On Trellis Shaping for PAR Reduction in OFDM Systems

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***Abstract*— The application of trellis shaping was recently proposed to reduce the peak-to-average power ratio (PAR) of orthogonal frequency division multiplexing (OFDM) signals. In this letter, we review the trellis shaping schemes presented in the literature and we introduce modifications such as a new decoding metric and the use of sequential decoding. We conduct a comprehensive complexity and performance comparison for the different schemes, and one interesting result of this work is that, in terms of PAR-reduction capability, trellis shaping with time-domain metrics is generally superior to trellis shaping with frequency-domain metrics. Furthermore, the proposed modifications enable trellis shaping for PAR reduction with a flexible performance-complexity tradeoff.**

***Index Terms*— Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAR) reduction, trellis shaping.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a popular technique for transmission over frequency-selective channels. A major drawback of OFDM, however, is the large peak-to-average power ratio (PAR) of the transmit signal. Therefore, PAR reduction techniques have been intensely studied in the literature, cf. e.g. [1] for an overview. Recently, Henkel and Wagner introduced a new PAR reduction method in [2], which is based on trellis shaping. The idea is that a decoder selects a sequence of a (convolutional) code such that, when "added" to the data signal, the PAR of the transmit signal is minimized. Trellis shaping can thus be classified as PAR reduction through multiple signal representation (MSR). Since the receiver does not need to be informed which code sequence was chosen, trellis shaping does not require explicit transmission of side information, which is a distinct advantage over other popular MSR schemes, cf. e.g. [3]. It was observed in [2] that trellis shaping using the Viterbi algorithm (VA) and a time-domain metric to select the code sequence achieves considerable PAR reductions. Recently, Ochiai [4] further elaborated on trellis shaping for PAR reduction and proposed a frequency-domain metric for the VA.

In this letter, we investigate trellis shaping for PAR reduction and our objectives are twofold. First, we extend the schemes from [2], [4] in that we propose (i) an additional metric and (ii) the application of the stack sequential decoding algorithm (ST-SDA) [5] to select the code sequence. The new metric facilitates low-complexity "adaptive" trellis shaping and the ST-SDA is considered as an alternative to the VA for a more flexible performance-complexity tradeoff. In this context, we also clarify that the VA and ST-SDA are generally suboptimum algorithms for the PAR reduction decoding problem. Second, we provide a performance and complexity comparison of the different metrics and selection algorithms considered for trellis shaping for PAR reduction. Such a comparison has not been presented in the literature yet.

As it is customary in the literature (e.g. [1], [4]), in the following we approximate the PAR of the OFDM transmit signal $x(t)$ by the PAR of the oversampled signal $x_n \triangleq x(nT/L)$, where T is the modulation interval and $L = 4$ is chosen for a good approximation [6]. Considering OFDM with N subcarriers and defining the vector $\boldsymbol{X} \triangleq [X_0 \ldots X_{N-1}]$ of N quadrature-amplitude modulation (QAM) data symbols X_i, $\boldsymbol{x} \triangleq [x_0 \ldots x_{LN-1}] = \mathrm{IDFT}(\boldsymbol{X})$ is the corresponding oversampled time-domain vector.[1] The PAR for $\boldsymbol{x}$ is then given by ($\mathrm{E}\{\cdot\}$: expectation, $||\cdot||_\infty$: max norm)

$$\begin{aligned}\gamma(\boldsymbol{x}) &= \max_{0\le n<LN}\{|x_n|^2\} \,/\, \mathrm{E}\{|x_n|^2\} \\ &= ||\boldsymbol{x}||_\infty^2 \,/\, \mathrm{E}\{|x_n|^2\}\ .\end{aligned} \tag{1}$$

