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Characteristics of Power Line Networks: Diversity and Interference Alignment

Lutz Lampe, Md. Jahidur Rahman, H. Ebrahimzadeh Saffar
University of British Columbia, Vancouver, BC, Canada, lampe@ece.ubc.ca

***Abstract*—In this paper, we revisit the problem of cooperative communication in power line communication (PLC) networks in two important instances. First, we consider the power line relay channel and investigate possible diversity gains. Exploiting the underlying signal propagation characteristics we sharpen previous results presented on this topic. Second, we apply the insights on signal propagation to the application of blind interference alignment in PLC networks. We show that this concept, previously suggested for wireless communication applying antenna reconfiguration, does not not carry over to PLC using receiver impedance reconfiguration. The results of our work are fundamental in nature and emphasize on the importance of taking the specifics of signal propagation along transmission lines into account when designing cooperative communication methods for PLC.**

***Index Terms*—Power line communications (PLC), keyhole channel, diversity, relay channel, interference alignment, blind interference alignment**

I. INTRODUCTION

Power line communication (PLC) uses a wired infrastructure for transporting communication signals. It has therefore commonalities with other wired communication techniques, such as digital subscriber line (DSL) transmission. This supports the use of similar methods for channel characterization based on the physics of wave propagation along transmission lines, e.g. [1]–[7], or phenomenological properties, e.g. [8]. Since PLC is a reuse technique, it shares also many similarities with wireless communication. This manifests itself in phenomenological channel models, e.g. [1], [9]–[12], and networking and signal processing concepts adopted from the wireless domain, e.g. [13]–[16]. The latter often arise from the broadcasting nature of the PLC channel, which it has in common with wireless propagation.

One of the more recently adopted concepts is cooperative communications, in which information transfer between nodes is aided by intermediate nodes through retransmission or relaying. This area has seen significant contributions, from single-frequency networking [17], to incremental redundancy [18]–[20] and distributed space-time coding [21], [22], to relaying protocol optimization [23], [24] and network coding [25]–[29]. Related to cooperative communications is the concept of interference alignment that has experienced a spur of recent research activities in the wireless domain [30], [31]. Since PLC networks in which multiple transmitter and receiver pairs communicate can be represented as interference channels, it is not surprising that interference alignment has been shown to provide spatial multiplexing gains also for PLC systems [32].

However, despite the many similarities, there are also important differences between power line and wireless channels. In the context of cooperative communications, the tree structure of the power line infrastructure is one such difference. It leads to the so-called keyhole property of PLC relay channels, which has been reported in [33] and identified to limit spatial diversity gains for decode-and-forward relaying. Another difference is the interdependency between channel frequency responses of different point-to-point links in PLC relay networks, which is a result of the physical connection of power line infrastructure components, i.e., power lines and connected equipment.

In this paper, concentrating on PLC over two-conductor lines, we take a closer look at the channel characteristics in PLC networks and their effect on the above-mentioned advanced communication strategies. In particular, we demonstrate that the keyhole property would indeed prevent a spatial diversity gain for any relaying scheme, if independence of signal transfer characteristics in different parts of the network can be assumed. We also show, however, that generally this is not the case. Thus, we extend and clarify the result from [33], as well as post a new research problem, which is for what network structures and relaying schemes a diversity gain could be realized. Our second contribution is a negative result for the applicability of an interference alignment method that does not require channel state information (CSI) at the transmitters. In particular, we consider the blind interference alignment scheme based on receiver reconfiguration, which has been proposed for wireless communication in [34]. This method seems to be well suited for PLC-based interference alignment, as a receiver reconfiguration can be accomplished through adapting the receiver impedance. However, we show that the transmission characteristics of PLC render the spatial multiplexing gains seen in wireless communication impossible.

The remainder of this paper is organized as follows. In Section II, we introduce the power line network model, which is essential to derive our results. Then, the PLC relay channel and its diversity are studied in Section III. Section IV presents the analysis of spatial multiplexing with blind interference alignment. Finally, conclusions are given in Section V. For clarity of presentation, some details of the derivations are relegated to Appendices A and B.

