NOTES ON FILTERING, ROBUST TRACKING AND DISTURBANCE REJECTION USED IN MODEL BASED PREDICTIVE CONTROL SCHEMES

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Abstract. The paper aims to provide the reader with more insight to the problem of filtering within Model Based Predictive Control $MBPC$ schemes. The issue of ensuring robust tracking and/or disturbance rejection for a various class of signals is treated in an unified manner. As a result a novel approach is considered.

Keywords. Predictive Control ($MBPC$), Integral action, Filtering techniques

1. INTRODUCTION

Model Based Predictive Control ($MBPC$) (M.Morari, 1994; J.H.Lee and Z.H.Yu, 1994) has emerged as a powerful practical control technique during the last decade. This control technique is also known by several other names such as Model Predictive Control (MPC) (C.E.Garcia *et al.*, 1989), Receding Horizon Control (RHC) (J.Rawlings and K.Muske, 1993), General Predictive Control (GPC) (D.W.Clarke and C.Mohtadi, 1989), Dynamic Matrix Control (DMC) (D.M.Prett *et al.*, 1982).

Various authors provide extra degrees of freedom to the $MBPC$ schemes by the filtering of the set-point signal, the signals involved in the $MBPC$ cost function or by plant shaping. Although these techniques increase design freedom, we shall show that to some extent they are equivalent to a suitable choice of weights in the cost function.

Another aim of this paper is to describe the advantages and drawbacks associated with an unified approach to robust tracking and/or disturbance rejection to be provided by the $MBPC$ schemes. As a result a zero offset following will be ensured for a larger class of references than random steps coming at random times. This is a novel approach that generalises the conventional formulation of $MBPC$.

A brief summary of the $MBPC$ technique is provided in order to enable the reader to follow the statements made in the next two sections.

For further details the papers (D.W.Clarke and C.Mohtadi, 1989; M.Morari, 1994; C.E.Garcia *et al.*, 1989; D.W.Clarke and C.Mohtadi, 1989; D.M.Prett and C.E.Garcia, 1988; D.W.Clarke, 1994) should be consulted.

In $MBPC$ control method the future control movements are determined by minimising the following cost function:

$$J(k) = \sum_{l=1}^{N_y} \|(\tilde{y}(k+l) - r(k+l)\|^2_{Q(l)} + \\ + \sum_{l=1}^{N_u} \|\Delta u(k+l-1)\|^2_{R(l)} \tag{1}$$

subject to constraints on:

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- the inputs levels: $u_l(l) \leq u(l) \leq u_u(l)$ where $k \leq l \leq k + N_u - 1$
- the input increments: $\Delta u_l(l) \leq \Delta u(l) \leq \Delta u_u(l)$ where $k \leq l \leq k + N_u - 1$
- the output (in general other linear combinations of states): $y_l(l) \leq \tilde{y}(l) \leq y_u(l)$ where $k + N_1 \leq l \leq k + N_y$

where $r(k)$ is the a priori known vector reference signal for outputs (at time k), $Q(l)$ and $R(l)$ are weights (independent of time k), $\|\alpha\|_Q^2 = \alpha^T Q \alpha$ and k is the current time. It is assumed that $\Delta u(j) = 0$ for $j \geq t + N_u$. The model used for predictions is an LTI model. The on-line constrained $MBPC$ optimisation is usually carried out using a quadratic program solver.

For clarity the unconstrained state feedback case is considered in Figure 1 which shows a block diagram of $MBPC$ when states are available for feedback, and when there are no constraints.

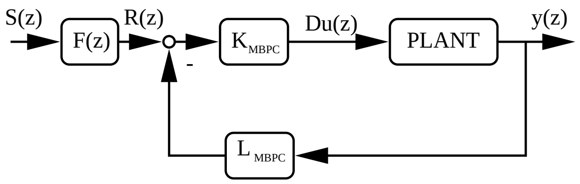

Fig. 1. Filtering problem formulation

For the computation of the matrices K_{MBPC} and L_{MBPC}, characterising the unconstrained $MBPC$, see (J.H.Lee and Z.H.Yu, 1994; M.Huzmezan and J.M.Maciejowski, 1996).

