Transceiver Design in Millimeter Wave Full-Duplex Multi-User Massive MIMO Communication Systems

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ABSTRACT We consider hybrid transmit precoding and receive combining in a millimeter wave (mmWave) multiple-input multiple-output (MIMO) multi-user cellular system, where a full-duplex (FD) base station (BS) serves multiple half-duplex (HD) uplink and downlink users simultaneously. Jointly deriving the four matrices of analog precoder, digital precoder, analog combiner, and digital combiner for mmWave FD communications is intractable. Hence, we propose two approaches for either designing joint analog and digital beamformers separately at the transmitters and receivers, or jointly deriving transmit precoders and receive combiners with separate designs of analog and digital beamforming matrices that maximize the sum-rate. In the first proposed method, at the first step, we derive an iterative solution for the fully digital minimum-mean-square-error (MMSE) transmit/receive beamforming. Then, exploiting the spatial structure of mmWave channels, sparse approximation problems are formulated to jointly design radio frequency (RF) and baseband filters approximating the fully digital beamformers via orthogonal-matchingpursuit (OMP) algorithm. To reduce the computational complexity of the first proposed method, which is mainly caused by the iterations between the transmit and receive beamformings, in the second presented scheme, transmit precoding and receive combining matrices are calculated jointly, while RF and digital beamformings are derived separately. More specifically, digital beamforming matrices are derived using the block diagonalization (BD) approach to mitigate the intra-user and inter-users interference, and analog filters are calculated via solving sum-rate maximization problem. In the simulation results, it is shown that our proposed OMP-based hybrid beamforming (HBF) scheme outperforms other HBF approaches for mmWave FD communications in the literature. In addition, it is shown that when the number of RF chains is at least double the number of data streams, the proposed BD-based HBF further improves over the OMPbased HBF in terms of sum-rate. Thus, choosing between OMP-based and BD-based HBF depends on the number of available RF chains.

INDEX TERMS Full-duplex (FD), millimeter wave communications, multi-user (MU)-MIMO, transceiver design, hybrid beamforming (HBF), orthogonal-matching-pursuit (OMP), block diagonalization (BD).

I. INTRODUCTION

M ILLIMETER wave (mmWave) bands from 30 GHz to 300 GHz are prime candidates for new spectrum in fifth-generation (5G) and beyond cellular systems, with the available unlicensed bandwidth reaching 10 GHz [1], [2]. The main obstacle in the successful deployment of mmWave cellular systems is the large path loss, as a result of the ten-fold decrease in the wavelength [3]. On the other hand, a large-scale antenna arrays which can be built in a small volume due to the short wavelength can be employed to provide significant beamforming gains to combat the path loss and to synthesize highly directional beams.

For traditional multiple-input multiple-output (MIMO) systems, precoding is fully accomplished at digital baseband through digital precoders, which can adjust both the magnitude and phase of the signals. However, fully digital precoding approach requires a radio frequency (RF) chain, including signal mixers, amplifiers, and analog-to-digital converters (ADCs), per antenna which is impractical and costly for mmWave base stations (BSs) equipped with large antenna

arrays. While the small wavelengths of mmWave frequencies facilitate the use of a large number of antenna elements, the prohibitive cost and power consumption of RF chains make digital precoding infeasible. Fully analog solutions, on the other hand, require only a single RF chain for the entire antenna array but have no capability of digital processing. Given such unique constraints in mmWave MIMO systems, a hybrid precoding architecture (an analog beamformer in RF band and a digital filter in baseband) has received much consideration, which only requires a small number of RF chains interfacing between a low-dimensional digital precoder and a high-dimensional analog precoder [4], and thus, reduces the required number of RF chains by precoding/combining of antenna signals. Therefore, hybrid beamforming (HBF) strikes a balance between these two solutions, wherein the number of RF chains can be designed to be between 1 (analog beamforming) and the number of antennas (digital beamforming).

Hybrid MIMO beamforming in mmWave communications have been proposed for single-user MIMO systems in [4] and [5]. The proposed algorithms are based on the orthogonal matching pursuit (OMP) method, which is popular for sparse approximation [6], where RF beamforming vectors are picked from certain candidate vectors, such as array response vectors pointing towards the angle of arrivals/departures (AoAs/AoDs), and/or discrete Fourier transform (DFT) beamformers. The authors in [7] introduce HBF design to cancel the self-interference (SI) in a pointto-point full-duplex (FD) mmWave communication system. However, with the realization that more profitable use of the large antenna arrays could be possible in certain scenarios, there have been some recent works on evaluating the system performance with different MIMO configurations like multiuser MIMO and spatial multiplexing in [8]-[11]. In [12], an HBF design is proposed for K-user FD MIMO interference channels, in which combiners and precoders are designed such that both the residual SI power and the multi-user interference power are minimized. In [13], an OMP-based hybrid precoding scheme is investigated for the multi-user MIMO system with the limited feedback assumption. Authors in [14] consider a downlink cellular network with multi-user MIMO communications and propose a block diagonalization (BD) method to design digital precoders and combiners for HBF to mitigate the inter-user and inter-stream interference signals. Furthermore, the equal gain transmission (EGT) method is used to derive analog beamforming matrices to harvest a large antenna array gain. In [15], a two-step HBF design is presented for the downlink of a massive MIMO communication, in which, in the analog stage, authors maximize the capacity of the baseband channel, and in the digital stage, the BD approach is applied for managing the interference signals. Authors in [16] propose a minimum mean square error (MMSE)-based HBF design for massive MIMO pointto-point communications in which a manifold optimizationbased scheme is introduced to deal with the fixed amplitude constraint of RF beamforming directly. In [17], a two-step

algorithm is proposed for deriving RF beamforming in an HBF structure. First, by formulating a convex problem, an initial beam set is determined to maximize the lower bound of the sum-rate. Then, based on this initial set, an iterative update approach is introduced to calculate a better beam set. Motivated by the low computational complexity of the OMP and BD approaches, in this paper, we apply these methods to design HBF matrices in a cellular network with multi-user MIMO communications in the uplink (UL) and downlink (DL) channel using the mmWave band.

FD transmission has attracted tremendous attention from both the academic and industrial fields to nearly double the spectral efficiency of the existing wireless communication systems by enabling transmission and reception of signals simultaneously over the same time and frequency resource [18]. The key challenge to realize the FD system is suppressing SI to a sufficiently low level. The authors in [19]- [23] studied fully digital transceiver design in FD cellular systems, in which a BS operating in FD mode communicates with UL and DL users operating in halfduplex (HD) mode simultaneously. In addition to SI at the BS, the optimization problems are exacerbated by the cochannel interference (CCI) caused by the UL users to the DL users. Since mmWave communications are inherently directional, in FD cellular systems, directional transmission and reception can be exploited [24]. The intuitive approach is to exploit the beamforming of transmission and reception antennas to reduce SI. More specifically, with the directions of transmit and receive antennas not directed towards each other, the SI can be reduced by beamforming [25]. In [26], an HBF design is developed at the BS to manage SI signal and minimize sum-MSE across all single antenna mobile users in an FD massive MISO system. A novel angularbased HBF method is introduced in [27] for a point-point FD mmWave massive MIMO network to minimize sum-MSE. In this paper, self-interference cancellation (SIC) is further improved by using angle-of-departure (AoD) and angle-ofarrival (AoA) information. Authors in [28] study HBF design for an FD mmWave MIMO relay network, in which by applying a zero-space beamforming scheme, SI signal is attenuated at the FD relay. However, although some works including [7], [12], [26], [27], and [28] have been done to demonstrate the viability of FD systems for mmWave bands, to the best of our knowledge, potential gains of HBF for FD cellular networks with multi-user MIMO communications in mmWave bands have not been reported yet. It is worth noting that authors in [12] have not considered a cellular set up with a BS, UL, and DL users, and have studied an interference multi-user MIMO channel instead. In addition, in [12], the SI and multi-user interference are minimized and authors have not directly dealt with the spectral efficiency which is an important key performance indicator in practice.

Motivated by the above, in this paper, we make an attempt to understand the benefits and actual gains that can be achieved by the use of HBF in the FD transceivers of a cellular network working in mmWave bands. Due to

TABLE 1. Our Contribution

Concepts of Contribution	This paper	[4], [5]	[7]	[12]
mmWave FD communications	\checkmark	×	\checkmark	\checkmark
Multi-user MIMO communications	\checkmark	×	×	\checkmark
OMP-based HBF	√	\checkmark	×	×
Cellular communications	\checkmark	×	×	×
Applying the BD method for interference management	\checkmark	×	×	×

the computational complexity of jointly deriving the four matrices of analog precoder, digital precoder, analog combiner, and digital combiner, we propose two policies for either calculating joint RF and baseband filters separately at the transmitters and receivers, or jointly deriving transmit precoders and receive combiners with a separate design for analog and digital beamforming matrices. Our contributions are illustrated in the Table 1 in comparison with the literature, and can be listed as follows:

- We develop an OMP-based algorithm to design hybrid filters for the receiver, which successively selects RF beamforming vectors from a set of candidate vectors using the MMSE criterion and optimizes the corresponding baseband processor in the least squares sense. Similar to [31], we employ uplink-downlink duality to compute the hybrid MIMO beamformers for the transmitters by reversing the system, and applying the same OMP-based algorithm.
- We propose a BD-based approach to jointly derive transmit precoding and receive combining filters with separate designs of analog and digital beamformings aiming to maximize the sum-rate of UL and DL users. In particular, RF and digital beamformings are designed separately in two stages to minimize information loss at each step.
- In the simulation results, we show that our two proposed HBF designs have a close performance to the fully digital transceiver design but with a low number of RF chains. In addition, it is shown that the proposed OMP-based HBF approach achieves higher spectral efficiency compared with the multi-user FD (MUFD) beamforming scheme proposed in [12]. Finally, we illustrate that if the number of RF chains is at least double the number of data streams, the proposed BD-based HBF scheme further enhances the spectral efficiency performance. Therefore, based on the number of available RF chains, we choose between OMP-based and BD-based HBF methods.

Notation: Matrices and vectors are denoted as bold capital and lowercase letters, respectively. $(\cdot)^T$ is the transpose, and $(\cdot)^H$ is the conjugate transpose. \mathbf{I}_N and $\mathbf{0}_{N \times M}$ are the $N \times N$ identity and $N \times M$ zero matrix, respectively; $\operatorname{tr}(\cdot)$ is the trace; $|\cdot|$ is the determinant; $\mathbb{E}[\cdot]$ denotes the expectation. The (i, j)-th element, *i*-th row, and *i*-th column of matrix \mathbf{A} are denoted by $\mathbf{A}(i, j)$, $\mathbf{A}(:, j)$, and $\mathbf{A}(i, :)$, respectively. $\|\mathbf{X}\|_F$ and $\|\mathbf{x}\|_2$ denote the Frobenius norm of matrix \mathbf{X} and the Euclidean norm of vector \mathbf{x} , respectively, and $\|\mathbf{x}\|_0$ denotes the number of non-zeros in vector x.

The remainder of the paper is organized as follows. Section II introduces the considered system and channel models. In Section III, the considered problem formulation is given. The proposed OMP-based HBF scheme for jointly designing analog and digital filters is derived in Section IV. In Section V, we introduce the BD-based HBF approach for jointly deriving transmit precoders and receive combiners. Simulation results for the proposed policies are presented in Section VI, and Section VII concludes the paper.