II. PAR Reduction Using Trellis Shaping

For a detailed description of trellis shaping for PAR reduction we refer to [2], [4]. We apply the "Type-I" constellation mapping from [4] where two M-ary signal points X_i with the same $(\log_2(M)-1)$ less significant bits (LSBs) are symmetric about the origin of the complex plane.[2] This means that flipping the most significant bit (MSB) will change the sign of the signal point X_i and thus strongly influence the PAR but not change the average power of the time-domain signal $\boldsymbol{x}$. The MSBs for the N symbols X_i in $\boldsymbol{X}$ are "shaped" by a binary sequence $\boldsymbol{y}$ of N bits, which is a code word of a rate-$1/n_s$ convolutional code. Due to the application of an inverse syndrome former at the transmitter and syndrome former at the receiver, shaping is transparent for data transmission (see [4, Fig. 1]). This means that no shaping side information needs to be transmitted to the receiver. The problem of trellis shaping

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[1] $\mathrm{IDFT}(\boldsymbol{X})$ returns the LN-dimensional inverse discrete Fourier transform (IDFT) of the appropriately zero-padded N-dimensional vector $\boldsymbol{X}$.

[2] It is interesting to note that also another mapping, referred to as "Type-II" mapping, is considered in [4], which, while inferior to Type-I mapping in PAR reduction, enables additional reduction of the average transmit power.

for PAR reduction is to select $\boldsymbol{y}$ from the set of code words $\mathcal{C}$ such that $\gamma(\boldsymbol{x})$ is minimized, i.e.

$$\hat{\boldsymbol{y}}_{\text{opt}} = \underset{\boldsymbol{y}\in\mathcal{C}}{\text{argmin}}\{\gamma(\boldsymbol{x})\} \ . \quad (2)$$

We note that the total redundancy inserted into one OFDM symbol by trellis shaping is N/n_s bits. Since the total number of transmitted bits is $N\log_2(M)$, the relative redundancy is

$$r = 1/(\log_2(M)n_s) \ . \quad (3)$$

A. PAR Metrics

In this section, we introduce three metrics for PAR reduction using trellis shaping. It is convenient to define a partial code sequence of length kn_s as $\boldsymbol{y}^{(k)} \triangleq [y_0 \ \dots \ y_{kn_s-1}]$, $1 \le k \le N/n_s$. $\boldsymbol{y}^{(k)}$ can be considered as the result of encoding the sequence $\boldsymbol{m}^{(k)} \triangleq [m_0 \ \dots \ m_{k-1}]$ by the shaping encoder. The partial shaped data sequence corresponding to $\boldsymbol{y}^{(k)}$ is denoted by $\boldsymbol{X}^{(k)} \triangleq [X_0 \ \dots \ X_{kn_s-1}]$, $1 \le k \le N/n_s$.

a) Metric 1: The first metric was suggested in [2] and applies the PAR criterion (2) directly to partial code sequences $\boldsymbol{y}^{(k)}$. Defining the zero-padded shaped data sequence $\boldsymbol{X}_z^{(k)} \triangleq [\boldsymbol{X}^{(k)} \, \boldsymbol{0}_{N-kn_s}]$ ($\boldsymbol{0}_K$: all-zero vector of length K) and the corresponding partial transmit sequence $\boldsymbol{x}_z^{(k)} = \text{IDFT}(\boldsymbol{X}_z^{(k)})$, the metric for $\boldsymbol{y}^{(k)}$ is given by

$$\Lambda_1(\boldsymbol{y}^{(k)}) = ||\boldsymbol{x}_z^{(k)}||_\infty^2 \ . \quad (4)$$

b) Metric 2: We propose a modification of Metric 1 where we always consider *all* subcarriers at each decoding step. Let $\boldsymbol{y}_d^{(k)}$ be the *full-length* (N-dimensional) code sequence obtained from encoding $[\boldsymbol{m}^{(k)} \, \boldsymbol{0}_{N/n_s-k}]$ by the shaping encoder, i.e., the encoder is driven into the zero-state. We note that $\boldsymbol{y}_d^{(k)}$ can be presented as $[\boldsymbol{y}^{(k)} \, y_{kn_s} \ \dots \ y_{kn_s+q-1} \, \boldsymbol{0}_{N-kn_s-q}]$, where q is the number of possible non-zero output bits, i.e., $q = \log_2(N_s)n_s$ if $kn_s < N - \log_2(N_s)n_s$ and $q = N - kn_s$ otherwise, and N_s is the number of decoder states. Let $\boldsymbol{X}_d^{(k)}$ be the resulting shaped data sequence, i.e., $\boldsymbol{X}_d^{(k)}$ follows from shaping the *full-length* data vector $\boldsymbol{X}$ with $\boldsymbol{y}_d^{(k)}$, and $\boldsymbol{x}_d^{(k)} = \text{IDFT}(\boldsymbol{X}_d^{(k)})$. The metric for $\boldsymbol{y}^{(k)}$ is obtained as