II. POWER LINE NETWORK MODEL

We consider a general power line network to which several PLC modems are connected. For our analysis it is sufficient to consider three PLC devices. Due to the tree configuration of the power line infrastructure, there will be a node in the

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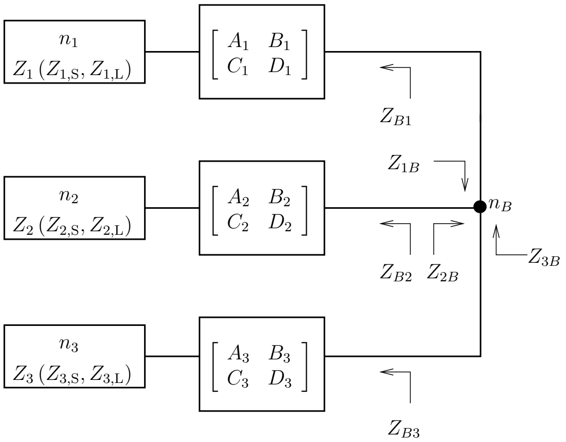

Fig. 1. PLC network with three communication nodes.

network through which all signals between the three PLC devices will pass. This node is identified as node n_B in Figure 1, which represents an abstract model of the physical power line network.

We assume a two-conductor PLC setup so that the network components between the three PLC devices, labeled as n_1, n_2, and n_3 in Figure 1, and node n_B are represented through ABCD transmission line parameters. The corresponding ABCD-matrices $\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}$ (see Appendix A for details) relate the appropriately oriented voltages and currents at node n_i and node n_B with each other. We note that they capture the aggregate effects of transmission lines, branches, loads etc. including those components that are physically behind nodes n_i from node n_B's perspective. For the following discussion it is irrelevant that the ABCD parameters are frequency-dependent, and we therefore do not indicate this dependency in our notation throughout this paper. Furthermore, we assume that channel frequency-selectivity is dealt with through the use of multicarrier modulation, such as orthogonal frequency-division multiplexing (OFDM), which allows us to consider a frequency-flat channel per subcarrier.

The benefit of the abstract model in Figure 1 is that it fully captures the interdependencies of signals communicated between the PLC devices. First, as mentioned above, all signals have to travel through node n_B, which therefore is also referred to as a keyhole [33]. Therefore, signals that are transmitted between for example node n_1 and nodes n_2 and n_3 have part of their transmission path in common, namely from node n_1 to node n_B in this case. Secondly, changes of transmission line parameters in one segment of the network, for example in the segment between nodes n_B and n_3, have an effect on the signal propagation in other segments of the network, for example in the segments between nodes n_1 and n_B and between n_2 and n_B. These features of multi-node transmission in PLC networks are different from wireless communication and cannot be ignored when analyzing any sort of cooperative communication. In particular, they are not taken into account by phenomenological channel modeling, which only describes point-to-point channels.

Let us now express the various relevant channel frequency responses and impedances for the network in Figure 1. To this end, we denote the source impedance when PLC device i is transmitting as $Z_{i,\mathrm{S}}$, $Z_{i,\mathrm{L}}$ is the load impedance when PLC device i is receiving, and when we do not need to specify whether a PLC device is transmitting or receiving, we refer to its impedance as Z_i, $i = 1, 2, 3$. Furthermore, we denote the impedance seen into/from node n_B from/to node n_i as Z_{iB} and Z_{Bi}, respectively. Then, from (41) (derived in Appendix A) it follows that

$$Z_{Bi} = \frac{D_i Z_i + B_i}{C_i Z_i + A_i}\ , \tag{1}$$

and

$$Z_{iB} = Z_{Bj} \| Z_{Bk} = \frac{1}{1/Z_{Bj} + 1/Z_{Bk}}\ , \tag{2}$$

where $j \neq k,\ j \neq i,\ i, j, k \in \{1, 2, 3\}$. Using (39) and (41), the voltage channel frequency response from node n_i to n_B and from node n_B and to n_i can be expressed as

$$H_{iB} \triangleq \frac{V_B}{V_{i,\mathrm{S}}} = \frac{Z_{iB}}{(A_i + C_i Z_{i,\mathrm{S}}) Z_{iB} + B_i + D_i Z_{i,\mathrm{S}}} \tag{3}$$

and

$$H_{Bi} \triangleq \frac{V_{i,\mathrm{L}}}{V_B} = \frac{Z_{i,\mathrm{L}}}{D_i Z_{i,\mathrm{L}} + B_i}\ , \tag{4}$$

respectively, where $V_{i,\mathrm{S}}$ is the voltage of the source before its internal impedance $Z_{i,\mathrm{S}}$, $V_{i,\mathrm{L}}$ is the voltage at load impedance $Z_{i,\mathrm{L}}$, and V_B is the voltage at node n_B.

