State-Space

$E3.1 \quad x = [i_{L1} \quad i_{L2} \quad v_{C1} \quad v_{C2}]^T$

$E3.2$

Define $u = i$, and let $k_1 = k_2 = 1$. Then,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ k_3 \end{bmatrix}, \text{and} \quad \mathbf{x} = \begin{pmatrix} y \\ v \end{pmatrix}$$

$E3.3 \quad -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

$E3.4$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -6 & -4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$E3.5$

$$\mathbf{A} = \begin{bmatrix} 0 & -k \\ a & -(fk+d) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 \\ f \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & b \end{bmatrix} \text{ and } \quad \mathbf{D} = [0]$$

$E3.9$

In state-variable form we have

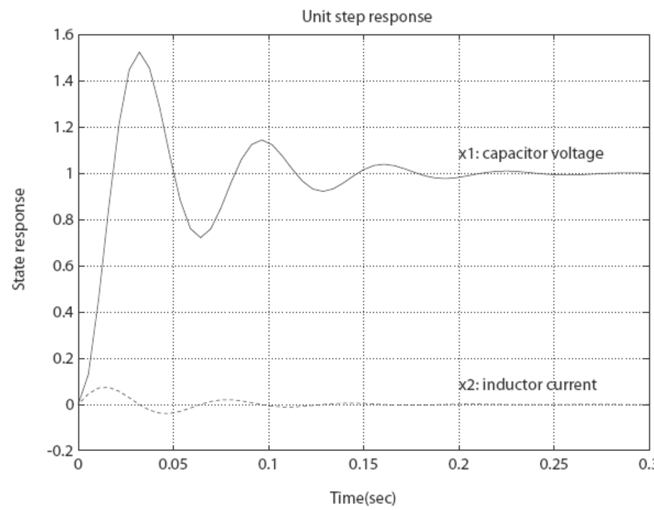
$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & \frac{1}{2} \\ 1 & -\frac{3}{2} \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} r, \quad y = \begin{bmatrix} 1 & -\frac{3}{2} \end{bmatrix}\mathbf{x} + \begin{bmatrix} -1 \end{bmatrix} r .$$

The characteristic equation is

$$s^2 + \frac{5}{2}s + 1 = (s+2)(s+\frac{1}{2}) = 0 .$$

$E3.12$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} v_{in} .$$



$E3.15$

In state variable form we have

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(k+k_1)}{m} & -\frac{b}{m} & \frac{k_1}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m} & 0 & -\frac{(k+k_1)}{m} & -\frac{b}{m} \end{bmatrix}\mathbf{x}$$

where $x_1 = x, x_2 = \dot{x}, x_3 = q$ and $x_4 = \dot{q}$.

$E3.16$

Let $x_1 = x, x_2 = \dot{x}, x_3 = q$ and $x_4 = \dot{q}$. Then,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & -\frac{(k_1+k_2)}{m_2} & -\frac{(b_1+b_2)}{m_2} \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u(t)$$

Since the output is $y(t) = q(t)$, then

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}\mathbf{x} .$$

*E*3.17

$$\dot{\mathbf{x}} = \begin{bmatrix} -\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right) & \frac{1}{R_2C_1} \\ -\frac{1}{R_2C_2} & -\left(\frac{1}{R_3C_2} + \frac{1}{R_2C_2}\right) \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{1}{R_1C_1} & 0 \\ 0 & \frac{1}{R_3C_2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix}$$

*E*3.18

Let $x_1 = i_1, x_2 = i_2, x_3 = v, u_1 = v_a$ and $u_2 = v_b$. Then,

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{R}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & 0 & -\frac{1}{L_2} \\ \frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \mathbf{u}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{x} + [0]\,\mathbf{u} \,.$$

*E*3.21

The transfer function is

$$G(s) = \mathbf{C}\,[s\mathbf{I} - \mathbf{A}]^{-1}\,\mathbf{B} + \mathbf{D} = \frac{-1}{s^2 + 2s + 1} \,.$$