II. SYSTEM MODEL

We consider a multi-user massive MIMO system, in which an FD BS serves K UL and J DL users simultaneously. As it is illustrated in Fig. 1, each node in the system is equipped with a hybrid MIMO beamformer. The BS is equipped with M_0 and N_0 transmit and receive antennas, respectively, and $M_0^{\rm RF}$ and $N_0^{\rm RF}$ RF chains at the transmit and receive chains, respectively. The number of transmit antennas and RF chains at the UL users are M and M^{RF} , respectively. The number of receive antennas and RF chains at the DL users are Nand N^{RF} , respectively. Each of the UL and DL users has $N_{\rm s}$ data streams to transmit or receive. In order to reap the benefits of the massive MIMO communication and to reduce the implementation complexity, we assume that the number of RF chains is constrained by $JN_s \leq M_0^{\text{RF}} \ll M_0$ and $KN_s \leq N_0^{\text{RF}} \ll N_0$ for the BS, $N_s \leq M^{\text{RF}} \leq M$ for the UL users, and $N_s \leq N^{\text{RF}} \leq N$ for the DL users. The source symbols $\mathbf{s}_k^{\text{UL}} \in \mathbb{C}^{N_s}$ and $\mathbf{s}_j^{\text{DL}} \in \mathbb{C}^{N_s}$ for the k-th UL and the *j*-th DL user, respectively are assumed to be independent and identically distributed (i.i.d.), and are zero mean with variance identity matrix. As it can be seen from Fig. 1, the hybrid MIMO beamformer for the data streams of the k-th UL user consists of baseband precoder $\mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}} \in \mathbb{C}^{M^{\mathrm{RF}} \times N_{\mathrm{s}}}$ followed by the RF beamformer $\mathbf{V}_{k,\mathrm{UL}}^{\mathrm{RF}} \in \mathbb{C}^{M \times M^{\mathrm{RF}}}$. Then, at the BS, the received signal is processed by RF combiner $\mathbf{U}_{\mathrm{UL}}^{\mathrm{RF}} \in \mathbb{C}^{N_0 \times N_0^{\mathrm{RF}}}$ and $\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}} \in \mathbb{C}^{N_0^{\mathrm{RF}} \times N_{\mathrm{s}}}$, respectively. The baseband MIMO beamformed for the second statement of the second statem hybrid MIMO beamformers for the data streams of the j-th DL user consists of baseband precoder $\mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}} \in \mathbb{C}^{M_0^{\mathrm{RF}} \times N_s}$ followed by the RF filter $\mathbf{V}_{\mathrm{DL}}^{\mathrm{RF}} \in \mathbb{C}^{M_0 \times M_0^{\mathrm{RF}}}$. Afterwards, at the *j*-th downlink user, the received signal is processed by RF combiner $\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}} \in \mathbb{C}^{N \times N^{\mathrm{RF}}}$ and $\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}} \in \mathbb{C}^{N^{\mathrm{RF}} \times N_{\mathrm{s}}}$, respectively.

A. CHANNEL MODEL

We assume a narrowband block-fading propagation channel, and denote $\mathbf{H}_{k}^{\mathrm{UL}} \in \mathbb{C}^{N_{0} \times M}$ and $\mathbf{H}_{j}^{\mathrm{DL}} \in \mathbb{C}^{N \times M_{0}}$ as the



FIGURE 1. The considered mmWave FD MIMO cellular systems.

k-th UL and the *j*-th DL user channel, respectively. The SI channel between the transmitter and receiver antennas of the BS is denoted as $\mathbf{H}_0 \in \mathbb{C}^{N_0 \times M_0}$, and the CCI channel from the *k*-th UL user to the *j*-th DL user is denoted as $\mathbf{H}_{jk}^{\text{DU}} \in \mathbb{C}^{N \times M}$ (see Fig. 1). We note that the channel state information (CSI) and the interference-plus-noise covariance matrix can be estimated at the receiver of each link and fed back to the transmitters. Therefore, the transmitters are able to obtain CSI of the links that are directly connected to them.

Due to high free-space path loss, the mmWave propagation environment is well characterized by a clustered channel model. We adopt the channel model introduced in [3], and a channel matrix $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ with N_r receive and N_t transmit antennas is represented as

where $N_{\rm cl}$ is the number of scattering clusters, $N_{\rm ray}$ is the number of rays per cluster, α_{mn} is the complex gain of the *n*th ray in the *m*th cluster, and $\theta_{mn}^{\rm r}(\phi_{mn}^{\rm r})$ and $\theta_{mn}^{\rm t}(\phi_{mn}^{\rm t})$ are the elevation (azimuth) AoAs/AoDs, respectively. The vectors $\mathbf{a}_{\rm r}(\theta_{mn}^{\rm r},\phi_{mn}^{\rm r}) \in \mathbb{C}^{N_{\rm r}}$ and $\mathbf{a}_t(\theta_{mn}^{\rm t},\phi_{mn}^{\rm t}) \in \mathbb{C}^{N_{\rm t}}$ are the array response vectors at the receiver and the transmitter to the angular arrivals and departures, respectively. The small-scale coefficients $\alpha_{mn} \sim \mathcal{CN}\left(0, \gamma_m 10^{-\frac{PL}{10}}\right)$, where PL is the path loss and γ_m is the fraction of power in *m*th cluster,

which is given by

$$\gamma_m = \frac{\gamma'_m}{\sum_{n=1}^{N_{\rm cl}} \gamma'_n},\tag{2}$$

where γ_m' is the power of the cluster m.

B. CSI ACQUISITION

As shown in Fig. 1, the system model contains three types of CSI: (i) channels between BS and users, (ii) a SI channel at the BS, and (iii) CCI channels between UL and DL users. To acquire the first type of CSI, a similar approach to the Long-Term Evolution (LTE) system can be adopted. The BS broadcasts the cell-specific reference signal (CRS) to the DL users, and each DL user can estimate the BS to user channels using this reference signal. CSI is sent back to the BS via the control channel [33]. Furthermore, UL users exploit demodulation reference signals (DRS) for estimating CSI of UL channels [34]. A similar approach to DL users can be applied at the BS using CRS to estimate the SI channel. However, the CRS-based SI channel estimation can be degraded by the interference from DRS transmitted by UL users. In order to tackle this issue, no DRS is allowed to be transmitted by UL users during SI channel estimation period. It is also known that the SI channel has a strong lineof-sight (LoS) component, and therefore, it can be modelled as Rician distribution with a large K-factor before analog domain cancellation. It was experimentally shown that the strong LoS component of the SI channel is attenuated by applying an analog domain cancellation or antenna isolation. With enough SI suppression via analog cancellation,

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the residual SI channel follows a Rician distribution with a small K-factor or a Rayleigh distribution [35]. Based on this, we can assume that the residual SI channel follows the Rayleigh distribution. The CCI between the UL and DL users are problematic to acquire since there is no direct signaling between users. However, a neighbor discovery mechanism can be exploited at each user to estimate CCI via sounding reference signals in the LTE system [36]. A same technique is proposed in [37] for device-to-device communications to acquire CCI information.

III. PROBLEM FORMULATION

In this paper, we aim to design the four matrices of analog precoder, digital precoder, analog combiner, and digital combiner to maximize the uplink and downlink sum-rate in mmWave FD communications. To do this, the signal received by the BS and the j-th DL user can be written, respectively, as

$$\mathbf{y}_{0} = \sum_{k=1}^{K} \mathbf{H}_{k}^{\mathrm{UL}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{RF}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}} \mathbf{s}_{k}^{\mathrm{UL}} + \mathbf{n}_{0}$$
$$+ \mathbf{H}_{0} \sum_{j=1}^{J} \mathbf{V}_{\mathrm{DL}}^{\mathrm{RF}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}} \mathbf{s}_{j}^{\mathrm{DL}}, \qquad (3)$$

$$\mathbf{y}_{j} = \mathbf{H}_{j}^{\mathrm{DL}} \sum_{j=1}^{J} \mathbf{V}_{\mathrm{DL}}^{\mathrm{RF}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}} \mathbf{s}_{j}^{\mathrm{DL}} + \mathbf{n}_{j}^{\mathrm{DL}} + \sum_{k=1}^{K} \mathbf{H}_{jk}^{\mathrm{DU}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{RF}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}} \mathbf{s}_{k}^{\mathrm{UL}}, \qquad (4)$$

where $\mathbf{n}_0 \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_0}^2 \mathbf{I}_{N_0}\right)$ and $\mathbf{n}_j^{\text{DL}} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{n_j}^2 \mathbf{I}_N\right)$ denote the additive white Gaussian noise (AWGN) vectors at the the BS and the *j*-th DL user, respectively. At the BS, the received UL signal (3) is first processed by the RF beamformer, $\mathbf{U}_{\text{UL}}^{\text{RF}} \in \mathbb{C}^{N_0 \times N_0^{\text{RF}}}$ followed by baseband MIMO filter $\mathbf{U}_{k,\text{UL}}^{\text{BB}} \in \mathbb{C}^{N_0^{\text{RF} \times N_s}}$. At the *j*-th DL user, the received DL signal (4) is first processed by the RF beamformer, $\mathbf{U}_{j,\text{DL}}^{\text{RF}} \in \mathbb{C}^{N \times N^{\text{RF}}}$ followed by baseband filter $\mathbf{U}_{j,\text{DL}}^{\text{BB}} \in \mathbb{C}^{N^{\text{RF}} \times N_s}$. Using these receivers, the estimate of data streams of the *k*-th UL and the *j*-th DL user are given by

$$\hat{\mathbf{s}}_{k}^{\text{UL}} = \left(\mathbf{U}_{k,\text{UL}}^{\text{BB}}\right)^{H} \left(\mathbf{U}_{\text{UL}}^{\text{RF}}\right)^{H} \mathbf{y}_{0},\tag{5}$$

$$\hat{\mathbf{s}}_{j}^{\text{DL}} = \left(\mathbf{U}_{j,\text{DL}}^{\text{BB}}\right)^{H} \left(\mathbf{U}_{j,\text{DL}}^{\text{RF}}\right)^{H} \mathbf{y}_{j}^{\text{DL}}.$$
 (6)