Using (3) and (4), the channel frequency responses between the three nodes n_1, n_2, and n_3 can be written as

$$H_{12} = H_{1B} H_{B2}, \tag{5}$$

$$H_{13} = H_{1B} H_{B3}, \tag{6}$$

$$H_{23} = H_{2B} H_{B3}. \tag{7}$$

III. RELAY CHANNEL AND DIVERSITY

We first consider the network in Figure 1 as a relay channel, where nodes n_1, n_2, and n_3 correspond to source, relay and destination node, respectively.

We are interested in finding possible asymptotic gains in terms of spatial diversity that could be achieved by the use of relaying. We note that spatial diversity manifests itself in reducing the probability of experiencing a low instantaneous signal-to-noise ratio (SNR) at a specific location given an average SNR in a geographic neighborhood. Due to mobility, spatial diversity in wireless communication is transformed into temporal diversity, which can be exploited through interleaving and coding or retransmission for a single link. This is typically not the case for PLC. While due to impedance changes of devices connected to the grid different channel realizations can also occur for PLC over time, these changes are either infrequent or periodic with the mains electric power cycle. Hence, spatial diversity for PLC would be observed across different instances of power line networks.

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A. Noise and SNR

PLC signals are contaminated by different types of additive noise that are usually classified as background noise, narrowband noise, and impulsive noise [35]–[37]. The former two may be modelled as additive white Gaussian noise (AWGN) with a certain variance for a given OFDM subcarrier channel (so noise coloring in frequency direction, which is not exploited though). Impulsive noise may be represented through a Gaussian mixture noise model, whose different components can be interpreted as impulse noise processes with different variances [38]–[40]. If the impulsive noise is significant and if the impulse events at multiple receiving nodes (relay and destination nodes in our case) are independent, then spatial diversity would be observed, even in a single PLC network realization. However, since impulse events are caused by loads connected to the power line network, it is not likely that impulse noise realizations at different nodes are independent. Furthermore, in this work we are interested in the effect of the keyhole channel on diversity. We thus proceed by considering the SNR at an OFDM subchannel as

$$\gamma = S|H|^2/N \ , \tag{8}$$

where S and N are the transmit and noise power spectral density (PSD) and H is the frequency response value in this subchannel, respectively. We assume S and N to be the same for all communication links and define the equivalent transmitter SNR as

$$\gamma_{\mathrm{t}} = S/N \ . \tag{9}$$

B. Outage and Diversity

Diversity is defined by how outage probability depends on average SNR for asymptotically large SNR. Denoting the target communication rate between two nodes i and j as R, an outage event occurs if R exceeds the mutual information I_{ij} supported by the channel between the two nodes. The mutual information I_{ij} is a random variable which is a function of the channel gain $|H_{ij}|$. As a result, the outage probability is given by

$$p_{\mathrm{out}} = \Pr\{R > I_{ij}\} \ . \tag{10}$$

The diversity order of the channel is then defined as

$$d \triangleq \lim_{\gamma_{\mathrm{t}}\to\infty} \frac{-\log(p_{\mathrm{out}})}{\log(\gamma_{\mathrm{t}})} \ . \tag{11}$$

C. Diversity Analysis

We start with the conventional point-to-point channel between source and destination node, and consider the frequency-flat channel experienced for the transmission over one OFDM subcarrier. Since in this case

$$I_{13} = \log(1 + |H_{13}|^2\gamma_{\mathrm{t}}) \ , \tag{12}$$

the outage probability (10) can be expressed as

$$p_{\mathrm{o,PTP}} = \Pr\left\{|H_{13}|^2 \le (2^R - 1)/\gamma_{\mathrm{t}}\right\} = \Pr\{|H_{13}|^2 \le \alpha\}, \tag{13}$$

where $\alpha \triangleq (2^R - 1)/\gamma_{\mathrm{t}}$.

