Multiuser Two-Way Filter-and-Forward Relaying for Ultra-Wideband Communications

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***Abstract*—In this paper, a multiuser two-way filter-and-forward relaying scheme for wireless communication over wideband channels is considered. We propose pre/post-rake processing in conjunction with optimized filtering at the relay to reduce the signal processing burden at the source and destination nodes. Two relay filter design problem formulations are introduced, namely (a) a convex optimization problem formulation with closed-form solutions and (b) the more general case, which is a non-convex problem solvable via an alternating optimization algorithm. For both design alternatives widely linear formulations are devised. The presented numerical results demonstrate the capability of the proposed designs to establish reliable two-way communication links between nodes with limited signal processing power and in the absence of a direct link.**

I. Introduction

Wireless communication solutions that satisfy criteria such as low power consumption, security, and reliability, in a network of low complexity nodes, will play an important role in the internet of things revolution and with regard to machine-to-machine (M2M) communications. Application examples for such a network setup include multimedia communication in wireless personal area networks, where wireless links serve e.g. for cable replacement, intra-vehicle data communication, and wireless communication in M2M sensor networks, for the exchange of sensor information (on the reverse link from the sensors to a central controller) as well as for distributing firmware updates (on the forward link).

Relaying can play a significant role in extending the range and throughput of ultra-wideband (UWB) communication systems, where the communication range is typically limited to less than 10 meters due to restrictions on the average transmit power spectral density. In particular, multi-user two-way relaying, which is a special case of multi-way relaying, has the potential to achieve higher spectral efficiencies compared to one-way and two-way relaying schemes [1], by establishing multiple simultaneous pair-wise two-way links through the relay. However, to this end efficient techniques are required for self-interference mitigation, multiuser interference (MUI) suppression, and – due to the UWB nature of the wireless links – intersymbol interference (ISI) cancelation, so as to achieve a certain end-to-end link quality, e.g. in terms of the resulting bit error rate (BER).

Relaying over frequency-flat channels and pre-processing at the relay has been widely studied in the past, see e.g. [2]–[4]. However, the literature on relaying over frequency-selective channels – as in the case of UWB links – is fairly limited. In particular, the presence of ISI differentiates relaying over frequency-selective channels from that over frequency-flat fading channels. A simple relaying scheme for frequency-flat fading is amplify-and-forward (AF) relaying. Filter-and-forward (FF) relaying, on the other hand, can be regarded as an extension of AF relaying to frequency-selective channels, as it attempts to partially suppress the ISI prior to forwarding.

One-way FF relaying was first introduced in [5]. Reference [6] offers an extension of this work for one-way relaying, when a direct link exists between the source and the destination node and equalization is performed at the destination node. Two-way FF relaying with multiple antennas was considered in [7], [8]. In reference [7], two-way FF relaying was optimized according to a worst-case signal-to-interference-plus-noise ratio (SINR) maximization criterion, and an algorithm based on bisection search was proposed to solve the relaxed problem. The design of equalization filters at the destination node was also addressed as part of the design. In [8], worst-case SINR maximization as well as transmit power minimization design formulations were investigated.

In the UWB literature, one-way UWB relaying for time-hopped UWB transmission schemes was considered in references [9] and [10]. Two-way relaying for transmit-reference UWB communication was proposed in [11], [12]. Differential schemes with non-coherent AF for multiple-hop one-way relaying were developed in [13]. One-way relaying with pre/post-rake combining at the relay was considered in [14] for UWB signaling with guard intervals, and in reference [15] one-way decouple and forward relaying with rake receivers at the destination node was investigated. However, the above mentioned methods cannot handle multi-way relaying links. In fact, to the best of our knowledge multiuser two-way relaying for UWB communication has not been considered in the literature yet.

Motivated by this fact, in this paper multiuser two-way relaying schemes for pairwise internode UWB communication are investigated. In particular, we consider direct-sequence UWB (DS-UWB) signaling, which facilitates coping both with ISI and with MUI, while supporting high data rate transmission. Different from the available literature on relaying over frequency-selective channels, we propose a combination

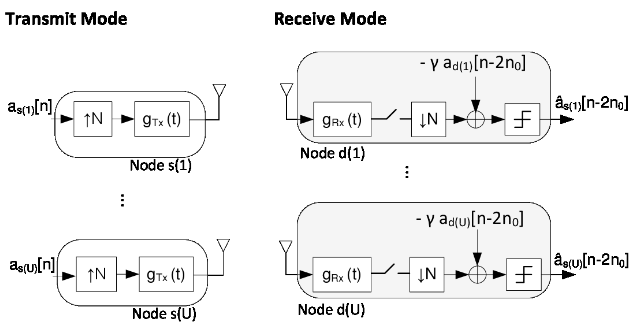

Fig. 1. Block diagram of the source/destination nodes in transmit and receive modes.

of post/pre-rake filtering with optimized equalization filtering at the relay for pairwise internode communication. The presence of post/pre-rake filtering reduces the optimized filter length required in dense UWB channels. The equalization filter optimization itself is based on a sum-mean-squared error (sum-MSE) minimization design criterion, which allows us to develop convex problem formulations for optimizing the system performance. Moreover, an alternative design based on a modified MSE formulation [16] is proposed that leads to further improved performance. The problem is, however, non-convex and we present an iterative solver according to the alternating optimization principle [17]. Finally, we extend our multiuser two-way relaying design strategies to the widely linear (WL) case, which exploits the fact that most DS-UWB systems use a real-valued modulation scheme [18], such as binary phase-shift keying (BPSK).