In addition, the estimated *i*-th data stream of the *k*-th UL and the *j*-th DL users are denoted by $\hat{s}_{k_i}^{\text{UL}}$ and $\hat{s}_{j_i}^{\text{DL}}$, respectively. Based on (5) and (6), we can write

$$\hat{s}_{k_{i}}^{\text{UL}} = \left(\mathbf{U}_{k,\text{UL}}^{\text{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{k}^{\text{UL}} \mathbf{V}_{k,\text{UL}}^{\text{BB}}(:,i) s_{k_{i}}^{\text{UL}}$$
(7)
+ $\sum_{l=1,l\neq i}^{N_{s}} \left(\mathbf{U}_{k,\text{UL}}^{\text{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{k}^{\text{UL}} \mathbf{V}_{k,\text{UL}}^{\text{BB}}(:,l) s_{k_{l}}^{\text{UL}}$ + $\sum_{m=1,m\neq k}^{K} \sum_{l=1}^{N_{s}} \left(\mathbf{U}_{k,\text{UL}}^{\text{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{k}^{\text{UL}} \mathbf{V}_{m,\text{UL}}^{\text{BB}}(:,l) s_{m_{l}}^{\text{UL}}$

$$+ \sum_{j=1}^{J} \sum_{l=1}^{N_{s}} \left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}}(i,:) \right)^{H} \bar{\mathbf{H}}_{0} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}}(:,l) s_{j_{l}}^{\mathrm{DL}} \\ + \left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}}(i,:) \right)^{H} \left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{RF}} \right)^{H} \mathbf{n}_{0}, \\ \hat{s}_{j_{i}}^{\mathrm{DL}} = \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:) \right)^{H} \bar{\mathbf{H}}_{j}^{\mathrm{DL}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}}(:,i) s_{j_{i}}^{\mathrm{DL}} \\ + \sum_{l=1,l\neq i}^{N_{s}} \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:) \right)^{H} \bar{\mathbf{H}}_{j}^{\mathrm{DL}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}}(:,l) s_{j_{l}}^{\mathrm{DL}} \\ + \sum_{m=1,m\neq j}^{J} \sum_{l=1}^{N_{s}} \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:) \right)^{H} \bar{\mathbf{H}}_{j}^{\mathrm{DL}} \mathbf{V}_{m,\mathrm{DL}}^{\mathrm{BB}}(:,l) s_{m_{l}}^{\mathrm{DL}} \\ + \sum_{k=1}^{K} \sum_{l=1}^{N_{s}} \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:) \right)^{H} \bar{\mathbf{H}}_{jk}^{\mathrm{DL}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}}(:,l) s_{k_{l}}^{\mathrm{DL}} \\ + \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:) \right)^{H} \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}} \right)^{H} \mathbf{n}_{j}^{\mathrm{DL}},$$

in which we define $\bar{\mathbf{H}}_{j}^{\text{DL}} = (\mathbf{U}_{j,\text{DL}}^{\text{RF}})^{H}\mathbf{H}_{j}^{\text{DL}}\mathbf{V}_{\text{DL}}^{\text{RF}}$, $\bar{\mathbf{H}}_{k}^{\text{UL}} = (\mathbf{U}_{\text{UL}}^{\text{RF}})^{H}\mathbf{H}_{k}^{\text{UL}}\mathbf{V}_{k,\text{UL}}^{\text{RF}}$, and $\bar{\mathbf{H}}_{jk}^{\text{DU}} = (\mathbf{U}_{j,\text{DL}}^{\text{RF}})^{H}\mathbf{H}_{jk}^{\text{DU}}\mathbf{V}_{k,\text{UL}}^{\text{RF}}$ as the *j*-th DL baseband channel, the *k*-th UL baseband channel, and the uplink to downlink baseband interference channel. The first term in (7) and (8) is the desired signal, the second term represents intra-user (inter-stream) interference, the third and fourth terms denote inter-user interference, and the last term is for the received noise.

Assuming Gaussian symbols are transmitted, the sum-rate of the UL and DL users follows as

$$R = \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{i=1}^{N_{s}} \log_{2}(1 + \text{SINR}_{j_{i}}^{\text{DL}}) + \log_{2}(1 + \text{SINR}_{k_{i}}^{\text{UL}}),$$
(9)

where $\text{SINR}_{j_i}^{\text{DL}}$ and $\text{SINR}_{k_i}^{\text{UL}}$ denote the signal-tointerference-and noise-ratio (SINR) of the *i*-th data stream for the *j*-th DL and the *k*-th UL user, respectively. $\text{SINR}_{k_i}^{\text{UL}}$ and $\text{SINR}_{j_i}^{\text{DL}}$ are shown in (10) and (11) at the bottom of next page. The optimization problem for maximizing the sumrate can then be formulated as

$$\max_{\mathbf{V}^{\mathsf{RF}}, \mathbf{U}^{\mathsf{RF}}} R \\ \mathbf{v}^{\mathsf{BB}}, \mathbf{U}^{\mathsf{BB}} \\ \text{s.t. } \operatorname{tr} \left\{ \mathbf{V}_{k,\mathsf{UL}}^{\mathsf{RF}} \mathbf{V}_{k,\mathsf{UL}}^{\mathsf{BB}} \left(\mathbf{V}_{k,\mathsf{UL}}^{\mathsf{RF}} \mathbf{V}_{k,\mathsf{UL}}^{\mathsf{BB}} \right)^{H} \right\} \leq P_{k}, \ k \in \mathcal{S}_{\mathsf{UL}} \\ (12a)$$

$$\sum_{j \in \mathcal{S}_{\text{DL}}} \operatorname{tr} \left\{ \mathbf{V}_{\text{DL}}^{\text{RF}} \mathbf{V}_{j,\text{DL}}^{\text{BB}} \left(\mathbf{V}_{\text{DL}}^{\text{RF}} \mathbf{V}_{j,\text{DL}}^{\text{BB}} \right)^{H} \right\} \leq P_{0}, \quad (12b)$$

$$\mathbf{V}^{\mathrm{RF}} \in \mathcal{V}_{RF}, \quad \mathbf{U}^{\mathrm{RF}} \in \mathcal{U}_{RF},$$
 (12c)

where P_k and P_0 are the transmit power constraints at the kth UL user and BS, respectively. Furthermore, S_{UL} and S_{DL} represent the set of K UL and J DL channels, respectively. V_{RF} and U_{RF} are the candidate sets of RF beamforming and RF combining matrices with the constant amplitude, respectively. Because of the non-convexity of the objective function and the constraint in (12c), optimizing the sumrate by jointly deriving the four matrices of analog precoder, digital precoder, analog combiner, and digital combiner is intractable. Hence, we propose the following two approaches: (i) jointly designing analog and digital filters separately at the transmitters and receivers, and (ii) deriving joint transmitter precoders and receiver combiners with separate designs for the analog and digital beamforming matrices. These two HBF procedures are presented in the following two sections.

IV. JOINT ANALOG AND DIGITAL BEAMFORMING DESIGN USING OMP ALGORITHM

Because of the appearance of the interference signals in the denominator of the objective function, the optimization problem in (12) is neither jointly convex over both the combiner and precoder filters nor convex over each of them. To handle the non-convexity, the optimization problem in (12) can be transformed into a minimization of the sum-MSE of the UL and DL channels as suggested by [38] and [39]. More specifically, it can be shown that the MSE function is related to the received SINR, and thus, the sum-rate maximization and sum-MSE minimization problems are equivalent. The derived sum-MSE minimization problem is convex over each of the precoder and combiner beamformings. Therefore, in this section, we minimize the sum-MSE as a proxy to the spectral efficiency maximization problem.

Using the estimates in (5) and (6), the MSE of the k-th UL and j-th DL user can be written in (13) and (14), respectively, shown at the bottom of the following page. In (13) and (14), Σ_k^{UL} and Σ_j^{DL} denote the covariance matrix of the aggregate interference-plus-noise terms of the k-th UL and the j-th DL

users, respectively, and they are given at the bottom of the following page. The considered optimization problem in this section can be formulated as

$$\min_{\substack{\mathbf{V}^{\mathsf{RF}},\mathbf{U}^{\mathsf{RF}}\\\mathbf{V}^{\mathsf{BB}},\mathbf{U}^{\mathsf{BB}}}} \sum_{k=1}^{K} \operatorname{tr}\left\{\mathbf{MSE}_{k}^{\mathsf{UL}}\right\} + \sum_{j=1}^{J} \operatorname{tr}\left\{\mathbf{MSE}_{j}^{\mathsf{DL}}\right\}$$
(17a)

s.t. tr
$$\left\{ \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{RF}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}} \left(\mathbf{V}_{k,\mathrm{UL}}^{\mathrm{RF}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}} \right)^{H} \right\} \leq P_{k}, \ k \in \mathcal{S}_{\mathrm{UL}},$$
(17b)

$$\sum_{j \in \mathcal{S}_{\text{DL}}} \operatorname{tr} \left\{ \mathbf{V}_{\text{DL}}^{\text{RF}} \mathbf{V}_{j,\text{DL}}^{\text{BB}} \left(\mathbf{V}_{\text{DL}}^{\text{RF}} \mathbf{V}_{j,\text{DL}}^{\text{BB}} \right)^{H} \right\} \leq P_{0}, \quad (17c)$$

$$\mathbf{V}^{\mathrm{RF}} \in \mathcal{V}_{RF}, \quad \mathbf{U}^{\mathrm{RF}} \in \mathcal{U}_{RF}.$$
(17d)

The optimization variables can be denoted as $\mathbf{V}^{\text{RF}(\text{BB})} = {\mathbf{V}_{k,\text{UL}}^{\text{RF}(\text{BB})}, \mathbf{V}_{j,\text{DL}}^{\text{RF}(\text{BB})} : k \in S_{\text{UL}}, j \in S_{\text{DL}}}$, and $\mathbf{U}^{\text{RF}(\text{BB})} = {\mathbf{U}_{k,\text{UL}}^{\text{RF}(\text{BB})}, \mathbf{U}_{j,\text{DL}}^{\text{RF}(\text{BB})} : k \in S_{\text{UL}}, j \in S_{\text{DL}}}$, which indicate the set of all precoding and combining matrices, respectively. Directly minimizing the problem (17) requires a joint optimization over the four matrix variables \mathbf{V}^{RF} , \mathbf{U}^{RF} , \mathbf{V}^{BB} , and \mathbf{U}^{BB} , which is found to be intractable. Furthermore, this optimization problem is non-convex due to the constant amplitude condition of RF beamforming and RF combining matrices in (17d). In order to eliminate this constraint and also to reduce the number of optimization variables, at the first step, we derive the solution for the case of fully digital beamforming. Then applying the OMP approach, we com-

$$\begin{aligned} \operatorname{SINR}_{k_{i}}^{\mathrm{UL}} &= \frac{S_{k_{i}}^{\mathrm{UL}}}{I_{k_{i}}^{\mathrm{UL}} + N_{k_{i}}^{\mathrm{UL}}} \\ S_{k_{i}}^{\mathrm{UL}} &= |\left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{k}^{\mathrm{UL}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}}(:,i) s_{k_{i}}^{\mathrm{UL}}|^{2} \\ I_{k_{i}}^{\mathrm{DL}} &= \sum_{l=1, l \neq i}^{N_{s}} |\left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{k}^{\mathrm{UL}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}}(:,l) s_{k_{l}}^{\mathrm{UL}}|^{2} + \sum_{m=1, m \neq k}^{K} \sum_{l=1}^{N_{s}} |\left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{k}^{\mathrm{UL}} \mathbf{V}_{m,\mathrm{UL}}^{\mathrm{BB}}(:,l) s_{m_{l}}^{\mathrm{UL}}|^{2} \\ &+ \sum_{j=1}^{J} \sum_{l=1}^{N_{s}} |\left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{0} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}}(:,l) s_{j_{l}}^{\mathrm{DL}}|^{2} \\ &N_{k_{i}}^{\mathrm{UL}} &= \sigma_{n_{0}}^{2} \|\left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}}(i,:)\right)^{H} \left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{RF}}\right)^{H} \|_{F}^{2} \end{aligned} \tag{10}$$

$$\begin{aligned} \operatorname{SINR}_{j_{i}}^{\mathrm{DL}} &= \frac{S_{j_{i}}^{\mathrm{DL}}}{I_{j_{i}}^{\mathrm{DL}} + N_{j_{i}}^{\mathrm{DL}}} \\ S_{j_{i}}^{\mathrm{DL}} &= |\left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{j}^{\mathrm{DL}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}}(:,i) s_{j_{i}}^{\mathrm{DL}}|^{2} \\ I_{j_{i}}^{\mathrm{DL}} &= \sum_{l=1, l \neq i}^{N_{s}} |\left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{j}^{\mathrm{DL}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}}(:,l) s_{j_{l}}^{\mathrm{DL}}|^{2} + \sum_{m=1, m \neq j}^{J} \sum_{l=1}^{N_{s}} |\left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{j}^{\mathrm{DL}} \mathbf{V}_{m,\mathrm{DL}}^{\mathrm{BB}}(:,l) s_{m_{l}}^{\mathrm{DL}}|^{2} \\ &+ \sum_{k=1}^{K} \sum_{l=1}^{N_{s}} |\left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:)\right)^{H} \bar{\mathbf{H}}_{jk}^{\mathrm{DU}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}}(:,l) s_{k_{l}}^{\mathrm{UL}}|^{2} \\ &N_{j_{i}}^{\mathrm{DL}} &= \sigma_{n_{j}}^{2} \left\|\left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}(i,:)\right)^{H} \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}}\right)^{H} \right\|_{F}^{2} \end{aligned}$$

$$\tag{11}$$

pute the hybrid MIMO beamformers with the decomposition of optimal conventional filters into the product of RF and baseband filters.

