort-flow representation) and has been well-exposed in classical texts by Paynter [3] and Karnopp et al [4]. The "force as current" mapping, introduced by Firestone [5] and Trent [6], is the common approach used in modern multi-domain model libraries implemented in behavioral languages such as VHDL-AMS and Verilog-AMS, and has the advantage of preserving the topology of the interconnections in an electrical circuit and its equivalent mechanical system. The correspondence between primitive elements in the mechanical and electrical domains are shown in Figure 1.

Regardless of how forces are mapped (as currents or voltages), the second law of dynamics must be applied with respect to an inertial reference system. In other words, the constitutive equation for the primitive mass element is restricted to motions relative to the inertial frame of reference:

$$\left(\overline{f_{total}} = m \cdot \frac{d\bar{v}}{dt}\right)_{inertial\ \mathrm{SR}} \quad (1)$$

This supplementary restriction on an inertial mass network component constrains the representation of mechanical systems in terms of equivalent Kirchhoff networks to a subset of the full class. Not all of the electrical network topologies built using analog primitive elements can be represented as equivalent

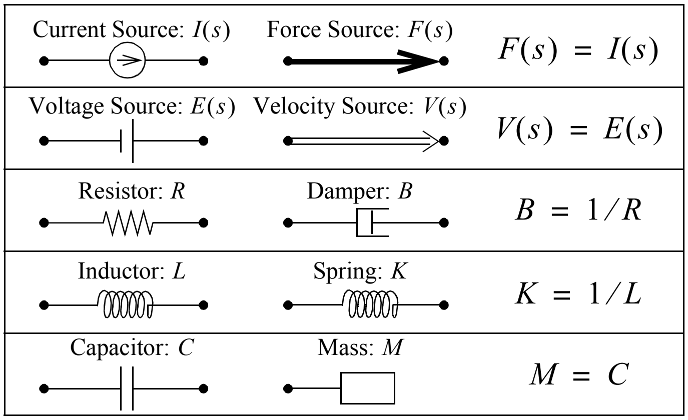


Fig. 1. Typical electro-mechanical equivalents.

mechanical systems. For instance, in the "force as current" representation used throughout this paper, the requirement to consider the motion of the inertial element relative to an inertial reference frame shows that the equivalent circuit of a mass is a grounded capacitor, as shown in Figure 2.

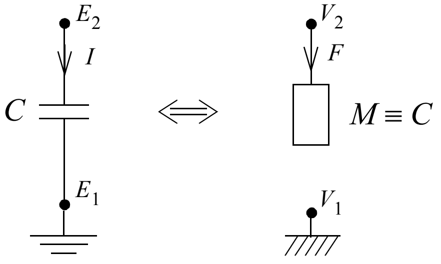


Fig. 2. Capacitor / mass equivalent models.

Any electrical filter using floating capacitors cannot be directly mapped to an equivalent mechanical system. Unfortunately, many of the optimum filter synthesis algorithms generate filters with both floating inductors and capacitors, especially in the case of bandpass filters. The limitation of the mass model induces a limitation in our ability to implement general transfer functions of mechanical filters and to take advantage of generic filter design methods used to develop electrical filters. Obviously, LC filters are optimal from an energy viewpoint since they do not have dissipative elements. The low-performance of inductors that are practically realizable using current CMOS technologies have created a trend towards active RC filters, switched-capacitors filters and active gyrator circuits which mimic the dynamic behaviour of an inductor.

The implementation of a signal processing subsystem in the micro-mechanical domain is a favorable alternative when one requires high-Q filters which operate at high frequencies and consume a small layout area. Ideally, a desired filter transfer function should be mapped by an almost automatic process from any passive electrical network to an equivalent micro-mechanical system. It is the purpose of the present paper to address the issue of bypassing the constraints of the inertial mass model, and make possible a complete mapping between generic LC circuit topologies and equivalent mechanical structures.

II. The Mass Pulley model as differential mass element

Ohm's law for a capacitor states that a differential voltage relationship exists when a current is driven through a capacitor (2). In order for a mechanical component to be fully interchangeable with a capacitor, it must have the corresponding differential velocity relationship shown in (3). The response of a conventional mass is exactly this relationship with V_1 implicitly set to 0 as indicated by Figure 2.

$$E_2(s) - E_1(s) = I(s)\frac{1}{sC} \quad (2)$$

$$V_2(s) - V_1(s) = F(s)\frac{1}{sM} \quad (3)$$

It is shown in [7] that the behaviour described by equation (3) can be obtained by connecting a mass to the output of a differential transmission. There are a number of ways to implement a differential transmission, one of which is the gear system shown in Figure 3. This system obeys the differential angular velocity relationship shown in (4).