$$\Lambda_2(\boldsymbol{y}^{(k)}) = ||\boldsymbol{x}_d^{(k)}||_\infty^2 \ . \quad (5)$$

Since, different from Metric 1, the whole data sequence, including the unshaped portion resulting from the all-zero tail of $\boldsymbol{y}_d^{(k)}$, is considered when forming the transmit signal and since $\boldsymbol{y}_d^{(k)}$ is a valid code sequence for all k, $1 \le k \le N/n_s$, we can terminate the selection algorithm as soon as the achieved PAR is below a predefined threshold value. Thus, Metric 2 facilitates adaptive PAR reduction [7], which considerably reduces the complexity of shaping.

c) Metric 3: Based on a relation between the PAR of $x(t)$ and the autocorrelation function of the frequency-domain data X_i derived in [8], Ochiai proposed in [4] to use the frequency-domain metric

$$\Lambda_3(\boldsymbol{y}^{(k)}) = \sum_{m=1}^{kn_s-1} \left| \sum_{i=0}^{kn_s-m-1} X_{i+m}X_i^* \right|^2 \ . \quad (6)$$

B. Selection Algorithms

We consider the application of the VA and the ST-SDA to select code sequences for PAR reduction.

a) Viterbi Algorithm (VA): Henkel and Wagner [2] and Ochiai [4] considered the VA employing, respectively, Metric 1 and Metric 3 for trellis shaping. If the two partial code sequences $\boldsymbol{y}_1^{(k)}$ and $\boldsymbol{y}_2^{(k)}$ enter the same trellis state, sequence $\boldsymbol{y}_1^{(k)}$ is selected if $\Lambda_l(\boldsymbol{y}_1^{(k)}) < \Lambda_l(\boldsymbol{y}_2^{(k)})$, $l \in \{1,3\}$ and vice versa. However, the VA does not necessarily find the optimum vector $\hat{\boldsymbol{y}}_{\text{opt}}$ since (i) Metric 1 is not additive and (ii) due to the unlimited memory of the metric increment for Metric 3 (see (6) and also [4, Eq. (28)]). It is worth noting that the suboptimality of the VA for Metric 3 is not mentioned in [4]. Since Metric 2 is not additive, the VA is also suboptimum in this case.

b) Stack Sequential Decoding Algorithm (ST-SDA): The application of sequential decoding algorithms (SDAs) to trellis shaping for PAR reduction was not considered in [2], [4]. In this letter, we consider the ST-SDA as one of the most commonly used SDAs [5]. The ST-SDA searches through the code tree rather than the code trellis with the advantage that the complexity is almost independent of the code memory. Since Metrics 1 and 2 are not additive, the ST-SDA does not find the code sequence $\hat{\boldsymbol{y}}_{\text{opt}}$. However, different from the VA, the ST-SDA finds the optimum solution $\hat{\boldsymbol{y}}_{\text{opt}}$ for Metric 3, which is additive and enables a tree search. A closer look at the application of the ST-SDA with Metric 2 reveals that the stack algorithm always expands the longest path. This fact leads to two consequences: (i) the algorithm stops after N/n_s steps and in total only $N/n_s + 1$ searches are needed; and (ii) a stack of size two is sufficient. This renders ST-SDA with Metric 2 very similar to an iterative bit-flipping algorithm proposed for PAR with partial transmit sequences (PTS) in [9].