For the relay channel, we note that the network can be seen as composition of a virtual single-input multiple-output (SIMO) and a multiple-input single-output (MISO) channel. That is, the flow of information has to pass through two cuts: 1) the cut dividing the source against the super-node consisting of the relay and the destination, and 2) the cut dividing the super-node consisting of the source and the relay against the destination. Using the max-flow min-cut theorem [41, Thm. 15. 2], the capacity of the network will be given by the mutual information of the minimum cut. We can write the mutual information for the SIMO and MISO channels as

$$I_{1(23)} = \log\left(1 + (|H_{12}|^2 + |H_{13}|^2)\gamma_{\mathrm{t}}\right) \tag{14}$$

and

$$I_{(12)3} = \log\left(1 + (|H_{23}|^2 + |H_{13}|^2)\,\gamma_{\mathrm{t}}\right) \tag{15}$$

where we have assumed the relay node's transmitter sends independently of and with the same power as the source transmitter in $I_{(12)3}$.

The outage probability (10) for the PLC relay channel can be lower-bounded as follows:

$$\begin{aligned} p_{\mathrm{o,RCH}} &= \Pr\left\{\min\left(I_{1(23)}, I_{(12)3}\right) < R\right\} && (16)\\ &\ge \min\left(\Pr\left\{I_{1(23)} < R\right\}, \Pr\left\{I_{(12)3} < R\right\}\right) \ . && (17)\end{aligned}$$

This means that the diversity order is bounded by the diversity orders $d_{1(23)}$ and $d_{(12)3}$ for the SIMO and MISO channels:

$$d_{\mathrm{RCH}} \le \min\left(d_{1(23)}, d_{(12)3}\right) \ . \tag{18}$$

We assume that the gains $|H_{12}|$, $|H_{23}|$, $|H_{13}|$ are identically distributed over different network realizations, i.e., we do not impose a specific structure for the network and locations of source, relay and destination nodes. Then, considering Eqs. (14) and (15), the diversity orders $d_{1(23)}$ and $d_{(12)3}$ will be identical. In the following, we will thus focus on the cut for the MISO channel. Using the expressions for the channel responses in Eqs. (6) and (7), we can rewrite (15) as

$$I_{(12)3} = \log\left(1 + |H_{B3}|^2(|H_{1B}|^2 + |H_{2B}|^2)\gamma_{\mathrm{t}}\right) \ . \tag{19}$$

Hence,

$$\begin{aligned} p_{\mathrm{o,RCH}} &\ge \Pr\left\{I_{(12)3} < R\right\} && (20)\\ &= \Pr(|H_{13}|^2 + |H_{23}|^2 \le \alpha) && (21)\\ &= \Pr(|H_{B3}|^2(|H_{1\mathrm{B}}|^2 + |H_{2B}|^2) \le \alpha) \ . && (22)\end{aligned}$$

The last expression can be lower bounded by

$$\begin{aligned} &\Pr\{|H_{B3}|^2(|H_{1\mathrm{B}}|^2 + |H_{2B}|^2) \le \alpha\} && (23)\\ &\quad\ge \Pr\{|H_{B3}|^2 \le \alpha, |H_{1\mathrm{B}}|^2 + |H_{2B}|^2 \le 1) && (24)\\ &\quad= \Pr\{|H_{B3}|^2 \le \alpha\}\\ &\qquad\times \Pr\{|H_{1\mathrm{B}}|^2 + |H_{2B}|^2 \le 1\big||H_{B3}|^2 \le \alpha\} && (25)\\ &\quad= p_{\mathrm{o,PTP}} \times \Pr\{|H_{1\mathrm{B}}|^2 + |H_{2B}|^2 \le 1\big||H_{B3}|^2 \le \alpha\}, && (26)\end{aligned}$$

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where the last step follows from noting that the channel from node n_B to n_3 is a point-to-point power line channel[1] and the first term in (25) is its outage probability.

Based on (26), we make two observations.