The remainder of the paper is organized as follows. The UWB system model including the FF relaying scheme is introduced in Section II. The equalization filter design formulations are introduced in Sections III and IV. Section V describes the WL variation of the design problems as well as an analytical performance estimation. Numerical results are presented in Section VI, followed by concluding remarks in Section VII.

We use the following notations: $\Re\{.\}$, $\Im\{.\}$, $[.]^T$, $[.]^H$ and $\mathcal{E}\{\}$, denote the real part of a complex number, the imaginary part of a complex number, matrix/vector transposition, Hermitian transposition, and statistical expectation, respectively. $[.] * [.]$ represents the linear convolution, $\mathbf{0}_n$ is the all-zero vector of length n and $\boldsymbol{I}_n$ is the identity matrix of dimension $n \times n$. Vectors and matrices are identified as bold lower case and bold upper case letters, respectively.

II. System Model

The block diagram of the source/destination nodes for the considered multiuser two-way relay network is shown in Figure 1. The nodes are assumed to be single antenna units with limited signal processing capability, while the central relay node, depicted in Figure 2, is equipped with multiple antennas ($M > 1$). The network uses a half-duplex two-phase multiple access and broadcast schedule as follows. During the first phase (uplink), all U nodes send their message to the central relay simultaneously. (Note that due to the two-way communication setup, U also denotes the number of simultaneous links.) In particular, the transmitted data symbols $a_{s(u)}[n]$ of source node $s(u)$ $(1 \leq u \leq U)$ are upsampled by a factor of N, filtered by a pulse-shaping filter $g_{Tx}(t)$, and transmitted via the antenna. Throughout this paper, the time index before the upsampling step is denoted by n, and the time index after the upsampling step is denoted by k. In the second phase (downlink), the relay processes and broadcasts the sum of all node messages through its M antennas. The received signal at destination node $d(u)$ $(1 \leq u \leq U)$ is in essence filtered by a noise-limiting filter $g_{Rx}(t)$ and then downsampled by a factor of N, and symbol detection is performed by means of a slicer. Optionally, a self-interference mitigation step can be incorporated prior to the detection step, as indicated in Figure 1 for the case of a single feedback tap with weight γ.

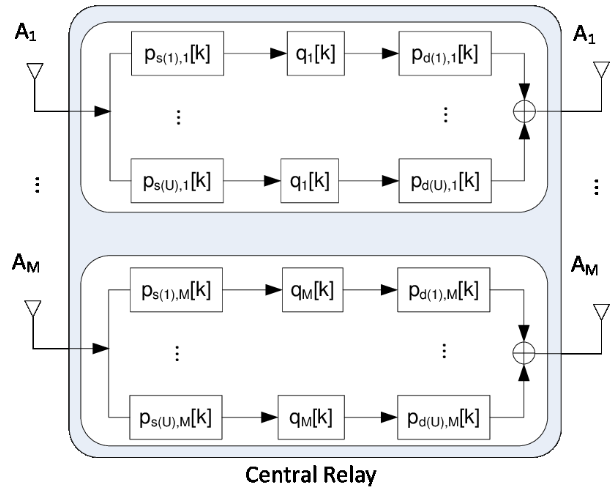

Fig. 2. Block diagram of the central relay with pre/post-rake filtering and equalization filtering for multiuser two-way FF relaying.

Note that the signal processing at the source/destination nodes is relatively simple, while inter-node communication is achieved through the central relay equipped with multiple antennas. The relay estimates the channel state information (CSI) between itself and the source/destination nodes, and handles the complexity associated with the different filtering steps. As it was mentioned in Section I, FF relaying can be considered as an extension of AF relaying to the case of frequency selective channels [5].

The details of the signal processing at the central relay for half-duplex FF relaying are shown in Figure 2. The received signal from source node $s(u)$ at the relay's m^{th} antenna is first (post-)rake combined, based on the estimated channel coefficients of the source-relay link, using a linear filter $\boldsymbol{p}_{s(u),m}$. Throughout this paper, we assume perfect CSI at the relay concerning all source-relay and relay-destination links. In the case of a 'full' rake filter, $\boldsymbol{p}_{s(u),m}$ is the complex conjugate, time-reversed version of the channel impulse response $h_{s(u),m}[k]$ associated with the source-relay link. In particular, the filter length L_p is equal to the length of the channel impulse response, L_h. Alternatively, a 'partial' rake filter may be employed, which is the complex conjugate, time-reversed version of the first L_p channel coefficients ($L_p < L_h$).