C. Complexity

Assuming that the computational complexity is dominated by the number of multiplications required to find the code sequence for shaping, in this section we quantify the complexities for the different trellis shaping metrics and selection algorithms.

a) Metrics 1 and 2: Trellis shaping with Metrics 1 and 2 involves computation of the partial time-domain vectors $\boldsymbol{x}_l^{(k)}$, $l \in \{z,d\}$, and a subsequent peak-amplitude search (see Eqs. (4) and (5)). In an efficient implementation, the vector update can be formulated as ($y_i \in \{0,1\}$ is the ith bit of $\boldsymbol{y}_l^{(k)}$)

$$\boldsymbol{x}_l^{(k)} = \boldsymbol{x}_l^{(k-1)} + \boldsymbol{v}_l^{(k)} \ , \quad l \in \{z,d\} \ , \quad (7)$$

with

$$\boldsymbol{v}_z^{(k)} = \sum_{i=(k-1)n_s}^{kn_s-1} (1-2y_i)\boldsymbol{v}_i \ , \quad \boldsymbol{v}_d^{(k)} = -2\sum_{i=(k-1)n_s}^{kn_s+q-1} y_i\boldsymbol{v}_i \ , \quad (8)$$

and

$$\boldsymbol{v}_i = \text{IDFT}([\boldsymbol{0}_i \ X_i \, \boldsymbol{0}_{N-i-1}]) \quad (9)$$

respectively. It should be noted that this update relies on the symmetric QAM labeling (cf. Section II) and multiplications are only required in (9). In particular, $2LN$ real multiplications

(RMs) are needed for each $\boldsymbol{v}_i$. If these vectors are saved for reuse (with Metric 1 we only need to save n_s such vectors at any time, and with Metric 2, at most $(\log_2(N_s)+1)n_s$ vectors), a total of $2LN^2$ RMs are required to generate time-domain vectors $\boldsymbol{x}_l^{(k)}$, $l \in \{z,d\}$, (the computations for an initial fast IDFT for Metric 2 are neglected). Each peak-amplitude search requires $2LN$ RMs. In VA, since in total $2N_s(N/n_s - \log_2(N_s))$ partial sequences are considered, the overall complexity is about $4(N_s/n_s)LN^2$. The ST-SDA has a very similar complexity per considered partial code sequence, or equivalently per peak-amplitude search, and therefore we consider the number searches when comparing the VA and ST-SDA in Section III.

b) Metric 3: A computationally efficient implementation of trellis-shaping with Metric 3 is described for the VA in [4]. It requires about $2(N_s/n_s)N^2$ RMs, i.e. by about a factor $2L$ less multiplications than with Metrics 1 and 2. For this implementation each state at time k has to store all partial autocorrelations $\rho_m^{(k)} \triangleq \sum_{i=0}^{kn_s-m-1} X_{i+m}X_i^*$ for $1 \le m \le kn_s - 1$ associated with the chosen path and all possible entries for $X_{i+m}X_i^*$ need to be tabulated to save computations. Further complexity reduction could be achieved by application of the trellis-window truncation proposed in [4], which in turn leads to a somewhat degraded PAR-reduction and is not further considered here. Similar conclusions apply if the ST-SDA is used and a comparable number of partial sequences are considered for the different metrics.

III. Performance Evaluation and Discussion

In this section, we discuss the performances for PAR reduction with trellis shaping and the different metrics and selection algorithms introduced in Section II. As it is customary (cf. e.g. [3], [7], [4]), we consider the complementary cumulative distribution function (CCDF) of the PAR ($\Pr\{\cdot\}$: probability):

$$Q_\gamma(\gamma_0) = \Pr\{\gamma(\boldsymbol{x}) > \gamma_0\}\ . \tag{10}$$

As an illustrative example, we assume OFDM transmission with $N = 128$ subcarriers and 16-QAM constellation. The shaping codes are maximum free distance convolutional codes. However, we found that the actual construction of the shaping code has only a small impact on performance, cf. also [2], [4].