A. FULLY DIGITAL BEAMFORMER

In order to derive the optimization problem for the fully digital beamforming, at the transmitter chains, the transmit precoders $\mathbf{V}_{k,\text{UL}}^{\text{RF}}\mathbf{V}_{k,\text{UL}}^{\text{BB}}$ and $\mathbf{V}_{\text{DL}}^{\text{RF}}\mathbf{V}_{j,\text{DL}}^{\text{BB}}$ have to be replaced with $\mathbf{V}_{k}^{\text{UL}}$ and $\mathbf{V}_{j}^{\text{DL}}$, respectively. Moreover, at the receiver chains, receive combiners $\mathbf{U}_{\text{UL}}^{\text{RF}}\mathbf{U}_{k,\text{UL}}^{\text{BB}}$ and $\mathbf{U}_{j,\text{DL}}^{\text{RF}}\mathbf{U}_{j,\text{DL}}^{\text{BB}}$ should be replaced with $\mathbf{U}_{k}^{\text{UL}}$ and $\mathbf{U}_{j}^{\text{DL}}$, respectively. Substituting these transmit and receive beamforming matrices into the optimization problem, (17) can be re-written as

$$\min_{\mathbf{V},\mathbf{U}} \quad \sum_{k=1}^{K} \operatorname{tr} \left\{ \mathbf{MSE}_{k}^{\mathrm{UL}} \right\} + \sum_{j=1}^{J} \operatorname{tr} \left\{ \mathbf{MSE}_{j}^{\mathrm{DL}} \right\}$$
(18a)

s.t.
$$\operatorname{tr}\left\{\mathbf{V}_{k}^{\mathrm{UL}}\left(\mathbf{V}_{k}^{\mathrm{UL}}\right)^{H}\right\} \leq P_{k}, \ k \in \mathcal{S}_{\mathrm{UL}},$$
 (18b)

$$\sum_{j \in \mathcal{S}_{\text{DL}}} \operatorname{tr} \left\{ \mathbf{V}_{j}^{\text{DL}} \left(\mathbf{V}_{j}^{\text{DL}} \right)^{H} \right\} \le P_{0}.$$
(18c)

Because of the multiplicative terms between the precoder and combiner matrices in the sum-MSE terms, the optimization problem of (18) is not jointly convex over both the combiner and precoder filters. However, this problem is convex over each of the precoder and combiner matrices. Hence, we can apply the alternating convex optimization technique to (18), alternating between the optimization for the precoder and the combiner matrix while keeping the other one fixed at each step. Using the Lagrangian multiplier method, the Lagrangian dual function can be written as

$$\mathcal{L} = \sum_{k=1}^{K} \operatorname{tr} \left\{ \mathbf{MSE}_{k}^{\mathrm{UL}} \right\} + \sum_{j=1}^{J} \operatorname{tr} \left\{ \mathbf{MSE}_{j}^{\mathrm{DL}} \right\}$$
(19)
$$+ \sum_{k=1}^{K} \lambda_{k}^{\mathrm{UL}} \left(\operatorname{tr} \left(\mathbf{V}_{k}^{\mathrm{UL}} (\mathbf{V}_{k}^{\mathrm{UL}})^{H} \right) - P_{k} \right)$$

$$+ \lambda_0 \bigg(\sum_{j=1}^J \operatorname{tr} \left(\mathbf{V}_j^{\mathrm{DL}} (\mathbf{V}_j^{\mathrm{DL}})^H \right) \bigg).$$

Taking the partial derivative of \mathcal{L} with respect to the matrices $\bar{\mathbf{U}}_{k}^{\text{UL}}, \bar{\mathbf{U}}_{j}^{\text{DL}}, \bar{\mathbf{V}}_{k}^{\text{UL}}$, and $\bar{\mathbf{V}}_{j}^{\text{DL}}$, we have

$$\frac{\partial \mathcal{L}}{\bar{\mathbf{U}}_{k}^{\text{UL}}} = \mathbf{R}_{0} \mathbf{U}_{k}^{\text{UL}} - \mathbf{H}_{k}^{\text{UL}} \mathbf{V}_{k}^{\text{UL}} = 0, \qquad (20)$$

$$\frac{\partial \mathcal{L}}{\bar{\mathbf{U}}_{j}^{\mathrm{DL}}} = \mathbf{R}_{j}^{\mathrm{DL}} \mathbf{U}_{j}^{\mathrm{DL}} - \mathbf{H}_{j}^{\mathrm{DL}} \mathbf{V}_{j}^{\mathrm{DL}} = 0, \qquad (21)$$

$$\frac{\partial \mathcal{L}}{\bar{\mathbf{V}}_{k}^{\text{UL}}} = \left(\sum_{j=1}^{K} \left(\mathbf{H}_{k}^{\text{UL}}\right)^{H} \mathbf{U}_{j}^{\text{UL}} \left(\mathbf{U}_{j}^{\text{UL}}\right)^{H} \mathbf{H}_{k}^{\text{UL}} + \sum_{j=1}^{J} \left(\mathbf{H}_{jk}^{DU}\right)^{H} \mathbf{U}_{j}^{\text{DL}} \left(\mathbf{U}_{j}^{\text{DL}}\right)^{H} \mathbf{H}_{jk}^{\text{DU}}\right) \mathbf{V}_{k}^{\text{UL}} - \left(\mathbf{H}_{k}^{\text{UL}}\right)^{H} \mathbf{U}_{k}^{\text{UL}} + \lambda_{k}^{\text{UL}} \mathbf{V}_{k}^{\text{UL}} = 0, \quad (22)$$

$$\frac{\partial \mathcal{L}}{\bar{\mathbf{V}}_{j}^{\text{DL}}} = \left(\sum_{k=1}^{J} \left(\mathbf{H}_{k}^{\text{DL}}\right)^{H} \mathbf{U}_{k}^{\text{DL}} \left(\mathbf{U}_{k}^{\text{DL}}\right)^{H} \mathbf{H}_{k}^{\text{DL}} + \sum_{k=1}^{K} \mathbf{H}_{0}^{H} \mathbf{U}_{k}^{\text{UL}} \left(\mathbf{U}_{k}^{\text{UL}}\right)^{H} \mathbf{H}_{0}\right) \mathbf{V}_{j}^{\text{DL}} - \left(\mathbf{H}_{j}^{\text{DL}}\right)^{H} \mathbf{U}_{j}^{\text{DL}} + \lambda_{0} \mathbf{V}_{j}^{\text{DL}} = 0.$$
(23)

Therefore, the optimal MMSE receive beamforming matrices for the problem (18) can be written as

$$\bar{\mathbf{U}}_{k}^{\mathrm{UL}} = (\mathbf{R}_{0})^{-1} \mathbf{H}_{k}^{\mathrm{UL}} \mathbf{V}_{k}^{\mathrm{UL}}, \qquad (24)$$

$$\bar{\mathbf{U}}_{j}^{\mathrm{DL}} = \left(\mathbf{R}_{j}^{\mathrm{DL}}\right)^{-1} \mathbf{H}_{j}^{\mathrm{DL}} \mathbf{V}_{j}^{\mathrm{DL}},\tag{25}$$

where the received covariance matrices \mathbf{R}_0 and \mathbf{R}_j^{DL} are expressed as

$$\mathbf{R}_{0} = \mathbf{H}_{k}^{\mathrm{UL}} \mathbf{V}_{k}^{\mathrm{UL}} \left(\mathbf{H}_{k}^{\mathrm{UL}} \mathbf{V}_{k}^{\mathrm{UL}} \right)^{H} + \boldsymbol{\Sigma}_{k}^{\mathrm{UL}}, \qquad (26)$$

$$\mathbf{R}_{j}^{\mathrm{DL}} = \mathbf{H}_{j}^{\mathrm{DL}} \mathbf{V}_{j}^{\mathrm{DL}} \left(\mathbf{H}_{j}^{\mathrm{DL}} \mathbf{V}_{j}^{\mathrm{DL}} \right)^{H} + \mathbf{\Sigma}_{j}^{\mathrm{DL}}.$$
 (27)

$$\mathbf{MSE}_{k}^{\mathrm{UL}} = \left(\left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}} \right)^{H} \left(\mathbf{U}_{\mathrm{UL}}^{\mathrm{RF}} \right)^{H} \mathbf{H}_{k}^{\mathrm{UL}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{RF}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}} - \mathbf{I}_{d_{k}^{\mathrm{UL}}} \right) \left(\left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}} \right)^{H} \left(\mathbf{U}_{\mathrm{UL}}^{\mathrm{RF}} \right)^{H} \mathbf{H}_{k}^{\mathrm{UL}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{RF}} \mathbf{V}_{k,\mathrm{UL}}^{\mathrm{BB}} - \mathbf{I}_{d_{k}^{\mathrm{UL}}} \right)^{H} \\ + \left(\mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}} \right)^{H} \left(\mathbf{U}_{\mathrm{UL}}^{\mathrm{RF}} \right)^{H} \mathbf{\Sigma}_{k}^{\mathrm{UL}} \mathbf{U}_{\mathrm{UL}}^{\mathrm{RF}} \mathbf{U}_{k,\mathrm{UL}}^{\mathrm{BB}}, \tag{13}$$

$$\mathbf{MSE}_{j}^{\mathrm{DL}} = \left(\left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}} \right)^{H} \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}} \right)^{H} \mathbf{H}_{j}^{\mathrm{DL}} \mathbf{V}_{\mathrm{DL}}^{\mathrm{RF}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}} - \mathbf{I}_{d_{j}^{\mathrm{DL}}} \right) \left(\left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}} \right)^{H} \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}} \right)^{H} \mathbf{H}_{j}^{\mathrm{DL}} \mathbf{V}_{\mathrm{DL}}^{\mathrm{RF}} \mathbf{I}_{d_{j}^{\mathrm{DL}}} \right)^{H} \\ + \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}} \right)^{H} \left(\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}} \right)^{H} \mathbf{\Sigma}_{j}^{\mathrm{DL}} \mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}} \mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}}. \tag{14}$$

$$\boldsymbol{\Sigma}_{k}^{\mathrm{UL}} = \sum_{j \neq k}^{K} \mathbf{H}_{j}^{\mathrm{UL}} \mathbf{V}_{j,\mathrm{UL}}^{\mathrm{RF}} \mathbf{V}_{j,\mathrm{UL}}^{\mathrm{BB}} \left(\mathbf{V}_{j,\mathrm{UL}}^{\mathrm{RF}} \mathbf{V}_{j,\mathrm{UL}}^{\mathrm{BB}} \right)^{H} \left(\mathbf{H}_{j}^{\mathrm{UL}} \right)^{H} + \sum_{j=1}^{J} \mathbf{H}_{0} \mathbf{V}_{\mathrm{DL}}^{\mathrm{RF}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}} \left(\mathbf{V}_{\mathrm{DL}}^{\mathrm{RF}} \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{BB}} \right)^{H} \mathbf{H}_{0}^{H} + \sigma_{n_{0}}^{2} \mathbf{I}_{N_{0}},$$
(15)

$$\boldsymbol{\Sigma}_{j}^{\text{DL}} = \sum_{i \neq j}^{J} \mathbf{H}_{j}^{\text{DL}} \mathbf{V}_{\text{DL}}^{\text{RF}} \mathbf{V}_{i,\text{DL}}^{\text{BB}} \left(\mathbf{V}_{\text{DL}}^{\text{RF}} \mathbf{V}_{i,\text{DL}}^{\text{BB}} \right)^{H} \left(\mathbf{H}_{j}^{\text{DL}} \right)^{H} + \sum_{k=1}^{K} \mathbf{H}_{jk}^{DU} \mathbf{V}_{k,\text{UL}}^{\text{RF}} \mathbf{V}_{k,\text{UL}}^{\text{BB}} \left(\mathbf{V}_{k,\text{UL}}^{\text{RF}} \mathbf{V}_{k,\text{UL}}^{\text{BB}} \right)^{H} \left(\mathbf{H}_{jk}^{DU} \right)^{H} + \sigma_{n_{j}}^{2} \mathbf{I}_{N_{j}}.$$
(16)