The rake-combined received signal is then passed through an optimized equalization filter $\boldsymbol{q}_m$ of length L_q, as shown in Figure 2. Note that we employ a common filter $q_m[k]$ for all U links, so that the complexity of our design is fairly limited. Prior to re-transmission, the filtered signal for destination node $d(u)$ is then pre-rake combined, based on the estimate of the channel impulse response $h_{d(u),m}[k]$ of the relay-destination link, using the linear filter $\boldsymbol{p}_{d(u),m}$.

At each of the relay's antennas, the sum received signal from all transmitting users passes through the U pairs of source and destination pre/post rake filters. Hence, we define the overall pre/post rake filter for relay antenna m as $R_{sd,m}[k] = \sum_{\ell=1}^{U} \left(p_{s(\ell),m}[k] * p_{d(\ell),m}[k]\right)$. Then, the overall channel consisting of the link between source node $s(\ell)$ and the m^{th} antenna at the relay, the post/pre-rake combining filters, and the link between the relay's m^{th} antenna and the destination node $d(u)$ is defined as $g_{s(\ell),m,d(u)}[k] = h_{s(\ell),m}[k] * R_{sd,m}[k] * h_{d(u),m}[k]$. Throughout, we assume that the individual links are reciprocal, i.e., the channel impulse responses are identical for the uplink (all nodes transmitting to the relay) and the downlink (all nodes receiving forwarded signals from the relay).

III. Problem Formulation

In this section the problem formulation for the design of optimized relay equalization filters $q_m[k]$ $(1 \leq m \leq M)$ is presented. We start by introducing a matrix-form representation of the received signal, which greatly simplifies the problem description. The sum-MSE minimization is selected as the design criterion of choice, as it enables convex problem formulations for optimizing the performance.

The received signal at the destination node can be written as

$$r_{d(u)}[n] = \sum_{\ell=1}^{U} \boldsymbol{q}^H \boldsymbol{G}^H_{s(\ell),d(u)} \boldsymbol{a}_{s(\ell)}[n] + z_{d(u)}[n] + v_{d(u)}[n] \ , \tag{1}$$

where

- $\boldsymbol{q} = [\boldsymbol{q}_1^T, \ldots, \boldsymbol{q}_M^T]^T$ is the concatenated vector of the filter coefficients across all antennas,
- $\boldsymbol{q}_m = [q_m[0], \ldots, q_m[L_q - 1]]^H$ is the vector of filter coefficients at the m^{th} antenna,
- $\boldsymbol{G}_{s(\ell),d(u)} = [\boldsymbol{G}_{s(\ell),1,d(u)}, \ldots, \boldsymbol{G}_{s(\ell),M,d(u)}]$ is a block-diagonal matrix with block components $\boldsymbol{G}_{s(\ell),m,d(u)}$,
- $\boldsymbol{G}_{s(\ell),m,d(u)}$ is formed by downsampling the rows of Toeplitz matrix $\bar{\boldsymbol{G}}_{s(\ell),m,d(u)}$ by factor N,
- $\bar{\boldsymbol{G}}_{s(\ell),m,d(u)}$ is defined by its first row vector $[(g_{s(\ell),m,d(u)}[k_f])^*, \boldsymbol{0}_{L_q-1}]$, where the sampling phase k_f is set as $k_f = L_p + L_h - 2 - N\lfloor \frac{L_p+L_h-2}{N} \rfloor$, and by its first column $\left[g_{s(\ell),m,d(u)}[k_f], g_{s(\ell),m,d(u)}[k_f+1], \ldots, g_{s(\ell),m,d(u)}[k_f + (2L_h + 2L_p - 4)], \boldsymbol{0}_{L_q-1}\right]^H$,
- $\boldsymbol{a}_{s(\ell)}[n] = \left[a_{s(\ell)}[n], \ldots, a_{s(\ell)}[n - L_g + 1]\right]^T$ is the $L_g \times 1$ vector of transmitted symbols affecting the received signal, where $L_g = \lceil \frac{2L_p+2L_h+L_q-4-k_f}{N} \rceil$,
- $z_{d(u)}[n] \sim \mathcal{N}(0, \sigma^2_{d(u)})$ is the additive white Gaussian noise (AWGN) at destination node $d(u)$,
- $v_{d(u)}[n] = \sum_{m=1}^{M} v_{d(u),m}[n]$ is the colored noise that is added at the relay (on the uplink) and which is being processed and forwarded to the destination node. It is defined as $v_{d(u)}[n] = \boldsymbol{q}^H \boldsymbol{\Upsilon}^H_{d(u)} \boldsymbol{z}_R$, where $\boldsymbol{\Upsilon}_{d(u)} = \text{diag}\{\boldsymbol{\Upsilon}_{d(u),1}, \ldots, \boldsymbol{\Upsilon}_{d(u),M}\}$ is of size $ML_v \times ML_q$, and its block components $\boldsymbol{\Upsilon}_{d(u),m}$ are Toeplitz matrices with first row $[\Upsilon_{d(u),m}[0], \boldsymbol{0}_{L_q-1}]$ and first column $[\Upsilon_{d(u),m}[k_f], \Upsilon_{d(u),m}[k_f + 1], \ldots, \Upsilon_{d(u),m}[k_f + (L_h + 2L_p - 3)], \boldsymbol{0}_{L_q-1}]^T$. Moreover, $\Upsilon_{d(u),m}$ is defined as $\Upsilon_{d(u),m} = R_{sd,m}[k] * h_{d(u),m}[k]$, $\boldsymbol{z}_R[n] = [\boldsymbol{z}_{R,1}[n], \ldots, \boldsymbol{z}_{R,M}[n]]$ and $\boldsymbol{z}_{R,m}[n]$ as $\boldsymbol{z}_{R,m}[n] = [z_{R,m}[n], z_{R,m}[n-1], \ldots, z_{R,m}[n - L_v + 1]]$. L_v is the length of the relay noise vector affecting the received colored noise at the destination node and is defined as $L_v = k_f + L_h + 2L_p - 3 + L_q$.

