E 2.5 The answer in the book is wrong $\frac{V_o}{V_i} = 1$

E 2.7 $$\frac{I(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

E 2.8 $$T(s) = \frac{KG_1(s)G_2(s)/s}{1 + G_1(s)G_2(s)\left[(H_2(s) + H_1(s)\right] + G_1(s)H_3(s) + KG_1(s)G_2(s)/s}.$$

E 2.23 $$\frac{Y(s)}{R(s)} = T(s) = \frac{10}{s^2 + 21s + 10}\ .$$

E 2.26 $$Y(s)/T_d(s) = \frac{G_2(s)}{1 + G_1G_2H(s)}\ .$$

E 2.28

(a) If

$$G(s) = \frac{1}{s^2 + 15s + 50} \quad \text{and} \quad H(s) = 2s + 15 ,$$

then the closed-loop transfer function of Figure E2.28(a) and (b) (in Dorf & Bishop) are equivalent.

(b) The closed-loop transfer function is

$$T(s) = \frac{1}{s^2 + 17s + 65} .$$

Note that part (a) has many possible correct answers.
Any answer that satisfies the following conditions is correct:
Exercise: try to derive this relationship on your own.

$$G = \frac{N_G}{D_G} \quad H = \frac{N_H}{D_H}$$

$$N_G = D_H = 1$$

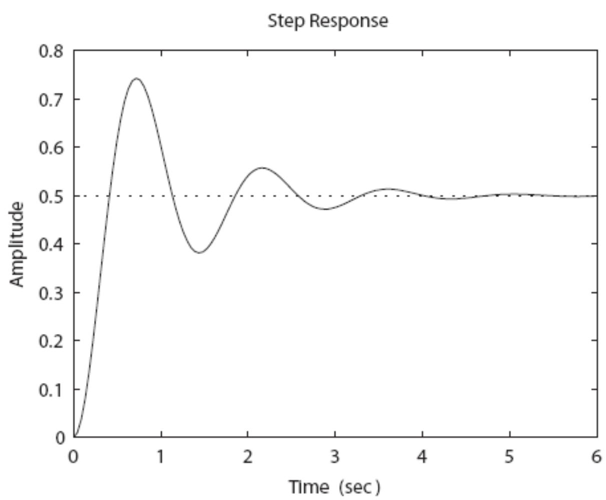
$$D_G + N_H = s^2 + 17s + 65$$

E2.29 (a) The closed-loop transfer function is

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{10}{s^2+2s+20} \quad \text{where} \quad G(s) = \frac{10}{s^2+2s+10} \ .$$

(b) The output $Y(s)$ (when $R(s) = 1/s$) is

$$Y(s) = \frac{0.12}{s} - \frac{0.24}{s+10} + \frac{0.12}{s+20} \ .$$



(c) The plot of $y(t)$ is shown in Figure E2.29. The output is given by

$$y(t) = 0.12 - 0.24e^{-10t} + 0.12e^{-20t} \ .$$

P 2.25

$$Y(s)/R(s) = G(s) \ .$$

P 2.32

Finally, solving for $Y_1(s)$ yields

$$Y_1(s) = T_1(s)R_1(s)$$

where

$$T_1(s) = \left[\frac{G_1(s)G_2(s)(1-G_5(s)G_6(s)H_2(s))}{(1+G_1(s)G_2(s)H_1(s))(1-G_5(s)G_6(s)H_2(s)) - G_3(s)G_4(s)G_6(s)H_2(s)}\right] \ .$$

Similarly, for $Y_2(s)$ we obtain

$$Y_2(s) = T_2(s)R_1(s) \ .$$

where

$$T_2(s) = \left[\frac{G_1(s)G_4(s)G_6(s)}{(1+G_1(s)G_2(s)H_1(s))(1-G_5(s)G_6(s)H_2(s)) - G_3(s)G_4(s)G_6(s)H_2(s)}\right] \ .$$

P2.34 The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = \frac{G_3(s)G_1(s)(G_2(s)+K_5K_6)}{1 - G_3(s)(H_1(s)+K_6) + G_3(s)G_1(s)(G_2(s)+K_5K_6)(H_2(s)+K_4)} \ .$$

P 2.35

$$\frac{Y_2(s)}{X(s)} = \frac{k_2(bs+k_1)}{(m_1s^2+bs+k_1)(m_2s^2+bs+k_1+k_2) - (bs+k_1)^2} \ .$$

P2.36 (a) We can redraw the block diagram as shown in Figure P2.36. Then,

$$T(s) = \frac{K_1/s(s+1)}{1+K_1(1+K_2s)/s(s+1)} = \frac{K_1}{s^2+(1+K_2K_1)s+K_2} \ .$$

P 2.47

$$\frac{X_1(s)}{F(s)} = \frac{\Delta_2(s)}{\Delta_1(s)\Delta_2(s) - (b_1 s + k_1)^2}.$$

$$\Delta_1 := m_v s^2 + b_1 s + k_1.$$

$$\Delta_2 := m_t s^2 + (b_1 + b_2)s + k_1 + k_2.$$

P2.50 (a) The closed-loop transfer function is

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{6205}{s^3 + 13s^2 + 1281s + 6205} .$$