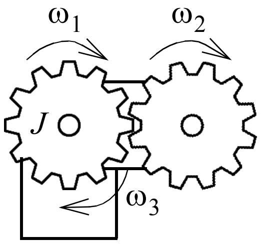


Fig. 3. Simple differential transmission.

$$\omega_1 = \omega_3 - \omega_2 \quad (4)$$

$$\omega_3(s) - \omega_2(s) = \tau(s)\frac{1}{sJ} \quad (5)$$

In Figure 3, the right gear (ω_2) and the rod connecting the two gears (ω_3) are the two inputs while the left gear (ω_1) is the output. All elements are massless and frictionless, except for the left gear, which has a rotational mass, or moment J to which a torque τ is applied. The response of this mechanical system (5) parallels that of an ungrounded capacitor (2).

Another implementation of a differential transmission with a mass at its output is the mass/pulley system shown in Figure 4. In this system, the ideal ropes are infinitely long and stiff and all ideal pulleys are massless and frictionless. The system operates in zero gravity and the ideal ropes can be compressed without buckling. It is shown in [7] that the mass is translated by the differential relationship shown in (3) which exactly mimics the ungrounded capacitor with the differential voltage relationship shown in (2). The mass/pulley system is more topologically consistent with a capacitor than the gear-based differential transmission, and is therefore the preferred analogy when used as a modelling tool.

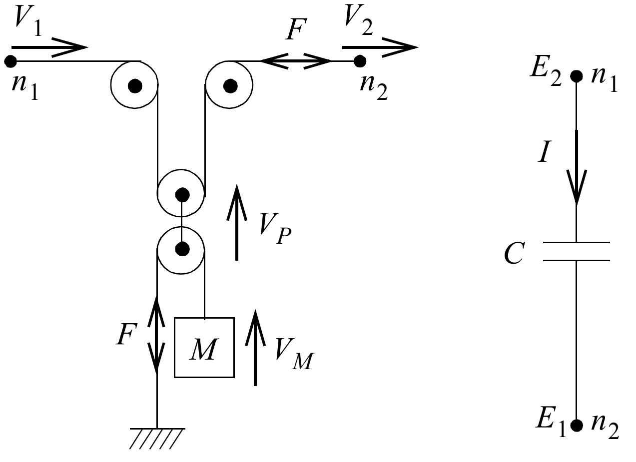


Fig. 4. Mass / pulley equivalent of a capacitor.

III. Differential mass element in MEM structures

In the context of MEMS, practical limitations dictate the types of transmission that can be implemented. Neither the differential gears nor the pulley mechanism are straightforward to implement at a micro-scale. However, some of the associated hurdles can be overcome by moving from a purely mechanical design to a hybrid electro-mechanical solution which relies on electrostatic forces for physical constraints and couplings. Such composite mechanical-electrical-mechanical (M-E-M) structures promise additional benefits due to the effective zero damping (i.e. zero loss) in an electro-static joint at low pressures.

Recently, electret materials have been used in MEMS applications such as miniature microphones [8] and capacitive-based energy scavenging systems [9]. Thin-film teflon, implanted with charges emitted by a pulsed electron gun, can achieve stable charge densities from $10^{-5}\ C/m^2$ to $8x10^{-4}\ C/m^2$. Unlike the attractive electrostatic force in a movable capacitive structure, electret layers can generate repulsive electrostatic forces without external excitation. Patterned charged electret thin films can be used to couple the motion of an inertial mass element to the differential motion of two physical input terminals, thus performing the analog function of a differential transmission or, in the electrical domain, an ideal transformer. Alternatively, an asymmetric field distribution could be exploited to generate lateral repulsive electrostatic forces, as reported in [10]. This alternative would, however, require an external source to generate the necessary forces.

As previously stated, a component equivalent to an ideal transformer would transform a grounded capacitor into an ungrounded capacitor (see Figure 5). The transformer ratio n scales the value of the input capacitance C_0 and changes the single-ended configuration of C_0 into a differential input across pins 1 and 2.

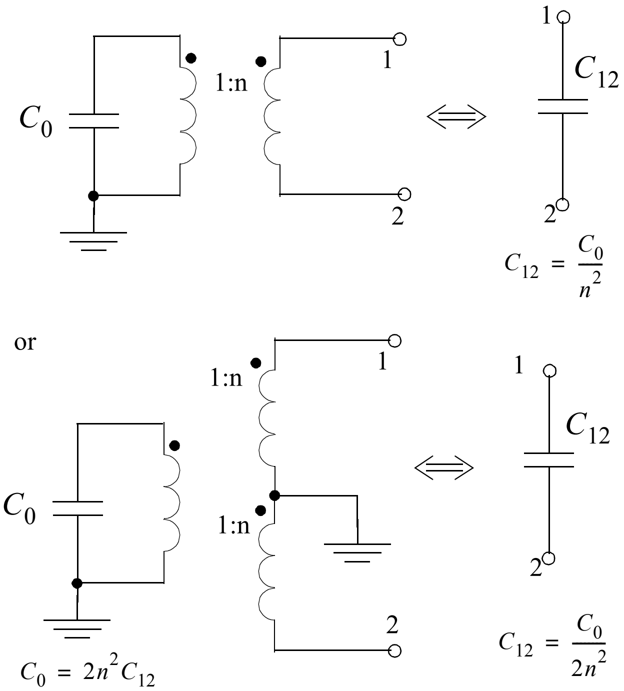


Fig. 5. Using an ideal transformer to convert a grounded capacitor to a floating one.