The average transmit power at the relay is the sum of the average power transmitted from the individual relay antennas. The transmit signal at the m^{th} antenna is $s_{R,m}[k] = (z_{R,m}[k] + \sum_{\ell=1}^{U} \tilde{a}_{s(\ell)}[k] * h_{\ell,m}[k]) * R_{sd,m}[k] * q_m[k]$, where $\tilde{a}_{s(\ell)}[k]$ represents the transmit symbol sequence upsampled by factor N, and $z_{R,m}[k]$ is the AWGN added at the m^{th} antenna of the relay with variance $\sigma^2_{z_R}$. Finally, the average transmit power at the relay can be written as

$$\begin{aligned} P_R &= \sum_{m=1}^{M} \mathcal{E}\{s_{R,m}[k] s^*_{R,m}[k]\} \\ &= \boldsymbol{q}^H \left(\sigma^2_{z_R} \boldsymbol{\Phi}_R + \sum_{\ell=1}^{U} \boldsymbol{\Phi}_{\Upsilon_{s(\ell)}} \right) \boldsymbol{q} \ , \end{aligned} \tag{2}$$

where the matrices $\boldsymbol{\Phi}_R$, and $\boldsymbol{\Phi}_{\Upsilon_{s(\ell)}}$ are block diagonal matrices with Hermitian Toeplitz block component matrices $\boldsymbol{\Phi}_{R,m}$ and $\boldsymbol{\Phi}_{\Upsilon_{s(\ell),m}}$, respectively. The first row of $\boldsymbol{\Phi}_{R,m}$ is defined as $[\phi_{R,m}[0], \phi_{R,m}[-1], \ \ldots, \phi_{R,m}[-L_q + 1]]$, where $\phi_{R,m}[k]$ is defined as $\phi_{R,m}[k] = R_{sd,m}[k] * R^*_{sd,m}[-k]$. The matrix $\boldsymbol{\Phi}_{\Upsilon_{s(\ell)}}$ is structured similarly to $\boldsymbol{\Phi}_R$, where $\phi_{R,m}[k]$ is replaced by $\phi_{\Upsilon_{s(\ell)}}[k] = \Upsilon_{s(\ell),m}[k] * \Upsilon^*_{s(\ell),m}[-k]$. The MSE with self-interference cancelation at the destination node prior to detection is defined as

$$\text{MSE}^{\text{IC}}_{d(u)} = \mathcal{E}\left\{ \left| \alpha r_{d(u)}[n] - a_{s(u)}[n - n_f] - \boldsymbol{\gamma}^H_{d(u)} \bar{\boldsymbol{a}}_{d(u)}[n] \right|^2 \right\} \tag{3}$$

where $\bar{\boldsymbol{a}}_{d(u)}[n]$ is an $L_t \times 1$ vector containing the stored transmitted symbols at the L_c indices selected for self-interference cancelation and zeros everywhere else, and the vector $\boldsymbol{\gamma}_{d(u)} \in \mathbb{C}^{L_g \times 1}$ contains the self-interference cancelation coefficients at the L_c self-interference cancelation indices and zeros everywhere else, and the delay, n_f, is set as $n_f = \lceil \lceil \frac{2L_h+2L_p+L_f-4-k_f}{N} \rceil / 2 \rceil$. Using the received signal representation from Eq. (1), the MSE is represented as

$$\begin{aligned}\mathrm{MSE}^{\mathrm{IC}}_{d(u)} = \Big\| \Big[&\alpha \boldsymbol{q}^H \boldsymbol{G}^H_{s(1),d(u)}, \ldots, \alpha \boldsymbol{q}^H \boldsymbol{G}^H_{s(u),d(u)} - \boldsymbol{e}_{n_f}, \ldots, \\ &\alpha \boldsymbol{q}^H \boldsymbol{G}^H_{d(u),d(u)} - \boldsymbol{\gamma}^H_{d(u)}, \ldots, \alpha \boldsymbol{q}^H \boldsymbol{G}^H_{s(U),d(u)}, \\ &\alpha \sigma_{d(u)}, \alpha \sigma_{z_R} \boldsymbol{q}^H \boldsymbol{\Upsilon}^H_{d(u)} \Big] \Big\|^2 , \end{aligned} \tag{4}$$