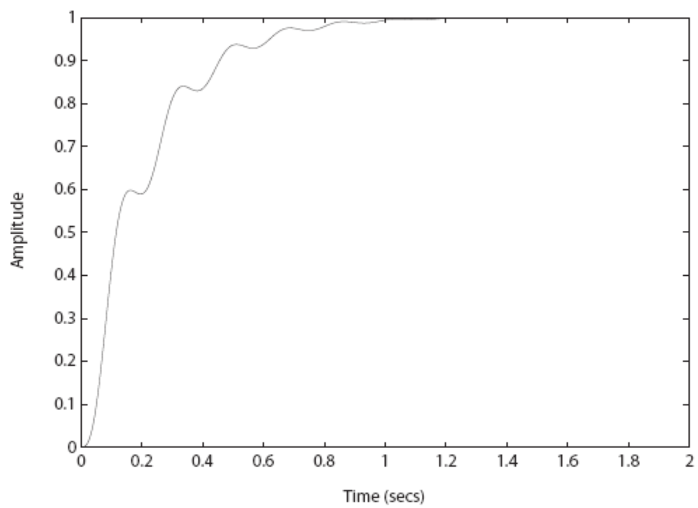


FIGURE P2.50
Step response.

(b) The poles of $T(s)$ are $s_1 = -5$ and $s_{2,3} = -4 \pm j35$.

(c) The partial fraction expansion (with a step input) is

$$Y(s) = 1 - \frac{1.0122}{s+5} + \frac{0.0061 + 0.0716j}{s+4+j35} + \frac{0.0061 - 0.0716j}{s+4-j35} .$$

P2.51 (a) The closed-loop transfer function is

$$T(s) = \frac{14000}{s^3 + 45s^2 + 3100s + 14500} .$$

(b) The poles of $T(s)$ are $s_1 = -5$ and $s_{2,3} = -20 \pm j50$.

(c) The partial fraction expansion (with a step input) is

$$Y(s) = 0.9655 - \frac{1.0275}{s+5} + \frac{0.0310 + 0.0390j}{s+20+j50} + \frac{0.0310 - 0.0390j}{s+20-j50} .$$

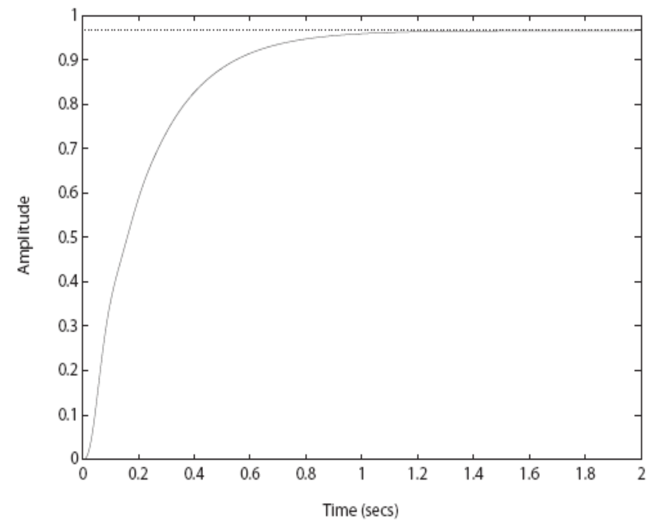


FIGURE P2.51
Step response.

(d) The step response is shown in Figure P2.51. The real root dominates the response.

(e) The final value of $y(t)$ is

$$y_{ss} = \lim_{s\to 0} Y(s) = \lim_{s \to 0} T(s) = 0.97 .$$

Block Diagram HW Soln.

1) $$T_{(s)} = \frac{G_1}{1 + G_1G_2H_2 + G_1G_3H_2} \begin{bmatrix} G_2 \\ G_3 \end{bmatrix}$$

2) $$T_{(s)} = \frac{G}{1 + K_2G} \begin{bmatrix} K_2 & 1 - K_1K_2 \end{bmatrix}$$

3) $$T_{(s)} = \begin{bmatrix} \frac{G_1G_3(G_2 + G_2G_5H_2 + G_2G_5G_7G_8 + G_5G_7G_9)}{(1+G_3H_1)(1+G_5H_2)} & \frac{G_3G_4G_5(G_2G_8 + G_9)}{(1+G_3H_1)(1+G_5H_2)} \\ \frac{G_1G_5G_6G_7}{1 + G_5H_2} & \frac{G_4G_5G_6}{1+G_5H_2} \end{bmatrix}$$

4) $$T_{(s)} = \frac{s^2M_2 + K_2}{s(s^2M_1M_2B_1(K_1+K_2) + sM_1M_2K_1K_2 + B_1K_1K_2(M_1+M_2)} \begin{bmatrix} 1 & \frac{K_2}{s^2M_2+K_2} \end{bmatrix}$$

5) $$T_{(s)} = \frac{s(s^2M + sB + K)}{s^4M^2 + s^3MB + s^2(3MK) + s(2KB) + K^2} \begin{bmatrix} 1 & \frac{sM}{sM+B} \end{bmatrix}$$

Block Diag. HW Cont.

6)

$$T(s) = \begin{bmatrix} \dfrac{G_1(1+G_2H)}{1+H(G_1+G_2)} & \dfrac{G_1G_2H}{1+H(G_2-G_1)} \\ \dfrac{G_1G_2H}{1+H(G_1-G_2)} & \dfrac{G_2(1+G_1H)}{1+H(G_1+G_2)} \end{bmatrix}$$