a) Comparison of Metrics: Fig. 1 depicts the CCDF $Q_\gamma(\gamma_0)$ as a function of γ_0 for trellis shaping with the VA and Metrics 1-3. A 4-state rate-1/2 code is used (Case 1 from Table I). As a reference, the CCDF for OFDM without shaping is also plotted. We observe that Metrics 1 and 2 show a very similar PAR reduction capability with the 0.1% PAR ($10\log_{10}(\gamma_0)$ at $Q_\gamma(\gamma_0) = 10^{-3}$) reduced by 4.5 dB compared to OFDM without shaping. This was expected as both metrics represent the same optimization criterion. Furthermore, it can be seen that Metric 3 compares unfavourably with Metrics 1 and 2 in terms of PAR reduction. For example, a gap of about 1 dB is observed for the 0.1% PAR.

This trend is confirmed by the figures in Table I for the 0.1% PAR and different convolutional codes. We consider only the VA for the moment. It can be seen that Metrics 1 and 2 consistently offer an improved PAR reduction over Metric 3.

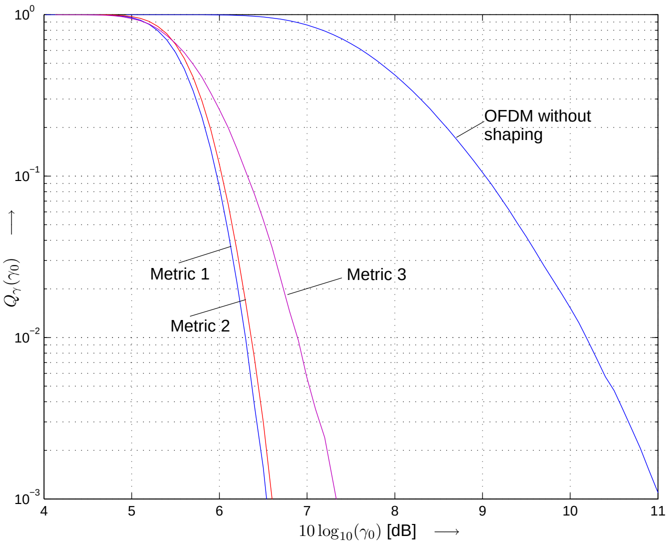

Fig. 1. Performance of trellis shaping with VA and Metrics 1-3. Case 1 from Table I.

We also observe that increasing the number of states N_s is beneficial for PAR reduction. In particular, a 0.1% PAR of about 6.5 dB is achieved for different redundancy ratios from $r = 1/8$ to $r = 1/32$, if the number of states are increased from $N_s = 4$ to $N_s = 64$.

In summary, we conclude that trellis shaping accomplishes significant PAR reduction and that Metrics 1 and 2 offer a superior PAR-reduction performance compared to Metric 3.

b) Comparison of Selection Algorithms: Although the ST-SDA finds the optimum code sequence for Metric 3, the average complexity for the tree search with the ST-SDA was found to be much larger than that for the trellis search with the VA. This is because longer paths accumulate larger metric values, and thus shorter paths are more likely to be extended than longer paths. We therefore applied a bias term to adjust the metrics when comparing paths of different lengths. Application of such a bias is well known from sequential decoding of convolutional codes. The achieved PAR reduction, however, is not comparable to that accomplished with the VA. We therefore concentrate on Metrics 1 and 2 in the following.

For the ST-SDA with Metric 1, a stack with a maximum of 100 entries was used and the expected metric value for a certain length was applied as a bias. The ST-SDA with Metric 2 requires a stack of size two and runs without bias (cf. Section II-B).

Table I shows the 0.1% PAR for the ST-SDA and Metrics 1 and 2 together with the average number of peak-power searches, which is the appropriate complexity measure when comparing with the VA. The respective complexity figures for the VA are also included. The figures show that (i) the ST-SDA performs considerably fewer searches than the VA, which results in a corresponding gap in PAR reduction capability, (ii) Metric 2 achieve a slightly better performance-complexity tradeoff than Metric 1, and (iii) the ST-SDA with Metrics 1 and 2 attains a better 0.1% PAR than the VA with Metric 3 (proposed in [4]), while computational complexity savings are likely due to the small number of searches. Hence, the ST-SDA

TABLE I

PERFORMANCE OF TRELLIS SHAPING WITH VA AND ST-SDA. "0.1% PAR [DB]" IS THE $10\log_{10}(\gamma_0)$ SUCH THAT $Q_\gamma(\gamma_0) = 10^{-3}$. "# OF SEARCHES" IS THE AVERAGE NUMBER OF PEAK-AMPLITUDE SEARCHES.