where $\boldsymbol{e}_{n_f}$ is a vector with the n_f^{th} element set as 1 and zeros elsewhere. Here, we have assumed that $\mathcal{E}\{|a_{s(u)}[n]|^2\} = \mathcal{E}\{|a_{d(u)}[n]|^2\} = 1$ for all $1 \leq u \leq U$. Assuming that factor α is identical for all users, and setting $\boldsymbol{\gamma}_{d(u)} = \alpha^H \boldsymbol{E}_{L_c} \boldsymbol{G}_{d(u),d(u)} \boldsymbol{q}$ with $\boldsymbol{E}_{L_c} = \mathcal{E}\{\boldsymbol{a}_{d(u)}[n] \bar{\boldsymbol{a}}^T_{d(u)}[n]\}$, the sum-MSE design problem is written as

$$\min_{\boldsymbol{q},\alpha} \quad \sum_{u=1}^{U} \mathrm{MSE}^{\mathrm{IC}}_{d(u)} , \tag{5a}$$

$$\text{s.t.} \quad \boldsymbol{q}^H \left(\sigma^2_{z_R} \boldsymbol{\Phi}_R + \sum_{u=1}^{U} \boldsymbol{\Phi}_{\Upsilon_{s(u)}} \right) \boldsymbol{q} \leq P_{\max} . \tag{5b}$$

The above problem is in convex form, and applying the Karush-Kuhn-Tucker (KKT) conditions, the closed-form solution is found as

$$\alpha = \sqrt{\frac{\boldsymbol{e}^T_{n_f} \sum\limits_{u=1}^{U} \boldsymbol{G}_{s(u),d(u)} \boldsymbol{T}^{\mathrm{FF^H}} \boldsymbol{D} \boldsymbol{T}^{\mathrm{FF}} \sum\limits_{u=1}^{U} \boldsymbol{G}^H_{s(u),d(u)} \boldsymbol{e}_{n_f}}{P_{\max}}} , \tag{6}$$

where $\boldsymbol{D} = \sigma^2_{z_R} \boldsymbol{\Phi}_R + \sum\limits_{u=1}^{U} \boldsymbol{\Phi}_{\Upsilon_{s(u)}}$,

$$\begin{aligned}\boldsymbol{T}^{\mathrm{FF}} = &\Big(\sum_{i=1}^{U} \sum_{j=1}^{U} \boldsymbol{G}^H_{s(i),d(j)} \boldsymbol{G}_{s(i),d(j)} + \sigma^2_{z_R} \sum_{u=1}^{U} \boldsymbol{\Upsilon}^H_{d(u)} \boldsymbol{\Upsilon}_{d(u)} \\ &- \sum_{u=1}^{U} \boldsymbol{G}^H_{d(u),d(u)} \boldsymbol{E}_{L_c} \boldsymbol{G}_{d(u),d(u)} + \boldsymbol{D} \sum_{u=1}^{U} \frac{\sigma^2_{d(u)}}{P_{\max}} \Big)^{-1} , \end{aligned}$$

and the (concatenated) relay equalization filter vector is given as

$$\boldsymbol{q} = \boldsymbol{T}^{\mathrm{FF}} \left(\sum_{u=1}^{U} \boldsymbol{G}^H_{s(u),d(u)} \right) \boldsymbol{e}_{n_f} / \alpha . \tag{7}$$

IV. Iterative Design for User Specific Scaling

In this section the design from Section III is extended to a more general case with user specific scaling factors $\alpha_{d(u)}$ for the individual source/destination nodes.

The MSE with self-interference cancelation and user specific scaling is defined as

$$\mathrm{MSE}^{\mathrm{IC}}_{d(u)} = \mathcal{E}\{|\alpha_{d(u)} r_{d(u)}[n] - a_{s(u)}[n - n_f] - \boldsymbol{\gamma}^H_{d(u)} \bar{\boldsymbol{a}}_{d(u)}|^2\} . \tag{8}$$

Using the received signal representation from (1), the MSE is derived as per Eq. (4), by replacing the common factor α by the user specific scaling factor $\alpha_{d(u)}$.