1) *If* the link gains $|H_{B3}|^2$ and $(|H_{1\mathrm{B}}|^2, |H_{2B}|^2)$ are independent of each other, i.e., $\Pr\{|H_{1\mathrm{B}}|^2 + |H_{2B}|^2 \leq 1\big||H_{B3}|^2 \leq \alpha\} = \Pr\{|H_{1\mathrm{B}}|^2 + |H_{2B}|^2 \leq 1\} = \text{const.}$, then from (22) and (26) we obtain
$$p_{\mathrm{o,RCH}} \geq p_{\mathrm{o,PTP}} \times \text{const.} \tag{27}$$
and thus
$$d_{\mathrm{RCH}} \leq d_{\mathrm{PTP}}\ . \tag{28}$$
Since d_{PTP} is also a trivial lower bound for d_{RCH}, the equality $d_{\mathrm{RCH}} = d_{\mathrm{PTP}}$ would follow. Hence, because of the keyhole property, PLC relay channels would not provide a diversity gain.
2) However, considering (45) and (46) from Appendix B, we note that $|H_{1\mathrm{B}}|^2$ and $|H_{2B}|^2$ depend on Z_{B3}. According to (1), this impedance is a function of the network parameters C_3, D_3 and $Z_{3,\mathrm{L}}$, which also determine $|H_{B3}|^2$ as per (4). Hence, independence of the frequency responses for different links *cannot* generally be concluded for PLC networks.

In summary, the keyhole property of cooperative power line transmission suggests that the diversity order of PLC relay channels would be limited by the diversity of point-to-point channels, as argued in [33]. However, due to the interdependence of the instantaneous gains of the different links of the relay channel, this conclusion does not hold true in general. Hence, a relevant problem for future studies is to determine under which circumstances there could be a diversity gain.

IV. BLIND INTERFERENCE ALIGNMENT

The network model in Figure 1 also allows us to gain insight into the applicability of interference alignment to PLC networks. We first briefly introduce the considered *blind* interference alignment scheme and analyze its use in PLC.

A. Interference Alignment Scheme

We consider the blind interference alignment scheme from [34] and for concreteness we focus on the case of two senders transmitting messages to two users, which is the 2-user 2×1 scenario in [34]. The setup is illustrated in Figure 2, which is also known as the X-channel [30].

The X-channel is a 4-node network where two nodes n_i, $i \in \{1, 2\}$, intend to communicate data u_{ij} to two different nodes n_j, $j \in \{3, 4\}$. It is shown in [34] that the maximum multiplexing gain of this channel can be achieved without CSI at the transmitters as follows. Nodes n_1 and n_2 transmit
$$\boldsymbol{x}_1 = \begin{bmatrix} u_{13} + u_{14} \\ u_{13} + u_{14} \\ 0 \end{bmatrix} \tag{29}$$

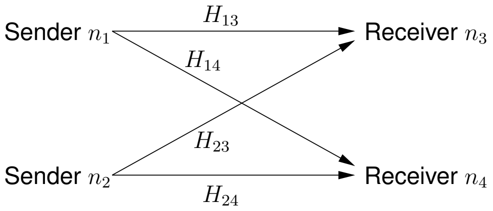

Fig. 2. Transmission scenario of interference alignment (X-channel setting).

and
$$\boldsymbol{x}_2 = \begin{bmatrix} u_{23} + u_{24} \\ 0 \\ u_{23} + u_{24} \end{bmatrix}, \tag{30}$$
respectively, in three successive time slots. Denoting the channel gains from node n_i to node n_j in time slot k by $H_{ij}(k)$, node n_j receives
$$\boldsymbol{y}_j = \begin{bmatrix} H_{1j}(1) \\ H_{1j}(2) \\ H_{1j}(3) \end{bmatrix} \circ \boldsymbol{x_1} + \begin{bmatrix} H_{2j}(1) \\ H_{2j}(2) \\ H_{2j}(3) \end{bmatrix} \circ \boldsymbol{x_2} + \boldsymbol{w}, \tag{31}$$
where $\circ$ represents element-wise multiplication and $\boldsymbol{w}$ is additive noise. Defining $\boldsymbol{H}_j(k) = [H_{1j}(k)\, H_{2j}(k)]$, $\boldsymbol{u}_j = [u_{1j}\, u_{2j}]^T$ and $\mathbf{0} = [0\ 0]$, the received signal at node n_3 can be rewritten as
$$\boldsymbol{y}_3 = \underbrace{\begin{bmatrix} \boldsymbol{H}_3(1) \\ \boldsymbol{H}_3(2) \\ \mathbf{0} \end{bmatrix}}_{\boldsymbol{M}_1} \boldsymbol{u}_3 + \underbrace{\begin{bmatrix} \boldsymbol{H}_3(1) \\ \mathbf{0} \\ \boldsymbol{H}_3(3) \end{bmatrix}}_{\boldsymbol{M}_2} \boldsymbol{u}_4 + \boldsymbol{w}\ . \tag{32}$$
For node n_3 to recover $\boldsymbol{u}_3$ from $\boldsymbol{y}_3$ without interference from the signal intended for node n_4, the trick of interference alignment is to ensure that
$$\begin{array}{rcl} \boldsymbol{H}_3(1) & \neq & \kappa \cdot \boldsymbol{H}_3(2) \\ \boldsymbol{H}_3(1) & = & \boldsymbol{H}_3(3) \end{array} \tag{33}$$
for any constant κ. Then, matrices $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ in Eq. (32) have ranks 2 and 1, respectively, so that interference can completely be cancelled at node n_3 without cancelling the desired signal $\boldsymbol{u}_3$. The conditions in (33) can be accomplished blindly, i.e., without CSI at the transmitters, by changing the configuration of the receiver at node n_3 during the second transmission slot. Rewriting Eq. (31) for node n_4, we find that the same interference cancellation can be achieved by enforcing $\boldsymbol{H}_4(1) \neq \kappa \cdot \boldsymbol{H}_4(3)$ and $\boldsymbol{H}_4(1) = \boldsymbol{H}_4(2)$. Since four data symbols are transmitted in three time slots, a multiplexing gain of 4/3 is achieved, which is the maximum possible for this network [34].