The unit step response is

$$y(t) = -1 + e^{-t} + te^{-t} \,.$$

*P*3.1

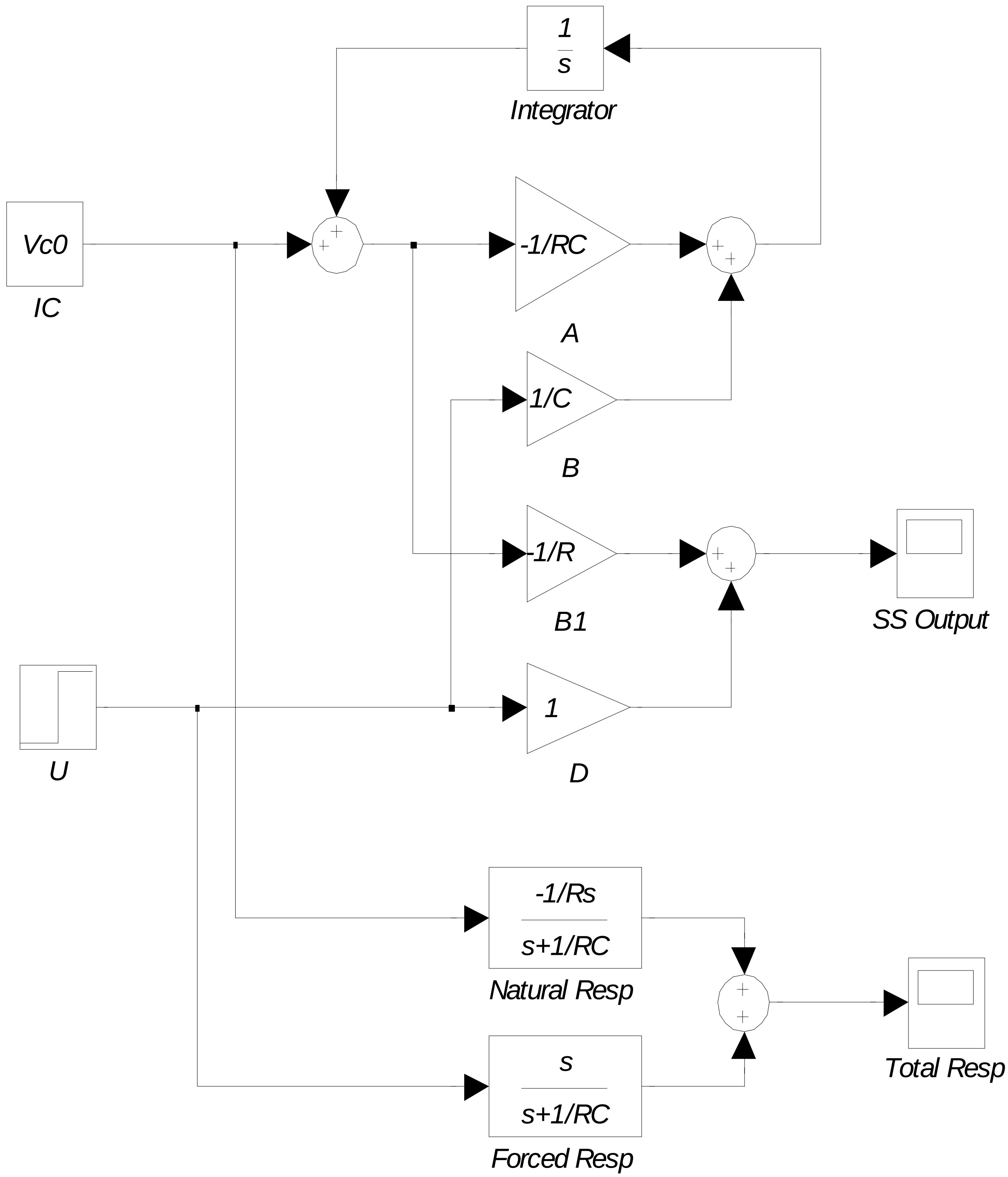
(a) Select the state variables as $x_1 = i$ and $x_2 = v_c$.

(b) The corresponding state equations are

$$\dot{x}_1 = \frac{1}{L}\,v - \frac{R}{L}\,x_1 - \frac{1}{L}\,x_2$$
$$\dot{x}_2 = \frac{1}{C}\,x_1 \,.$$

(c) Let the input $u = v$. Then, in matrix form, we have

$$\dot{\mathbf{x}} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u$$



1
s
Integrator
Vc0
IC
-1/RC
A
1/C
B
-1/R
B1
SS Output
U
1
D
-1/Rs
s+1/RC
Natural Resp
s
s+1/RC
Forced Resp
Total Resp