Next we use the alternating optimization (AO) approach to arrive at a solution for the sum MSE minimization problem subject to the maximum relay transmit power constraint. The AO method is an iterative procedure for optimizing a function jointly over a number of variables. The method is applicable to problems which are convex with respect to individual subsets of the variables, and it is based on dividing the parameter space into a number of non-overlapping subsets and alternating between restricted minimizations over each subset of variables [17]. For a non-convex optimization problem, the AO algorithm does not guarantee convergence to a global optimum. In our case, the parameter space is divided as $\boldsymbol{\chi}_1 = [\alpha_1, \ldots, \alpha_U]$, $\boldsymbol{\chi}_2 = \boldsymbol{q}$ and $\chi_3 = \lambda$, where λ is a scalar parameter. Setting $\boldsymbol{\gamma}_{d(u)} = \alpha^H_{d(u)} \boldsymbol{E}_{L_c} \boldsymbol{G}_{d(u),d(u)} \boldsymbol{q}$, and applying the KKT conditions, the strict minimizers at each iteration index, t, are found as

$$\alpha^{(t+1)}_{d(u)} = \frac{\Re\{\boldsymbol{q}^{(t)H} \boldsymbol{G}^H_{s(u),d(u)} \boldsymbol{e}_{n_f}\}}{\boldsymbol{q}^{(t)H} \boldsymbol{\Gamma}_{d(u)} \boldsymbol{q}^{(t)} + \sigma_{d(u)}} , \tag{9}$$

where

$$\begin{aligned}\boldsymbol{\Gamma}_{d(u)} = &\sum_{\ell=1}^{U} \boldsymbol{G}^H_{s(\ell),d(u)} \boldsymbol{G}_{s(\ell),d(u)} - \boldsymbol{G}^H_{d(u),d(u)} \boldsymbol{E}_{L_c} \boldsymbol{G}_{d(u),d(u)} \\ &+ \sigma^2_{z_R} \boldsymbol{\Upsilon}^H_{d(u)} \boldsymbol{\Upsilon}_{d(u)} , \end{aligned}$$

$$\boldsymbol{q}^{(t+1)} = \boldsymbol{T}_{\alpha}^{(t+1)} \left(\sum_{u=1}^{U} \alpha^{(t+1)}_{d(u)} \boldsymbol{G}^H_{s(u),d(u)} \boldsymbol{e}_{n_f} \right) , \tag{10}$$

where

$$\boldsymbol{T}^{(t+1)}_{\alpha} = \Big(\sum_{u=1}^{U} |\alpha^{(t+1)}_{d(u)}|^2 \boldsymbol{\Gamma}_{d(u)} + \lambda^{(t)} \boldsymbol{D} \Big)^{-1} ,$$

and

$$\lambda^{(t+1)} = \frac{\sum\limits_{u=1}^{U} |\alpha^{(t+1)}_{d(u)}|^2 \sigma^2_{d(u)}}{P_{\max}} .$$

For the initial solution, the parameters from the optimal solution of the design with identical parameter α for all users from Eqs. (6) and (7) are used to solve Eq. (9), and (10) in the first iteration. After each iteration two conditions are checked as stopping criteria, (i) $\|\boldsymbol{\chi}^{(t+1)} - \boldsymbol{\chi}^t\| \leq \epsilon$, where $\boldsymbol{\chi}^t$ is defined as $\boldsymbol{\chi}^t = [\boldsymbol{\chi}^t_1, \boldsymbol{\chi}^t_2, \chi^t_3]$ and ϵ is a small threshold value, (ii) $t < N_{\mathrm{iter}}$, checking if the number of iterations has reached the maximum allowable number of iterations.

V. Widely Linear Filtering and BER Analysis

The widely linear counterparts of the filter design schemes introduced in Section III and Section IV are obtained by incorporating the real part of the received signal in the MSE definitions of (3) and (8). The real part of the received signal is written as

$$y_{d(u)}[n] = \sum_{\ell=1}^{U} \tilde{\boldsymbol{q}}^T \tilde{\boldsymbol{G}}^T_{s(\ell),d(u)} \boldsymbol{a}_{s(\ell)}[n] + \tilde{z}_{d(u)}[n] + \tilde{v}_{d(u)}[n] , \tag{11}$$

where $\tilde{\boldsymbol{q}}$ is defined as $\tilde{\boldsymbol{q}} = [\Re\{\boldsymbol{q}\}, \Im\{\boldsymbol{q}\}]$, $\tilde{\boldsymbol{G}}_{s(\ell),d(u)} = [\Re\{\boldsymbol{G}_{s(\ell),d(u)}\}, -\Im\{\boldsymbol{G}_{s(\ell),d(u)}\}]$, and the AWGN noise term $\tilde{z}_{d(u)}[n]$ is Gaussian distributed with zero mean and variance $\sigma^2_{d(u)}/2$. The real part of the colored noise forwarded through the relay is written as $\tilde{v}_{d(u)}[n] = \tilde{\boldsymbol{q}}^T \tilde{\boldsymbol{\Upsilon}}^T_{d(u)} \tilde{\boldsymbol{z}}_R$, where $\tilde{\boldsymbol{z}}_R = [\Re\{\boldsymbol{z}_R\}, \Im\{\boldsymbol{z}_R\}]$ and

$$\tilde{\boldsymbol{\Upsilon}}_{d(u)} = \begin{bmatrix} \Re\{\boldsymbol{\Upsilon}_{d(u)}\} & -\Im\{\boldsymbol{\Upsilon}_{d(u)}\} \\ -\Im\{\boldsymbol{\Upsilon}_{d(u)}\} & -\Re\{\boldsymbol{\Upsilon}_{d(u)}\} \end{bmatrix}.$$