Moreover, the optimal transmit beamforming matrices can be computed as

$$\bar{\mathbf{V}}_{k}^{\mathrm{UL}} = \left(\sum_{j=1}^{K} \left(\mathbf{H}_{k}^{\mathrm{UL}}\right)^{H} \mathbf{U}_{j}^{\mathrm{UL}} \left(\mathbf{U}_{j}^{\mathrm{UL}}\right)^{H} \mathbf{H}_{k}^{\mathrm{UL}} + \lambda_{k}^{\mathrm{UL}} \mathbf{I}_{M_{k}} \right. \\ \left. + \sum_{j=1}^{J} \left(\mathbf{H}_{jk}^{DU}\right)^{H} \mathbf{U}_{j}^{\mathrm{DL}} \left(\mathbf{U}_{j}^{\mathrm{DL}}\right)^{H} \mathbf{H}_{jk}^{DU} \right)^{-1} \\ \left. \times \left(\mathbf{H}_{k}^{\mathrm{UL}}\right)^{H} \mathbf{U}_{k}^{\mathrm{UL}}, \qquad (28)$$

$$\bar{\mathbf{V}}_{j}^{\mathrm{DL}} = \left(\sum_{k=1}^{J} \left(\mathbf{H}_{k}^{\mathrm{DL}}\right)^{H} \mathbf{U}_{k}^{\mathrm{DL}} \left(\mathbf{U}_{k}^{\mathrm{DL}}\right)^{H} \mathbf{H}_{k}^{\mathrm{DL}} \\ \left. + \sum_{k=1}^{K} \mathbf{H}_{0}^{H} \mathbf{U}_{k}^{\mathrm{UL}} \left(\mathbf{U}_{k}^{\mathrm{UL}}\right)^{H} \mathbf{H}_{0} + \lambda_{0} \mathbf{I}_{M_{0}} \right)^{-1} \\ \left. \times \left(\mathbf{H}_{j}^{\mathrm{DL}}\right)^{H} \mathbf{U}_{j}^{\mathrm{DL}}, \qquad (29)$$

where λ_k^{UL} and λ_0 are the Lagrangian multipliers associated with the power constraints in (18b) and (18c), respectively. To simplify the presentation of deriving HBF, we will combine UL and DL transmissions using the following notations in the rest of the paper

$$\begin{pmatrix} \mathbf{R}_k, \ \tilde{N}_k^{\text{RF}}, \ \tilde{N}_k, \ \tilde{M}_k^{\text{RF}}, \ \tilde{d}_k^X, \ \tilde{M}_k \end{pmatrix}$$
(30)
$$= \begin{cases} \begin{pmatrix} (\mathbf{R}_0, \ N^{RF}, \ N_0, \ M_k^{\text{RF}}, \ d_k^{\text{UL}}, \ M_k \end{pmatrix}, & k \in \mathcal{S}_{\text{UL}}, \\ \begin{pmatrix} (\mathbf{R}_k^{\text{DL}}, \ N_k^{RF}, \ N_k, \ M^{\text{RF}}, \ d_k^{\text{DL}}, \ M_0 \end{pmatrix}, & k \in \mathcal{S}_{\text{DL}}. \end{cases}$$

B. CONVERGENCE ANALYSIS

Before extending the digital beamforming to the HBF design, we briefly consider the convergence of the former. For this, we note that since each of the sub-problems of optimizing precoder and combiner filters is convex, the alternating optimization process in (24)-(29) converges. In particular, as it is proved in [29], in each iteration of the fully digital step, the total sum-MSE is decreased by updating the combining and precoding matrices. Moreover, the MSE is lower bounded by zero, and thus, the fully digital scheme is convergent.

In Fig. 2, the convergence rate of fully digital beamforming step in the OMP-based HBF approach is assessed for different numbers of users and antennas. The simulation parameters used for this figure are the same as those adopted in Section VI. As this figure illustrates, the objective sum-MSE function in (12) decreases in each iteration and it converges to a local optimum beamforming. As it can be seen from this figure, the setup with a lower number of users converges faster with fewer number of iterations and with better performance. This is because the interference between users is decreased for a lower number of users and the proposed policy is more effective. Moreover, if the number of antennas increases, the performance will be improved. However, we need more iterations to converge due to the increased number of variables. As this figure shows, the fully digital step in the proposed OMP-based HBF converges, which leads to convergence of the proposed OMP-based HBF.



FIGURE 2. Convergence rate of the fully digital design of OMP-based HBF scheme.

C. HYBRID MIMO RECEIVER

We now turn to decomposing the optimal receiving matrices $\bar{\mathbf{U}}_k^X$ in (24) and (25) into the products of $\bar{\mathbf{U}}_{k,X}^{\text{RF}}$ and $\bar{\mathbf{U}}_{k,X}^{\text{BB}}$, $X \in \{\text{UL, DL}\}$. Note that $\bar{\mathbf{U}}_k^X$ cannot, in general, be expressed as $\bar{\mathbf{U}}_{k,X}^{\text{RF}} \bar{\mathbf{U}}_{k,X}^{\text{BB}}$, and thus, cannot be realized in the mmWave architecture of interest. Therefore, we aim to make the hybrid precoder $\bar{\mathbf{U}}_{k,X}^{\text{RF}} \bar{\mathbf{U}}_{k,X}^{\text{BB}}$ sufficiently "close" to the optimal receiver $\bar{\mathbf{U}}_k^X$ [4]. More specifically, the design problem of hybrid combiners for the mmWave receivers can be expressed as

$$\left(\bar{\mathbf{U}}_{k,X}^{\text{RF}}, \ \bar{\mathbf{U}}_{k,X}^{\text{BB}}\right) = \arg\min_{\mathbf{U}_{k,X}^{\text{RF}}, \mathbf{U}_{k,X}^{\text{BB}}} \mathbb{E}\left[\left\|\mathbf{s}_{k}^{X} - \hat{\mathbf{s}}_{k}^{X}\right\|_{2}^{2}\right],$$
(31)

where $\hat{\mathbf{s}}_k^X$ is defined in (5) and (6). As shown in [4, (23)], the problem (31) can be equivalently re-written as:

$$\left(\bar{\mathbf{U}}_{k,X}^{\text{RF}}, \bar{\mathbf{U}}_{k,X}^{\text{BB}} \right) = \arg \min_{\mathbf{U}_{k,X}^{\text{RF}}, \mathbf{U}_{k,X}^{\text{BB}}} \left\| \mathbf{R}_{k}^{1/2} \left(\bar{\mathbf{U}}_{k}^{X} - \mathbf{U}_{k,X}^{\text{RF}} \mathbf{U}_{k,X}^{\text{BB}} \right) \right\|_{F}^{2},$$
(32)

which amounts to finding the projection of the unconstrained MMSE combiner $\bar{\mathbf{U}}_k^X$ onto the set of hybrid combiners of the form $\mathbf{U}_{k,X}^{\text{RF}}\mathbf{U}_{k,X}^{\text{BB}}$.

To solve (32), we will leverage the fact that the columns of $\mathbf{R}_{k}^{1/2} \bar{\mathbf{U}}_{k}^{X}$ and $\mathbf{R}_{k}^{1/2} \mathbf{U}_{k,X}^{\text{RF}}$ are both located in the columnspace of $\mathbf{R}_{k}^{1/2}$, and thus, we can approximate $\mathbf{R}_{k}^{1/2} \bar{\mathbf{U}}_{k}^{X}$ by the linear combinations of the $\mathbf{R}_{k}^{1/2} \mathbf{U}_{k,X}^{\text{RF}}$, where the weights for the linear combination are $\mathbf{U}_{k,X}^{\text{BB}}$. The problem consists of selecting the "best" $\tilde{N}_{k}^{\text{RF}}$ array response vectors and finding their optimal baseband combination. To this end, we first design $\mathbf{U}_{k,X}^{\text{RF}}$ by choosing appropriate vectors from a set of candidate beamformers, whose elements pre-multiplied with $\mathbf{R}_{k}^{1/2}$ form the bases for the column space of $\mathbf{R}_{k}^{1/2}$. The authors in [4] provide some examples for beamformers as follows:

- 1) Eigen-beamformer: The set consists of \tilde{N}_k^{RF} dominant eigenvectors of $\mathbf{R}_k^{1/2}$, and use them for RF beamforming.
- 2) DFT beamformer: The set consists of the columns of $\tilde{N}_k \times \tilde{N}_k$ DFT matrix.
- 3) Discrete cosine transform (DCT) beamformer: The set consists of the columns of $\tilde{N}_k \times \tilde{N}_k$ DCT matrix.

The constraint that the receiver uses \tilde{N}_k^{RF} of the vectors from the matrices defined above can be embedded directly into the optimization problem (31), and can be equivalently written as

where the columns of $\mathbf{B}_k \in \mathbb{C}^{\tilde{N}_k \times N_b}$ consists of all the elements in the candidate beamformers, and N_b is the number of elements in the candidate beamformers. The sparsity constraint (33b) states that $\tilde{\mathbf{U}}_{k,X}^{\text{BB}}$ cannot have more than \tilde{N}_k^{RF} non-zero rows. When only \tilde{N}_k^{RF} rows of $\tilde{\mathbf{U}}_{k,X}^{\text{BB}}$ are non-zero, only \tilde{N}_k^{RF} columns of the matrix \mathbf{B}_k are effectively "selected". As a result, the baseband the optimal receive beamforming matrix $\bar{\mathbf{U}}_{k,X}^{\text{BB}}$ will be given by the \tilde{N}_k^{RF} non-zero rows of $\tilde{\mathbf{U}}_{k,X}^{\text{BB}}$ and the RF receiver $\bar{\mathbf{U}}_{k,X}^{\text{RF}}$ will be given by the spine provide the spine by the corresponding \tilde{N}_k^{RF} columns of \mathbf{B}_k .

The problem in (33) is equivalent to the problem of sparse signal recovery with multiple measurement vectors [30], and therefore, we can apply the well-known concept of OMP [6]. The summary of the algorithm to compute the hybrid receive beamformers is shown in Algorithm 1. Here, in Steps 4 and 5, the column of $\mathbf{R}_k^{1/2}\mathbf{B}_k$ that is most strongly correlated (the maximum projection) with the residual $\mathbf{R}_k^{1/2}\mathbf{U}_{k,\text{res}}^X$ is chosen, and then the selected column vector $\mathbf{B}_k[l]$ is appended to the RF receiver $\mathbf{U}_{k,X}^{\text{RF}}$ in Step 6. After the dominant vector is found, and the least squares solution to $\mathbf{U}_{k,X}^{\text{BB}}$ is calculated in Step 7, the contribution of the selected vector is removed in Step 8 and the algorithm proceeds to find the column along which the residual precoding matrix $\mathbf{U}_{k,\text{res}}^X$ has the largest projection. The process continues until all \tilde{N}_k^{RF} beamforming vectors have been selected.