Case	n_s	N_s	r	VA				ST-SDA			
				0.1% PAR [dB] Metric			# of searches Metrics	0.1% PAR [dB] Metric		# of searches Metric	
				1	2	3	1 and 2	1	2	1	2
1	2	4	1/8	6.55	6.55	7.35	496	6.80	7.10	174	65
2	4	4	1/16	6.95	6.85	7.90	240	7.20	7.40	83	33
3	4	16	1/16	6.45	6.55	7.60	896	7.25	7.45	83	33
4	8	8	1/32	7.15	7.05	8.25	208	7.70	7.85	40	17
5	8	64	1/32	6.50	6.55	7.90	1280	7.70	7.70	40	17

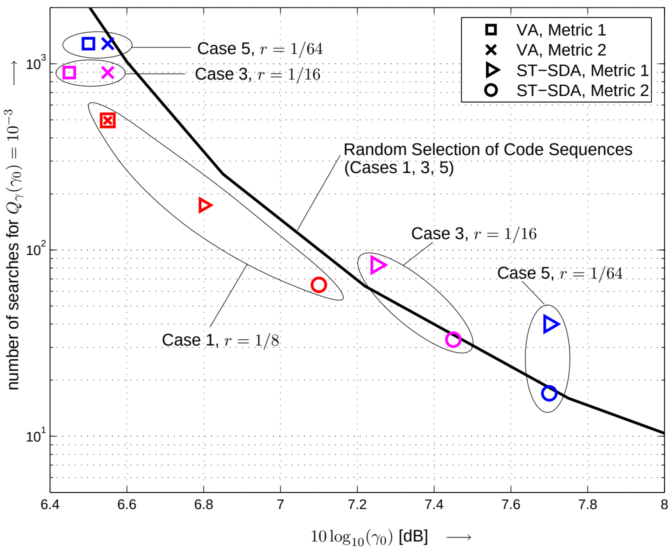

Fig. 2. Performance of trellis shaping. VA and ST-SDA with Metrics 1 and 2, respectively. Cases 1, 3, and 5 from Table I. Also shown: Random selection of code sequences (SLM-like PAR reduction).

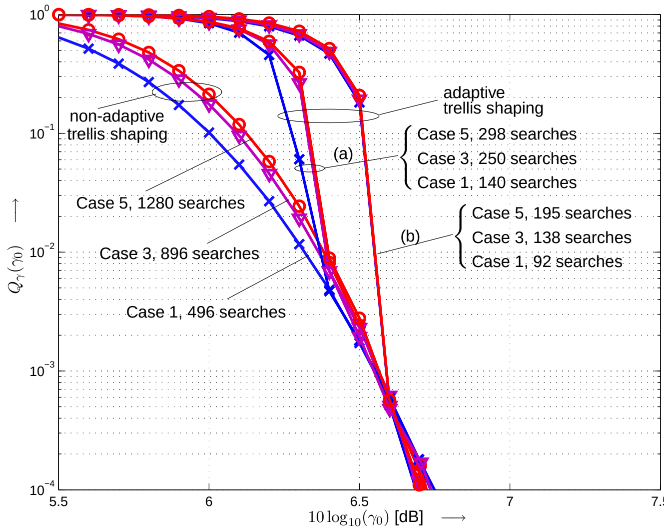

Fig. 3. Performance of non-adaptive and adaptive trellis shaping and the number of searches required. (a) Threshold chosen as the 1% PAR. (b) Threshold chosen as the 0.1% PAR. VA with Metric 2. Cases 1, 3, and 5 from Table I.

is an interesting alternative for low-complexity trellis shaping.