The matrices $\boldsymbol{\Phi}_R$ and $\boldsymbol{\Phi}_{\Upsilon_{s(u)}}$ are replaced with

$$\tilde{\boldsymbol{\Phi}}_R = \begin{bmatrix} \Re\{\boldsymbol{\Phi}_R\} & -\Im\{\boldsymbol{\Phi}_R\} \\ \Im\{\boldsymbol{\Phi}_R\} & \Re\{\boldsymbol{\Phi}_R\} \end{bmatrix},$$

and

$$\tilde{\boldsymbol{\Phi}}_{\Upsilon_{s(u)}} = \begin{bmatrix} \Re\{\boldsymbol{\Phi}_{\Upsilon_{s(u)}}\} & -\Im\{\boldsymbol{\Phi}_{\Upsilon_{s(u)}}\} \\ \Im\{\boldsymbol{\Phi}_{\Upsilon_{s(u)}}\} & \Re\{\boldsymbol{\Phi}_{\Upsilon_{s(u)}}\} \end{bmatrix},$$

respectively.

The BER for each of the source and destination pairs can be approximated using the resulting SINR for the corresponding link, assuming that the residual interference is Gaussian distributed. The SINR for the design with self-interference cancelation at the destination node $d(u)$ after the real operator is derived as per Eq. (12), where $\eta_{d(u)} = \frac{\sigma^2_{z_R}}{2} \tilde{\boldsymbol{q}}^T \tilde{\boldsymbol{\Upsilon}}^T_{d(u)} \tilde{\boldsymbol{\Upsilon}}_{d(u)} \tilde{\boldsymbol{q}} + \frac{\sigma^2_{d(u)}}{2}$.

Using this definition for the SINR, the corresponding BER for the BPSK DS-UWB signaling can be evaluated as

$$\mathrm{BER}_{d(u)} = \mathrm{Q}\left(\sqrt{\mathrm{SINR}_{d(u)}}\right), \tag{13}$$

where $Q(.)$ denotes the Gaussian Q-function.

VI. Numerical Results

In the following we describe and discuss a set of numerical results that demonstrate the performance of the two proposed relay filter design schemes. For the following numerical results, we consider the CM2 channel model for the residential non-line-of-sight environment, cf. [19], and the channel realizations are generated according to the procedure described in [20]. Note that the designs proposed in this paper are applicable to any UWB channel model, while CM2 from [19] is only chosen as an example. The signaling specifications include a center frequency of 6 GHz and a pulse bandwidth of 0.5 GHz using root-raised cosine pulses $g_{\mathrm{Tx}}(t)$ and $g_{\mathrm{Rx}}(t)$ with roll-off 0.7. Throughout, we consider 'full' pre/post-rake processing ($L_p = L_h$). Unless otherwise specified, results are averaged over 500 channel realizations. Furthermore, it is assumed that $\sigma^2_R = \sigma^2_{d(1)} = \ldots = \sigma^2_{d(U)}$.

In Figure 3, the pairwise BER for linear and WL schemes is shown for two design scenarios, namely with and without self-interference cancelation ($L_c = 0$). For the simulations $U = 4$ transceiver nodes, $M = 4$ antennas at the relay node and an equalization filter length of $L_q = 20$ were considered. As per the relay block diagram in Figure 2, the received signal at the relay was processed with a combination of post- and pre-rake filters in addition to the equalizing filter prior to retransmission. In case of self-interference cancelation, L_c was

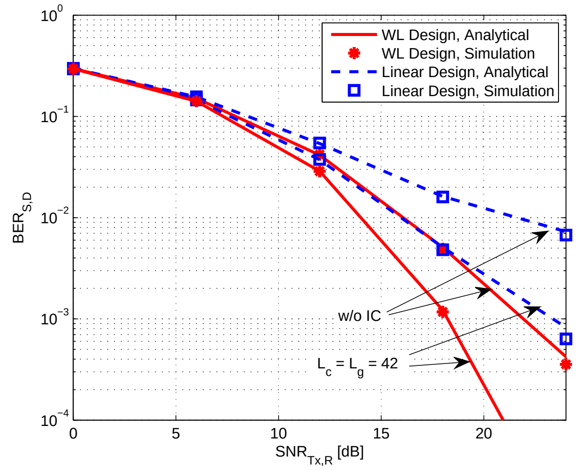

Fig. 3. BER between a pair of source and destination nodes versus relay transmit SNR, $\mathrm{SNR}_{\mathrm{Tx,R}} = P_{\max}/\sigma^2_{d(u)}$, shown for multiuser two-way FF relaying with $U = 4$ users, $M = 4$ antennas at the relay, and an equalization filter length of $L_q = 20$. Comparison between linear and widely linear designs, with and without self-interference cancelation, and identical scaling factor for all users.