D. HYBRID MIMO TRANSMITTER

The hybrid filters at the transmitter are designed based on the equivalent system, that models the reverse of the FD channel using uplink-downlink duality [31]. In particular, in the reverse system, the DL users transmit their signals to the BS, and the UL users receive signal from the BS. Moreover, the SI channel at the BS is from its receive to transmit antennas, and the CCI channel is from DL to the UL users. The estimated symbols of the jth DL and kth UL users are given as

$$\hat{\mathbf{s}}_{j}^{\text{DL}} = (\mathbf{V}_{j,\text{DL}})^{H} \left(\sum_{k=1}^{J} \left(\mathbf{H}_{k}^{\text{DL}} \right)^{H} \mathbf{U}_{k,\text{DL}} \mathbf{s}_{k}^{\text{DL}} \right. \\ \left. + \mathbf{H}_{0}^{H} \sum_{k=1}^{K} \mathbf{U}_{k,\text{UL}} \mathbf{s}_{k}^{\text{UL}} + \hat{\mathbf{n}}_{0} \right), \qquad (34)$$
$$\hat{\mathbf{s}}_{k}^{\text{UL}} = \left(\mathbf{V}_{k,\text{UL}} \right)^{H} \left(\sum_{j=1}^{J} \left(\mathbf{H}_{jk}^{DU} \right)^{H} \mathbf{U}_{j,\text{DL}} \mathbf{s}_{j}^{\text{DL}} \right. \\ \left. + \left(\mathbf{H}_{k}^{\text{UL}} \right)^{H} \sum_{j=1}^{K} \mathbf{U}_{j,\text{UL}} \mathbf{s}_{j}^{\text{UL}} + \hat{\mathbf{n}}_{k}^{\text{UL}} \right). \qquad (35)$$

Note that the reverse system is defined so that the optimal transmit filter of the forward system in (28) and (29) can be obtained from the optimal receive filter of the reverse system. It is easy to show that after some simple calculations the optimal MMSE receive filters obtained from the symbol estimates in (34) and (35) are same as the ones in (28) and (29) if and only if the noise vectors $\hat{\mathbf{n}}_0$ and $\hat{\mathbf{n}}_k^{\text{UL}}$ are distributed as $\mathcal{CN}(\mathbf{0}, \lambda_0 \mathbf{I}_{M_0})$ and $\mathcal{CN}(\mathbf{0}, \lambda_k^{\text{UL}} \mathbf{I}_{M_k})$, respectively.

Now, the objective is to decompose the optimal transmitting matrices $\bar{\mathbf{V}}_k^X$ in (28) and (29) into the products of $\bar{\mathbf{V}}_{k,X}^{\text{RF}}$ and $\bar{\mathbf{V}}_{k,X}^{\text{BB}}$, $X \in \{\text{UL}, \text{DL}\}$ at the receiver side of the reverse system. Similar to the analysis in Section IV-C, the design problem of hybrid combiners for the mmWave receivers in the reverse system can be expressed as

where the received covariance matrix $\hat{\mathbf{R}}_k$ in the reverse system is given as

$$\hat{\mathbf{R}}_{k} = \begin{cases} \sum_{k=1}^{J} \left(\mathbf{H}_{k}^{\mathrm{DL}}\right)^{H} \mathbf{U}_{k,\mathrm{DL}} \left(\mathbf{U}_{k,\mathrm{DL}}\right)^{H} \mathbf{H}_{k}^{\mathrm{DL}} \\ +\mathbf{H}_{0}^{H} \sum_{k=1}^{K} \mathbf{U}_{k,\mathrm{UL}} \left(\mathbf{U}_{k,\mathrm{UL}}\right)^{H} \mathbf{H}_{0}, & k \in \mathcal{S}_{\mathrm{DL}} \\ \sum_{j=1}^{J} \left(\mathbf{H}_{jk}^{\mathrm{DU}}\right)^{H} \mathbf{U}_{j,\mathrm{DL}} \left(\mathbf{U}_{j,\mathrm{DL}}\right)^{H} \mathbf{H}_{jk}^{\mathrm{DU}} \\ + \left(\mathbf{H}_{k}^{\mathrm{UL}}\right)^{H} \sum_{j=1}^{K} \mathbf{U}_{j,\mathrm{UL}} \left(\mathbf{U}_{j,\mathrm{UL}}\right)^{H} \mathbf{H}_{k}^{\mathrm{UL}}, & k \in \mathcal{S}_{\mathrm{UL}}. \end{cases}$$

Due to the similarity between the problems (36) and (32), we can again design $\bar{\mathbf{V}}_{k,X}^{\text{RF}}$ by choosing appropriate vectors from a set of candidate beamformers, and a sparse approximation problem can be formulated as before, i.e., the objective function in (36) can be written as $\left\|\hat{\mathbf{R}}_{k}^{1/2}\left(\bar{\mathbf{V}}_{k}^{X} - \hat{\mathbf{B}}_{k}\tilde{\mathbf{V}}_{k,X}^{\text{BB}}\right)\right\|_{F}^{2}$, where the columns of $\hat{\mathbf{B}}_{k} \in \mathbb{C}^{\tilde{M}_{k} \times N_{b}}$ consists of all the elements in the candidate beamformers, and $\tilde{\mathbf{V}}_{k,X}^{\text{BB}} \in \mathbb{C}^{N_{b} \times \tilde{d}_{k}^{X}}$ having $\tilde{M}_{k}^{\text{RF}}$ nonzero rows that constitute $\bar{\mathbf{V}}_{k,X}^{\text{BB}}$. The positions of the nonzero rows correspond to the selected columns of $\tilde{\mathbf{B}}_{k}$.

The steps of the proposed algorithm to compute the hybrid transmit processors are given in Algorithm 2 below. Algorithm 2 is similar to Algorithm 1 with the exception of Steps 10 and 11, which are used to realize the power constraints at the UL users and the BS.

Algorithm 1 Receiver hybrid MIMO beamformer

1:
$$\mathbf{U}_{k,\mathrm{res}}^{\mathrm{RF}} = []$$

2: $\mathbf{U}_{k,\mathrm{res}}^{X} = \bar{\mathbf{U}}_{k}^{X}$
3: for $i \leq \tilde{N}_{k}^{\mathrm{RF}}$ do
4: $\mathbf{\Phi}_{k}^{X} = \left(\mathbf{R}_{k}^{1/2}\mathbf{B}_{k}\right)^{H}\left(\mathbf{R}_{k}^{1/2}\mathbf{U}_{k,\mathrm{res}}^{X}\right)$
5: $l = \arg\max_{m=1,...,N_{b}}\left(\mathbf{\Phi}_{k}^{X}\left(\mathbf{\Phi}_{k}^{X}\right)^{H}\right)_{m,m}$
6: $\mathbf{U}_{k,X}^{\mathrm{RF}} = \left[\mathbf{U}_{k,X}^{\mathrm{RF}} \mid \mathbf{B}_{k}[l]\right]$
7: $\mathbf{U}_{k,X}^{\mathrm{BB}} = \left(\left(\mathbf{U}_{k,X}^{\mathrm{RF}}\right)^{H}\mathbf{R}_{k}\mathbf{U}_{k,X}^{\mathrm{RF}}\right)^{-1}\left(\mathbf{U}_{k,X}^{\mathrm{RF}}\right)^{H}\mathbf{R}_{k}\bar{\mathbf{U}}_{k}^{X}$
8: $\mathbf{U}_{k,\mathrm{res}}^{X} = \frac{(\bar{\mathbf{U}}_{k}^{X} - \mathbf{U}_{k,X}^{\mathrm{RF}}\mathbf{U}_{k,X}^{\mathrm{BB}})}{\|\bar{\mathbf{U}}_{k}^{X} - \mathbf{U}_{k,X}^{\mathrm{RF}}\mathbf{U}_{k,X}^{\mathrm{BB}}\|_{F}}$
9: end for
10: return $\mathbf{U}_{k,X}^{\mathrm{RF}}, \mathbf{U}_{k,X}^{\mathrm{BB}}$

Algorithm 2 Transmit hybrid MIMO beamformer
1:
$$\mathbf{V}_{k,X}^{\text{RF}} = []$$

2: $\mathbf{V}_{k,\text{res}}^{X} = \bar{\mathbf{V}}_{k}^{X}$
3: for $i \leq \tilde{M}_{k}^{\text{RF}}$ do
4: $\Phi_{k}^{X} = (\hat{\mathbf{R}}_{k}^{1/2}\hat{\mathbf{B}}_{k})^{H} (\hat{\mathbf{R}}_{k}^{1/2}\mathbf{V}_{k,\text{res}}^{X})$
5: $l = \arg\max_{m=1,...,N_{b}} (\Phi_{k}^{X} (\Phi_{k}^{X})^{H})_{m,m}$
6: $\mathbf{V}_{k,X}^{\text{RF}} = [\mathbf{V}_{k,X}^{\text{RF}} | \hat{\mathbf{B}}_{k}[l]]$
7: $\mathbf{V}_{k,X}^{\text{BB}} = ((\mathbf{V}_{k,X}^{\text{RF}})^{H} \hat{\mathbf{R}}_{k}\mathbf{V}_{k,X}^{\text{RF}})^{-1} (\mathbf{V}_{k,X}^{\text{RF}})^{H} \hat{\mathbf{R}}_{k}\bar{\mathbf{V}}_{k}^{X}$
8: $\mathbf{V}_{k,\text{res}}^{X} = \frac{(\bar{\mathbf{V}}_{k}^{X} - \mathbf{V}_{k,X}^{\text{RF}}\mathbf{V}_{k,X}^{\text{BB}})}{\|\bar{\mathbf{V}}_{k}^{X} - \mathbf{V}_{k,X}^{\text{RF}}\mathbf{V}_{k,X}^{\text{BB}}\|_{F}}$
9: end for
10: $\mathbf{V}_{k,\text{UL}}^{\text{BB}} = \mathbf{V}_{k,\text{UL}}^{\text{BB}} \frac{\sqrt{P_{k}}}{\|\mathbf{V}_{k,\text{UL}}^{\text{RF}}\mathbf{V}_{k,\text{UL}}^{\text{BB}}\|_{F}}, k \in S^{\text{UL}}$
11: $\mathbf{V}_{k,\text{DL}}^{\text{BB}} = \mathbf{V}_{k,\text{DL}}^{\text{BB}} \frac{\sqrt{P_{0}}}{\sum_{k \in S_{\text{DL}}} \|\mathbf{V}_{k,\text{DL}}^{\text{RF}}\mathbf{V}_{k,\text{DL}}^{\text{BB}}\|_{F}}}$
12: return $\mathbf{V}_{k,X}^{\text{RF}}, \mathbf{V}_{k,X}^{\text{BB}}$

V. JOINT TRANSMIT AND RECEIVER BEAMFORMING DESIGN USING BD APPROACH

In this section, we maximize the sum-rate of UL and DL users via jointly deriving transmit precoding and receive combining matrices with separate designs for analog and digital beamformers. By removing the iteration between the transmitter and receiver optimization, the computational complexity of the BD-based HBF scheme can be reduced compared to the OMP-based HBF approach. We solve the optimization problem in (12) indirectly by deriving analog and digital beamformings in two steps such that an information loss associated with each step is minimized. To maximize the sum-rate, we first solve the optimization problem for the case that number of RF chains is at least double the number of data streams. Then, we extend the solution to the situation that the number of RF chains is at least equal to the number of data streams.