The complexity-performance tradeoff is more clearly illustrated in Fig. 2, which shows the number of searches required for the 0.1% PAR and the Cases 1, 3, and 5 corresponding to relative redundancies of $r = 1/8$, $1/16$, and $1/32$, respectively (see Table I). It is interesting to observe that the VA achieves about the same PAR reduction also for lower shaping redundancies at the cost of higher complexity. The ST-SDA, on the other hand, performs fewer searches with decreasing redundancy, but also PAR reduction degrades. Also included in Fig. 2 is the curve for a random selection of code sequences, whose performance was found to be rather independent of the applied code and practically identical for Cases 1 and 5. Trellis shaping with random selection could be interpreted as a form of selected mapping (SLM) [3] with implicit side information embedding. It is an interesting benchmark case, but its implementation would require a full IDFT for each tested code sequence. It can be seen that the two trellis-shaping selection algorithms perform quite similar to SLM-like random selection while facilitating a more efficient implementation.

c) Adaptive Shaping: Finally, Fig. 3 shows the results for adaptive trellis shaping using the VA with Metric 2 (similar results are obtained with the ST-SDA). The threshold for each case is chosen as the (a) 1% PAR and (b) 0.1% PAR for the conventional VA (cf. Table I). A consistent complexity reduction in terms of number of searches by more than (a) 70% and (b) 80% can be observed, while the same (a) 1% PAR and (b) 0.1% PAR is achieved. Clearly, the complexity savings increase with larger thresholds, i.e., lower clipping rate. This renders adaptive shaping, which is only feasible with Metric 2, quite an attractive feature for trellis shaping and practical scenarios, where power amplifiers are operated with a certain power backoff.

IV. CONCLUSION

In this letter, we have studied the application of trellis shaping for PAR reduction in OFDM systems. To this end, we have compared different metrics and selection algorithms. It is found that the time-domain Metrics 1 and 2 achieve a better PAR-reduction than the frequency-domain Metric 3 at the price of increased computational complexity if the VA algorithm is used for selection of the code sequence. The application of the VA and the ST-SDA and codes with different redundancies are shown to enable trellis shaping for PAR reduction with a flexible complexity-performance tradeoff comparable to that of SLM-like PAR reduction. The new Metric 2 furthermore facilitates adaptive shaping with favorably low complexity.

References

[1] S. Han and J. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Trans. Wireless Commun.*, vol. 12, no. 2, pp. 56–65, Apr. 2005.

[2] W. Henkel and B. Wagner, "Another application for trellis shaping: PAR reduction for DMT (OFDM)," *IEEE Trans. Commun.*, vol. 48, no. 9, pp. 1471–1476, Sept. 2000.

[3] S. Müller, R. Bäuml, R. Fischer, and J. Huber, "OFDM with reduced peak-to-average power ratio by multiple signal representation," *Annals of Telecom.*, vol. 52, no. 1-2, pp. 58–67, Feb. 1997.

[4] H. Ochiai, "A novel trellis-shaping design with both peak and average power reduction for OFDM systems," *IEEE Trans. Commun.*, vol. 52, no. 11, pp. 1916–1926, Nov. 2004.

[5] S. Lin and D.J. Costello, Jr., *Error Control Coding*, 2nd ed. Upper Saddle River, N.J: Pearson-Prentice Hall, 2004.

[6] C. Tellambura, "Computation of the continuous-time PAR of an OFDM signal with BPSK subcarriers," *IEEE Commun. Lett.*, vol. 5, no. 5, pp. 185–187, May 2001.

[7] A. Jayalath and C. Tellambura, "Adaptive PTS approach for reduction of peak-to-average power ratio of OFDM signal," *Electr. Lett.*, vol. 36, no. 14, pp. 1226–1228, July 2000.

[8] C. Tellambura, "Upper bound on peak factor of N-multiple carriers," *Electr. Lett.*, vol. 33, pp. 1608–1609, Sept. 1997.

[9] L. J. Cimini, Jr. and N. R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," *IEEE Commun. Lett.*, vol. 4, no. 3, pp. 86–88, Mar. 2000.