B. Application to PLC Networks

For the case of wireless communications considered in [34], the requirement (33) has been met through staggered antenna switching at the receiving nodes. That is, a first receiver antenna configuration is chosen during slots $k = 1$ and $k = 3$, and a second antenna configuration is applied during $k = 2$. In the case of PLC considered here, a receiver modem connected to two conductors can change its input impedance.

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[1] Note also that the channel response H_{B3} can be written as the product of partial channel responses for the path from n_B to n_3 alike H_{ij}, $i, j = 1, 2, 3$ in (5)-(7).

Since the frequency response to a device is dependent on this impedance, see e.g. expression (4), blind interference alignment as described above seems to be directly applicable.

However, the specific characteristics of PLC signal propagation thwart our attempt to apply this blind interference alignment. To show this, we consider nodes n_1 and n_2 in Figure 1 as the two transmitters and node n_3 as one of the receivers. Then, we can write the channel vector from n_1 and n_2 to n_3 as

$$\begin{aligned} \boldsymbol{H}_3(k) &= [H_{13}(k)\ \ H_{23}(k)] && (34)\\ &= [H_{1B}(k)H_{B3}(k)\ \ H_{2B}(k)H_{B3}(k)]\,, && (35)\end{aligned}$$

where time variation with k is accomplished through modifying $Z_{3,\mathrm{L}}$ at node n_3. We note that $Z_{3,\mathrm{L}}$ directly affects $H_{B3}(k)$ via (4), but also $H_{1B}(k)$ and $H_{2B}(k)$ through the dependency chain:

$$Z_{3,\mathrm{L}} \xrightarrow{\text{Eq.(1)}} Z_{B3} \xrightarrow{\text{Eq.(2)}} (Z_{1B}, Z_{2B}) \xrightarrow{\text{Eq.(3)}} (H_{1B}(k), H_{2B}(k))\,.$$

On the other hand, we can rearrange (34) into

$$\boldsymbol{H}_3(k) = H_{23}(k)[H_{13}(k)/H_{23}(k)\ \ 1]\,. \tag{36}$$

In Appendix B we show that the *ratio* $H_{13}(k)/H_{23}(k)$ is independent of Z_{B3}. Hence, $H_{13}(k)/H_{23}(k) = H_{13}/H_{23} \triangleq c$ independent of k, and thus (see (32))

$$\begin{aligned} \boldsymbol{M}_1 &= \begin{bmatrix} \boldsymbol{H}_3(1) \\ \boldsymbol{H}_3(2) \\ \boldsymbol{0} \end{bmatrix} && (37)\\ &= \begin{bmatrix} c \cdot H_{23}(1) & H_{23}(1) \\ c \cdot H_{23}(2) & H_{23}(2) \\ 0 & 0 \end{bmatrix}. && (38)\end{aligned}$$

The rank of matrix $\boldsymbol{M}_1$ is one, regardless of how $H_{23}(k)$ is changed due to reconfiguration at node n_3. Hence, spatial multiplexing is not achieved.