A. DL TRANSMISSION

The data transmission by the BS can be decomposed into two consecutive stages of digital and analog beamformings. First, in the digital beamforming step, the BS applies power allocation and digital precoding to the data symbol s and transmits it through the baseband channel. At each DL user, using a digital combiner, the received symbol is transformed to \hat{s} . Second, in the analog beamforming step, the BS applies analog precoding to the digitally precoded symbol x and transmits it through the channel, and the received symbol at each DL user is transformed to \hat{x} using an analog combiner. Considering the downlink channel, the two steps of analog and digital beamformings are

$$\hat{\mathbf{s}}_{j}^{\text{DL}} = (\mathbf{U}_{j,\text{DL}}^{\text{BB}})^{H} \bar{\mathbf{H}}_{j}^{\text{DL}} \mathbf{V}_{j,\text{DL}}^{\text{BB}} \mathbf{s}_{j}^{\text{DL}} + \sum_{\substack{m=1\\m\neq j}}^{J} (\mathbf{U}_{j,\text{DL}}^{\text{BB}})^{H} \bar{\mathbf{H}}_{j}^{\text{DL}} \mathbf{V}_{m,\text{DL}}^{\text{BB}} \mathbf{s}_{m}^{\text{DL}} + \sum_{\substack{k=1\\m\neq j}}^{K} (\mathbf{U}_{j,\text{DL}}^{\text{BB}})^{H} \bar{\mathbf{H}}_{jk}^{\text{DU}} \mathbf{V}_{k,\text{UL}}^{\text{BB}} \mathbf{s}_{k}^{\text{UL}} + (\mathbf{U}_{j,\text{DL}}^{\text{BB}})^{H} \mathbf{n}_{j}^{\prime \text{DL}}, \quad (38)$$

$$\hat{\mathbf{x}}_{j}^{\text{DL}} = (\mathbf{U}_{j,\text{DL}}^{\text{RF}})^{H} \mathbf{H}_{j}^{\text{DL}} \mathbf{V}_{j,\text{DL}}^{\text{RF}} \mathbf{x}_{j}^{\text{DL}} + \sum_{\substack{m=1\\m\neq j}}^{J} (\mathbf{U}_{j,\text{DL}}^{\text{RF}})^{H} \mathbf{H}_{j}^{\text{DL}} \mathbf{V}_{m,\text{DL}}^{\text{RF}} \mathbf{x}_{m}^{\text{DL}} + \sum_{\substack{k=1\\m\neq j}}^{K} (\mathbf{U}_{j,\text{DL}}^{\text{RF}})^{H} \mathbf{H}_{jk}^{\text{DU}} \mathbf{V}_{k,\text{UL}}^{\text{RF}} \mathbf{x}_{k}^{\text{UL}} + (\mathbf{U}_{j,\text{DL}}^{\text{RF}})^{H} \mathbf{n}_{j}^{\text{DL}},$$
(39)

in which $\mathbf{x}_{j}^{\text{DL}} = \mathbf{V}_{j,\text{DL}}^{\text{BB}} \mathbf{s}_{j}^{\text{DL}}$ and $\mathbf{x}_{k}^{\text{UL}} = \mathbf{V}_{k,\text{UL}}^{\text{BB}} \mathbf{s}_{k}^{\text{UL}}$ are the digitally precoded symbols, $(\mathbf{U}_{j,\text{DL}}^{\text{BB}})^{H} \hat{\mathbf{x}}_{j}^{\text{DL}} = \hat{\mathbf{s}}_{j}^{\text{DL}}$, and $\mathbf{n}_{j}^{\prime \text{DL}} = (\mathbf{U}_{j,\text{DL}}^{\text{RF}})^{H} \mathbf{n}_{j}^{\text{DL}}$. Also, we have $\mathbf{H}^{\text{DL}} = [(\mathbf{H}_{1}^{\text{DL}})^{T}, (\mathbf{H}_{2}^{\text{DL}})^{T}, \dots, (\mathbf{H}_{J}^{\text{DL}})^{T}]^{T}$. The information flow for the downlink channel can be described as [15]

$$\mathbf{s}_{j}^{\mathrm{DL}} \to \mathbf{x}_{j}^{\mathrm{DL}} \xrightarrow{\bar{\mathbf{H}}^{\mathrm{DL}}} \hat{\mathbf{x}}_{j}^{\mathrm{DL}} \to \hat{\mathbf{s}}_{j}^{\mathrm{DL}}.$$
 (40)

Based on the above information flow, we can write [15]

$$R \stackrel{a}{\leq} I(\mathbf{s}^{\mathrm{DL}}; \hat{\mathbf{s}}^{\mathrm{DL}}) \stackrel{b}{\leq} C_{\mathrm{BC}}(\bar{\mathbf{H}}^{\mathrm{DL}}) \stackrel{c}{\leq} C_{\mathrm{BC}}(\mathbf{H}^{\mathrm{DL}}), \quad (41)$$

where $I(\cdot)$ denotes mutual information and C_{BC} represents the broadcast capacity of the respective channel. Therefore, in order to maximize the sum-rate, we attempt to force the above inequalities to equalities. Inequality (a) becomes an equality by mitigating the intra-user and inter-user interference via applying digital beamformings at the BS and the DL users using an extended version of the BD method introduced in [14]. More specifically, in our proposed BD method, in addition to the interference between the DL users, the interference from the UL users to the DL users is also cancelled. Furthermore, it is proved in [40] and [15] that exploiting massive MIMO technology leads to the orthogonality of user channels which makes inequality (b) equality. For inequality (c), we calculate analog precoder and combiner filters under the constant amplitude constraint to maximize $I(\mathbf{s}^{DL}; \hat{\mathbf{s}}^{DL})$. To do this, we exploit extra RF chains to handle the fixed modulus condition and derive RF beamforming matrices for

the case when the number of RF chains is at least double the number of data streams. Thus, in our proposed BD-based HBF scheme, interference is assumed to be dealt with in the digital domain in which the joint digital precoding and combining matrices are derived to achieve equality in (a), and the analog precoder and combiner filters are jointly designed to achieve equality in (c).

The operation of baseband BD beamforming cancels the intra-user and intra-user interference. Thus, the digital precoder at the BS should lie in the null space of the user channels and the digital combiner at the DL user should lie in the null space of interference channels between the UL and DL users. In order to find the null spaces, we apply singular value decomposition (SVD) to the DL channels and interference channels. The digital precoder can be derived as

$$\begin{split} \tilde{\mathbf{H}}_{j}^{\text{DL}} &= [(\bar{\mathbf{H}}_{1}^{\text{DL}})^{T}, \dots, (\bar{\mathbf{H}}_{j-1}^{\text{DL}})^{T}, (\bar{\mathbf{H}}_{j+1}^{\text{DL}})^{T}, \dots, (\bar{\mathbf{H}}_{J}^{\text{DL}})^{T}]^{T}, \\ \tilde{\mathbf{H}}_{j}^{\text{DL}} &= \tilde{\mathbf{U}}_{j,\text{DL}} \sum_{j,\text{DL}} [\tilde{\mathbf{V}}_{j,\text{DL}}^{(1)}, \tilde{\mathbf{V}}_{j,\text{DL}}^{(0)}]^{H}, \\ \tilde{\mathbf{V}}_{j,\text{DL}}^{(1)} &= \tilde{\mathbf{V}}_{j,\text{DL}}(:, 1: (J-1)M_{0}^{\text{RF}}), \\ \tilde{\mathbf{V}}_{j,\text{DL}}^{(0)} &= \tilde{\mathbf{V}}_{j,\text{DL}}(:, (J-1)M_{0}^{\text{RF}} + 1: \text{end}), \\ \bar{\mathbf{H}}_{j}^{\text{DL}} \tilde{\mathbf{V}}_{j,\text{DL}}^{(0)} &= \mathbf{U}_{j,\text{DL}} \sum_{j,\text{DL}} \mathbf{V}_{j,\text{DL}}^{H}, \\ \mathbf{V}_{j,\text{DL}}^{\text{BB}} &= \tilde{\mathbf{V}}_{j,\text{DL}}^{(0)} \mathbf{V}_{j,\text{DL}}^{(N_{s})}, \end{split}$$
(42)

in which $\mathbf{V}_{j}^{(N_{s})}$ denotes the first N_{s} columns of \mathbf{V}_{j} . Also, the digital combiner can be calculated as

$$\begin{split} \tilde{\mathbf{H}}_{\mathbf{j}}^{\mathrm{DU}} &= [(\bar{\mathbf{H}}_{j1}^{\mathrm{DU}}), \dots, (\bar{\mathbf{H}}_{jK}^{\mathrm{DU}})]^{T}, \\ \tilde{\mathbf{H}}_{\mathbf{j}}^{\mathrm{DU}} &= \tilde{\mathbf{U}}_{j,\mathrm{DU}} \sum_{j,\mathrm{DU}} [\tilde{\mathbf{V}}_{j,\mathrm{DU}}^{(1)}, \tilde{\mathbf{V}}_{j,\mathrm{DU}}^{(0)}]^{H}, \\ \tilde{\mathbf{V}}_{j,\mathrm{DU}}^{(0)} &= \tilde{\mathbf{V}}_{j,\mathrm{DU}}(:, N^{\mathrm{RF}} - N_{\mathrm{s}} + 1 : N^{\mathrm{RF}}), \\ \mathbf{U}_{j,\mathrm{DL}}^{\mathrm{BB}} &= \tilde{\mathbf{V}}_{j,\mathrm{DU}}^{(0)}. \end{split}$$
(43)

In [41], it is shown that two RF chains with amplitude constraint can be combined to create one RF chain without constant modulus condition. Thus, if the number of RF chains is at least double the number of data streams, we can eliminate the constant modulus constraint in (12c). In this case, in order to turn inequality (c) into an equality, the RF beamformings can be derived similar to [15] as

$$\begin{split} \mathbf{H}_{j}^{\mathrm{DL}} &= \mathbf{U}_{j,\mathrm{DL}} \sum_{j,\mathrm{DL}} \mathbf{V}_{j,\mathrm{DL}}^{H}, \\ \mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}} &= \mathbf{U}_{j,\mathrm{DL}}(:,1:N^{\mathrm{RF}}), \\ \mathbf{H}_{\mathrm{comp}}^{\mathrm{DL}} &= (\mathbf{U}_{\mathrm{DL}}^{\mathrm{RF}})^{H} \mathbf{H}^{\mathrm{DL}}, \\ \mathbf{H}_{\mathrm{comp}}^{\mathrm{DL}} &= \mathbf{U}_{\mathrm{comp},\mathrm{DL}} \sum_{\mathrm{comp},\mathrm{DL}} \mathbf{V}_{\mathrm{comp},\mathrm{DL}}^{H}, \\ \mathbf{V}_{j,\mathrm{DL}}^{\mathrm{RF}} &= \mathbf{V}_{\mathrm{comp},\mathrm{DL}}(:,1:N^{\mathrm{RF}}) \end{split}$$
(44)

Otherwise, if the number of RF chains at the BS is insufficient to eliminate the constant amplitude constraint in (12c), the optimal solution for analog beamforming would be the nearest point to the solution for unconstrained sumrate optimization [15]. Hence, we can write

$$\mathbf{U}_{j,\mathrm{DL}}^{\mathrm{RF}} = \frac{1}{\sqrt{N_j}} e^{\angle \mathbf{U}_{j,\mathrm{DL}}(:,1:N^{\mathrm{RF}})},$$

 TABLE 2.
 Computational complexities of the proposed methods in terms of big-O notation

Algorithm	Computational Complexity
Deriving the fully digital	$\mathcal{O}\left(UI(N^3 + M^3 + dM^2 + dN^2)\right)$
beamforming solution	
Applying the OMP method	$\mathcal{O}\left(N^{\mathrm{RF}}U(d^2N+d^2M+M^3+N^3)\right)$
for deriving HBF design	
BD-based HBF scheme	$\mathcal{O}(U(N^{\mathrm{RF}^3} + dN^{\mathrm{RF}^2} + dN^{\mathrm{RF}}M))$
	$+\min(MN^2, M^2N)$

$$\mathbf{V}_{j,\mathrm{DL}}^{\mathrm{RF}} = \frac{1}{\sqrt{M_0}} e^{\angle \mathbf{V}_{\mathrm{comp},\mathrm{DL}}(:,1:N^{\mathrm{RF}})}$$
(45)

B. UL TRANSMISSION

Based on the uplink-downlink duality, the HBF for the uplink channel can be derived by reversing the system and using the results for the downlink channel [31]. We omit the details of the derivations for brevity.