IV. Example of LC filter structure mapping

A simple example that uses a floating capacitor is a 3rd order Chebyshev bandpass filter such as that shown in Figure 6.

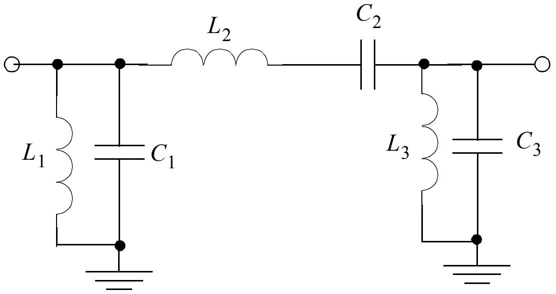
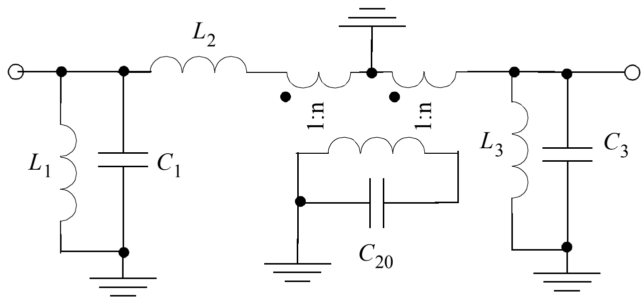


Fig. 6. Third order Chebyshev bandpass filter.

It would be useful to be able to implement this filter mechanically since Chebyshev filters are high-Q filters that provide steep transitions between the pass band and stop bands. The downside of these filters is that they introduce ripples of controllable magnitude into the pass band. As seen from Figure 6, the presence of a floating capacitor C_2 makes this filter impossible to implement in the absence of an analog mechanical component. By adding an ideal differential transformer, C_2 can be implemented using a grounded capacitor C_{20}, which is the electrical analog of a conventional mass scaled by the transformer ratio n. The schematic of the transformed Chebyshev bandpass filter is shown in Figure 7. Here, the LC filter can be mapped to a micro-mechanical layout, provided that there exists an associated component which simulates the ideal transformer.



$$C_{20} = 2n^2C_2$$

Fig. 7. Transformed Chebyshev filter schematic.

A mechanical filter such as this could also be used to condition electrical signals by including comb drives at the input and output terminals. Comb drives transform an electrical signal into a mechanical one, and vice versa. The constitutive equations for a general capacitive coupling element with one degree of freedom (DOF), generically called q, are given by (6) and (7) with the equivalent large signal model shown in Figure 8. This system results in an E-M-E implementation of an electrical filter.

$$f_q = -\frac{1}{2}u^2\frac{dC}{dq}(q) \tag{6}$$

$$i = \frac{dQ}{dt} = C(q)\frac{du}{dt} + u\frac{dC}{dq}(q)v_q \tag{7}$$

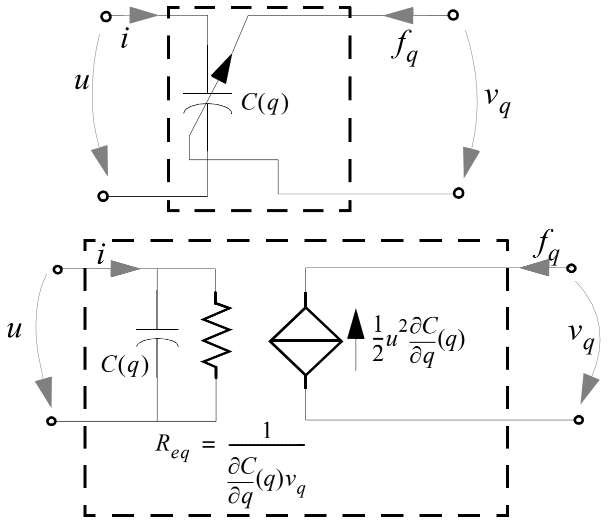


Fig. 8. Electro-mechanical diport (capacitor with 1DOF) and its equivalent large signal model.

V. Conclusions

A complete topological equivalence between passive electrical and mechanical networks requires the existence of a floating (ungrounded) capacitor in the mechanical domain. MEMS offers the capability to couple M-E-M blocks and convert inertial mass elements into equivalent floating mechanical capacitors. This opens the door to the realization of arbitrary passive filter architectures in the mechanical domain. Two paths are presently being investigated for such blocks; the use of thin-film electrets and/or asymmetrical electrical fields effects.

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