We thus have shown that blind interference alignment through receiver reconfiguration, a seemingly attractive scheme for PLC, is not possible in principle due to the properties of the transmission-line signal propagation.

V. CONCLUSIONS

In this paper, we have first discussed the diversity gain achievable with PLC relaying. We have pointed out that the keyhole property of the relay channel suggests the absence of a diversity gain, but also found that this statement cannot be made rigorously due to the dependency of signal transfer in one part of the network on elements in another. Our second contribution uses the same underlying result to establish that blind interference alignment cannot be accomplished with receiver impedance reconfiguration, which would seem to follow as an analogous solution to blind interference alignment in wireless communication. While our results are in part nonconclusive and in part negative in nature, we believe that the insights and methodology presented here are fundamentally useful for further studies and the meaningful design of cooperative communication schemes in PLC networks.

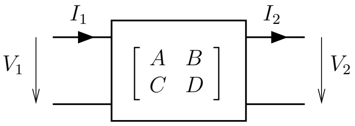

Fig. 3. ABCD-matrix representation of a two-port network.

APPENDIX A
ABCD MATRIX REPRESENTATION

The ABCD-matrix representation of a two-port network relates the voltages and currents identified in Figure 3 as

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}. \tag{39}$$

The same ABCD parameters can be used when input and output are swapped, i.e., transmission in the other direction is considered. Then we have

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} D & B \\ C & A \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}. \tag{40}$$

If the reciprocity property holds, $AD-BC = 1$ is true. Hence, in this case (40) simplifies to

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} D & B \\ C & A \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}. \tag{41}$$

We note that reciprocity can be assumed for power line networks, where the overall ABCD-matrix is a cascade of reciprocal ABCD matrices, e.g. [42].

APPENDIX B
PROPERTY OF THE PLC KEYHOLE CHANNEL

We consider the model in Figure 1 and assume that either nodes n_1 and n_2 are transmitting simultaneously, or that source and load impedance are identical for those two nodes, so that $Z_1 = Z_{1,\mathrm{S}}$ and $Z_2 = Z_{2,\mathrm{S}}$. In this appendix, we show that then the ratio of the channel frequency responses H_{13} and H_{23} is independent of the network elements located between node n_B and node n_3.

From (6) and (7) we can write the ratio as

$$\frac{H_{13}}{H_{23}} = \frac{H_{1B}}{H_{2B}}\,. \tag{42}$$

Let us consider the numerator H_{1B} first. Starting from (3) we obtain

$$\begin{aligned} H_{1B} &= \frac{1}{A_1 + C_1 Z_{1,\mathrm{S}} + (B_1 + D_1 Z_{1,\mathrm{S}})/Z_{1B}} && (43)\\ &\overset{(a)}{=} \frac{1}{A_1 + C_1 Z_{1,\mathrm{S}} + (B_1 + D_1 Z_{1,\mathrm{S}})\left(\frac{1}{Z_{B2}} + \frac{1}{Z_{B3}}\right)} && (44)\\ &\overset{(b)}{=} \frac{1}{(B_1 + D_1 Z_{1,\mathrm{S}})\left(\frac{1}{Z_{B1}} + \frac{1}{Z_{B2}} + \frac{1}{Z_{B3}}\right)}\,, && (45)\end{aligned}$$

where (a) follows from substituting Z_{1B} using (2) and (b) from (1). Applying the same transformations to H_{2B} leads to

$$H_{2B} = \frac{1}{(B_2 + D_2 Z_{2,\mathrm{S}})\left(\frac{1}{Z_{B2}} + \frac{1}{Z_{B1}} + \frac{1}{Z_{B3}}\right)}\,. \tag{46}$$

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Finally, substituting (45) and (46) into (42) gives us

$$\frac{H_{13}}{H_{23}} = \frac{H_{1B}}{H_{2B}} = \frac{B_2 + D_2 Z_{2,\mathrm{S}}}{B_1 + D_1 Z_{1,\mathrm{S}}} , \qquad (47)$$

which only depends on the parameters of the network elements between nodes n_1 and n_B and nodes n_2 and n_B, respectively.

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