C. COMPUTATIONAL COMPLEXITY ANALYSIS

In this section, the computational complexities of the proposed OMP-based and BD-based HBF schemes are investigated. For ease of notation, suppose that all the nodes have the same number of transmit antennas, M, receive antennas, N, data streams, d, and RF chains, N^{RF} . Also, assume that the number of UL and DL users and the number of iterations in the digital beamforming step of OMP are U and I, respectively. The asymptotic behavior of the computational complexities of our proposed policies are shown in Table 2, where the expression for the OMP-based HBF scheme is equal to the sum of the complexities of the fully digital beamforming step and the OMP method. Since the BD-based HBF does not include any iterations between the transmitters and receivers, it has low complexity. In line with this, as it can be seen in Table 2, the big-O computational complexity of the BD-based HBF method has a polynomial with degree 4. In contrast, the computational complexity of the OMPbased HBF approach includes a polynomial with degree 5. Therefore, the BD-based HBF scheme has lower computational complexity than the OMP-based HBF approach.

VI. SIMULATION RESULTS

In this section, we investigate the performance of our proposed OMP-based and BD-based HBF schemes in an FD MIMO multi-user mmWave cellular system through simulations. First, we compare the achievable spectral efficiency of the proposed OMP-based HBF scheme with the fully digital one, and also contrast beamformer designs for the FD system with that of a traditional HD scheme having the relevant hardware configurations. Moreover, the performance of the OMP-based HBF is compared with the HBF approach for MUFD communications introduced in [12]. Then, the average spectral efficiency and the computational complexity of the BD-based HBF approach are compared with those for the OMP-based HBF method.

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TABLE 3. Simulation Parameters

Parameter	Settings
Cell Radius	80m
Carrier Frequency	28GHz
Bandwidth	1GHz
BS Transmission power	40dBm-50dBm
User Transmission power	20dBm-30dBm
Thermal Noise Density	-174dBm/Hz
Noise Figure	BS: 5dB, User: 7dB
Path Loss (dB) (d in m)	$PL(d) = \alpha + 10\beta \log_{10}(d) + \psi$
	$\psi \sim \mathcal{N}\left(0,\sigma^{2} ight)$
NLoS parameters	$\alpha = 72.0, \ \beta = 2.92, \ \sigma = 8.7 \mathrm{dB}$
LoS parameters	$\alpha = 61.4, \ \beta = 2.0, \ \sigma = 5.8 \mathrm{dB}$
Probability of LoS	$P_{LoS}(d) = e^{(-a_{LoS}d)}, \ 1/a_{LoS} = 67.1 \text{m}$
BS antenna	$64 \times 64 \ \lambda/2$ uniform linear array
UE antenna	$8 \times 8 \lambda/2$ uniform linear array
Number of Clusters	$N_{cl} \sim \max \left\{ Poisson(\chi), 1 \right\}, \ \chi = 1.8$
Cluster power fraction	$\gamma'_{m} = U_{m}^{\tau-1} 10^{-\frac{Z_{m}}{10}}, Z_{m} \sim \mathcal{N}(0,\varsigma^{2})$
	$U_m \sim Uniform[0,1], \ \tau = 2.8, \ \varsigma = 4$

Since the application with the most potential for FD technology in cellular and mmWave systems is in small cells. We evaluate the performance of the proposed algorithms in a mmWave small cell deployments. The channel parameters and the path loss model are taken from [3] and listed in Table 3. We assume a cell area of radius r = 80 m with the mmWave BS at its center, and 8 uplink users and 8 downlink users randomly placed within this area. Each user has $N_{\rm s} = 2$ data streams to transmit or receive. We then compute a random path loss between the BS and users based on the model in [3]. An accurate model for the SI channel plays an important role in evaluating the performance of FD systems. For the SI channel, we assume a LoS channel where the distance between transmit and receive antennas is d = 1m. We assume a set of two dimensional antenna arrays at both the BS and the users. On the BS and user sides, the arrays consist of 64×64 , and 8×8 elements, respectively. The spacing of the elements is set at half the signal wavelength. These antenna patterns were used in [3] showing excellent system capacity for small cell urban deployments. We consider two cases for the number of RF chains, including case-1 with 2 RF chains at each user and 16 RF chains at the BS, and case-2 with 3 RF chain at each user and 24 RF chains at the BS. It is worth mentioning that the maximum number of iterations of fully digital beamforming step in the OMP-based HBF policy is set to 50, and the results are averaged over 10^5 independent channel realizations.

A. COMPARISON OF THE PROPOSED OMP-BASED HBF WITH FULLY DIGITAL BEAMFORMING

In this section, the results for the UL and DL channels are given when the power expenditure of the users and BS are 20 dBm and 40 dBm, respectively. The average spectral efficiency of each UL and DL users in our proposed OMPbased policy and the optimal unconstrained precoding and combining scheme are shown in Fig. 3. In the HBF design, we consider two cases for the number of RF chains at the BS and users. As it can be seen from this figure, by adopting HBF



FIGURE 3. The average spectral efficiency of each user for fully digital beamforming and two cases of OMP-based HBF scheme with 20 dBm and 40 dBm transmit powers at the users and the BS, respectively.

technique, there is a loss in the spectral efficiency. However, by increasing the number of RF chains to the case-2, the spectral efficiency loss is reduced. Although a spectral loss is observed in HBF approach, fully digital is impractical for large scale antenna arrays and has a much higher cost.

B. COMPARISON OF OUR PROPOSED OMP-BASED HBF POLICY WITH OTHER APPROACHES

In Fig. 4 and Fig. 5, the average spectral efficiency of our proposed OMP-based HBF is compared with that for the scheme introduced in [12], considering case-1 for the number of RF chains. Since authors in [12] have not considered a cellular set up and have studied an interference multi-user MIMO channel, in order to be able to do this comparison, we extended the proposed policy in [12] to a cellular set up with a BS, UL, and DL users. As this figure depicts, our proposed OMP-based HBF approach outperforms the proposed scheme in [12], since our method directly deals with spectral efficiency and authors in [12] have tried to minimize the SI and multi-user interference. For a comparison of computational complexities, we refer to Section VI-D below.

C. COMPARISON OF FD AND HD TRANSMISSIONS USING OMP-BASED HBF

In Fig. 6, the achievable spectral efficiency of the FD and HD transmission modes and also mode selection approach using OMP-based HBF are depicted when the transmit power of BS is 40 dBm and the transmit power of users varies in the range of [20 dBm, 30 dBm]. In the mode selection scheme, at each transmission time slot, the scheduler compares the performance of HD-DL, HD-UL, and FD modes and chooses best of them [32]. As this figure illustrates the achievable spectral efficiency of the mode selection method is higher than the sum-rate of FD and HD transmission modes. The reason is that by using mode selection method, we can take advantage of all the available degree of freedom. In Fig. 7, the



FIGURE 4. The average spectral efficiency of each UL user for FD transmission with $N_{\rm s}=2$ data streams and 40 dBm transmit power at the BS.



FIGURE 5. The average spectral efficiency of each DL user for FD transmission with $N_{\rm s}=2$ data streams and 20 dBm transmit power at the UL users.

achievable spectral efficiency of the FD and HD transmission modes and also mode selection approach are depicted when the transmit power of UEs is 20 dBm and the transmit power of BS varies in the range of [40 dBm, 50 dBm]. As this figure illustrates mode selection method outperforms FD and HD transmission modes in terms of sum-rate.

D. COMPARISON OF OMP-BASED AND BD-BASED HBF APPROACHES

In Fig. 8, the performances of the proposed OMP-based and BD-based HBF methods are compared for the two cases of $M^{\rm RF} = N^{\rm RF} = 2N_{\rm s}$ and $M^{\rm RF} = N^{\rm RF} = N_{\rm s}$ for the DL transmission. As it can be seen from these figures, when



FIGURE 6. The average spectral efficiency of each user for mode selection, FD and HD transmissions with $N_{\rm s}=2$ data streams and 40 dBm transmit power at the BS.



FIGURE 7. The average spectral efficiency of each user for mode selection, FD and HD transmissions with $N_{\rm s}=2$ data streams and 20 dBm transmit power at the users.

there exist $2N_{\rm s}$ RF chains, the achievable spectral efficiency of BD-based scheme is higher than that for the OMP-based beamforming approach. However, when the number of RF chains is as low as $N_{\rm s}$, the OMP-based method outperforms the BD-based beamforming method. The reason is that in the BD-based approach, for the large number of RF chains, the fixed amplitude condition can be removed and the digital and RF beamformings are designed to minimize information loss. Therefore, choosing between OMP-based and BDbased HBF depends on the amount of available RF chains.



FIGURE 8. Comparison of the proposed OMP-based and BD-based HBF methods in the DL channel for FD transmission with 20 dBm transmit power at the UL users.



FIGURE 9. Computational complexity in the proposed OMP-based and BD-based HBF schemes.

Finally, in Fig. 9, the number of complex multiplications required for the proposed OMP-based and BD-based HBF policies and the scheme introduced in [12] are shown considering case-1 for the number of RF chains and maximum 50 iterations for the OMP-based method. As we expect based on the computational complexity analysis presented in Section V-C, this figure illustrates that our proposed BD-based HBF scheme has about 11% and 40% lower computational complexity compared to the proposed OMP-based HBF approach and the policy introduced in [12], respectively.

VII. CONCLUSION

In this paper, we have considered the design of HBF filters for mmWave multi-user MIMO FD cellular systems and have proposed the two approaches of OMP-based and BDbased HBF designs that maximize the sum-rate. In the first method, the RF and baseband beamformers are optimized jointly at each transmitter and the receiver. For this, we first design the fully digital beamforming to minimize the sum-MSE and then, by exploiting the spatial structure of mmWave channels and uplink-downlink duality, sparse approximation problems are formulated to design hybrid MIMO filters approximating the MMSE transmit/receive processors. In the second proposed method, we have jointly derived transmit precoders and receive combiners with a separate designs for analog and digital beamformers. In this method, the iterations between the transmitters and receivers are eliminated, and hence, the computational complexity is reduced compared to OMP-based HBF scheme. In the simulation results, it has been shown that the proposed OMP-based HBF schemes outperform HD HBF technique and has better performance compared with other HBF approaches in the literature. Furthermore, the proposed BD-based HBF scheme has been found to improve over the OMP-based HBF in terms of sumdata rate if the number of RF chains is at least double the number of data streams. Hence, according to the number of RF chains, we select either OMP-based or BD-based HBF policy.

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