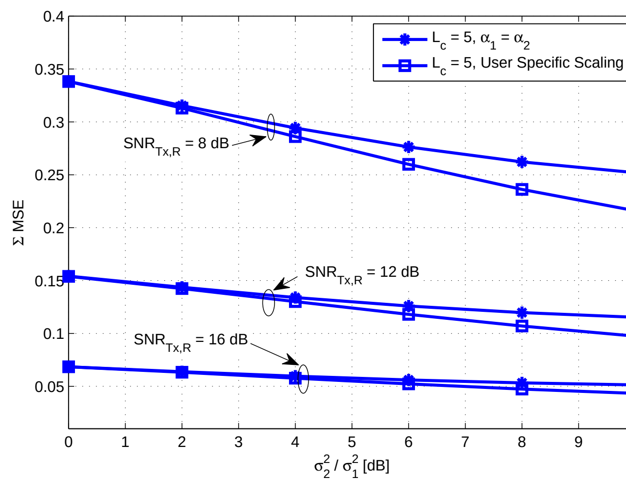

Fig. 4. Sum MSE versus user's noise level difference σ^2_2/σ^2_1 in dB, for FF relaying with $U = 2$ users, $M = 4$ antennas at the relay, and an equalization filter length of $L_q = 20$. Comparison between WL designs with self-interference cancelation and identical receiver scaling factor for all users and the WL design with user specific scaling factors (with self-interference cancelation).

set to $L_c = 42$, which corresponds to full self-interference cancelation. The close match of the simulated BER results with those obtained from the analytical evaluation from (13) confirms the validity of the derivations in Section V. It is observed that applying the WL design without self-interference cancelation achieves a BER that is comparable to the BER for the linear design with full self-interference cancelation. Considering that the gains achieved by the WL design come without any transmission overhead, unlike the self-interference cancelation scheme that requires feedback of information and storing of the transmitted symbols, applying the WL design is clearly advantageous.

Next, we proceed to evaluate the effect of the iterative

$$\mathrm{SINR}_{d(u)} = \frac{|\tilde{\boldsymbol{q}}^T \tilde{\boldsymbol{G}}^T_{s(u),d(u)} \boldsymbol{e}_{n_f}|^2}{\sum_{\ell=1}^{U} \tilde{\boldsymbol{q}}^T \tilde{\boldsymbol{G}}^T_{s(\ell),d(u)} \tilde{\boldsymbol{G}}_{s(\ell),d(u)} \tilde{\boldsymbol{q}} - \|[\tilde{\boldsymbol{q}}^T \tilde{\boldsymbol{G}}^T_{s(u),d(u)} \boldsymbol{e}_{n_f}, \tilde{\boldsymbol{q}}^T \tilde{\boldsymbol{G}}^T_{d(u),d(u)} \boldsymbol{E}_{L_c}]\|^2 + \eta_{d(u)}} \tag{12}$$

design with user specific scaling with regard to the resulting sum-MSE. In Figure 4, the sum-MSE is plotted versus the noise level difference for a FF relaying network consisting of $U = 2$ nodes, using $M = 4$ antennas at the relay, and an equalizing filter length of $L_q = 20$. The sum-MSE is plotted for three transmit SNR levels of $\mathrm{SNR}_{\mathrm{Tx,R}} = 8, 12, 16$ dB. It is observed that at higher transmit SNR levels, the iterative design with user specific scaling achieves a sum-MSE that is comparable to that achieved by the non-iterative convex design with identical receiver scaling for all users. The effect of user specific scaling factors is more pronounced at lower transmit SNR levels and when the two users operate at different receive SNR levels. Note that the difference in noise levels translates to a difference in received SNR. Therefore, the convex design scheme with identical scaling factor for all users can be used as the default FF relaying design procedure. Once the optimal solution of the design problem in (5) is obtained, the received SINR can be evaluated analytically using (12). Combining the information about the relay transmit SNR and the destination node SINR levels, the relay can make a decision about whether or not switching to the iterative design is useful.

VII. Conclusion

In this paper, we have developed novel multiuser two-way relaying techniques for a network of low-complexity direct-sequence ultra-wideband (DS-UWB) nodes communicating via a more powerful central relay. The designs are novel in that we consider relaying over frequency-selective fading channels and use post/pre-rake combining at the relay. Considering binary phase-shift keying DS-UWB signaling, widely linear (WL) counter-parts of the proposed filter design schemes were devised, and the superiority of the WL design was demonstrated via numerical performance evaluation. We formulated convex optimization problems with closed-form solution and also a more general formulation with an iterative design approach using alternating optimization. Based on numerical evaluations, the benefits of the iterative design are notable when users operate at different signal-to-noise levels. Using our bit error rate analysis, we suggest adopting the convex design with identical receiver scaling for all users as the default design procedure and switching to the iterative design, when signaling conditions change accordingly.

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