From the Figure 1:

$$\Delta u(k) = K_{MBPC} \left[R(k) - L_{MBPC} \begin{bmatrix} \Delta x(k) \\ \tilde{y}(k) \end{bmatrix} \right]$$

where:

$$\begin{aligned} K_{MBPC} &= \begin{bmatrix} I & 0 & \dots & 0 \end{bmatrix} \\ &\quad \left[(\mathcal{C}^u)^T \mathcal{Q}^T \mathcal{Q} \mathcal{C}^u + \mathcal{R}^T \mathcal{R} \right]^{-1} \mathcal{R}^T \mathcal{Q}^T \mathcal{Q}, \\ L_{MBPC} &= \begin{bmatrix} \mathcal{C}^x & \mathcal{C}^y \end{bmatrix}, \end{aligned}$$

$y(k) = \begin{bmatrix} \Delta x(k) \\ \tilde{y}(k) \end{bmatrix}$, matrices $\mathcal{C}^u$, $\mathcal{C}^x$ and $\mathcal{C}^y$ are defined using a linear internal model of the plant and $\mathcal{R}$, $\mathcal{Q}$ are the cost-function weighting matrices over the prediction (N_y) and control (N_u) horizon, respectively. $S(k)$ is the set-point and $R(k)$ the reference vectors across the N_y horizon at time k.

2. FILTERING AND PLANT SHAPING

In the literature, see (C.E.Garcia *et al.*, 1989), filtering of the future set-point is often recommended as shown in Figure 2 and is considered as an additional degree of freedom used together with the traditional ones which are horizons and weighting matrices.

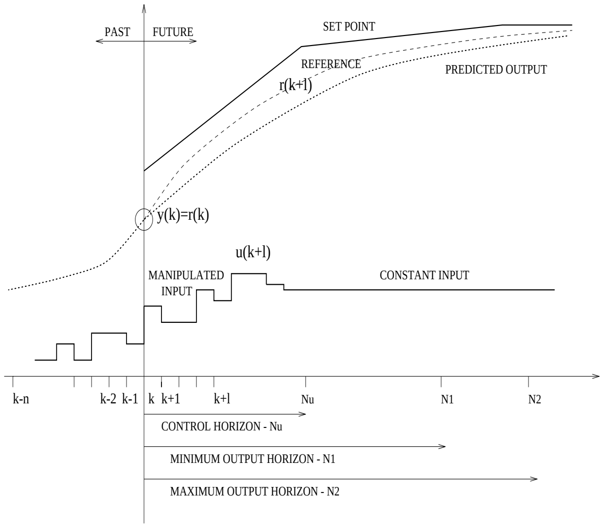

Fig. 2. Prediction Strategy

The reference trajectory generation assumes a certain knowledge of the set-point to be followed which is embedded in the pre-filter design. By making an a priori decision on the pre-filter structure it is possible to degrade the achievable performance because the pre-filter is usually chosen to give the fastest possible response during typical demands.

For example a simple way to generate the reference trajectory $r(k + l)$ is via a first order filter:

$$\begin{aligned} r(k) &= y(k) \\ r(k + l) &= \alpha r(k + l - 1) + (1 - \alpha) s(k) \end{aligned}$$

where $y(k)$ is the current output, $s(k)$ is current set-point at time k for $l = 0 \dots N_y$ and $\alpha \in [0, 1]$.

In (J.A.Rossiter and B.G.Grinnell, 1996) the authors developed an algorithm with better tracking based on minimising a cost function which is a measure of the tracking across the whole simulation.

A similar approach developed in (A.Naganawa and K.Aida, 1996) enhances the tracking performance by adapting a pre-filter on-line with the aim of achieving good tracking and low sensitivity properties for the closed loop.

In other papers, such as (L.Chisci and G.Zappa, 1991), the authors have used filtered set-points, inputs and measurements in the cost function.

In the following we investigate the cases when the above formulations are equivalent:

Theorem 2.1. Consider the $MBPC$ problem:

$$J(k) = \sum_{l=1}^{N_y} \|(\tilde{y}_f(k+l) - r(k+l)\|^2_{Q(l)} + + \sum_{l=1}^{N_u} \|\Delta u_f(k+l-1)\|^2_{R(l)} \tag{2}$$

If $e_f(z) = W_y(z)e(z)$ with $e(z) = \tilde{y}(z) - s(z)$, $r(z) = F(z)s(z)$, $\tilde{y}_f(z) = W_y(z)\tilde{y}(z)$, $\Delta u_f(z) = W_u(z)^{-1}\Delta u(z)$ and moreover $W_y(z) = F(z)$ with $W_y(z)$ and $W_u(z)^{-1}$ being FIR filters having a causal and anti-causal part:

$$W_y(z) = a_{-N_p}z^{-N_p} + \cdots + a_{-1}z^{-1} + \\ +a_0z^0 + a_1z^1 + \cdots + a_{N_f}z^{N_f}$$

$$W_u(z)^{-1} = b_{-N_q}z^{-N_q} + \cdots + b_{-1}z^{-1} + \\ +b_0z^0 + b_1z^1 + \cdots + b_{N_g}z^{N_g}$$

where $N_f \leq N_y$ and $N_g \leq N_u$ then the $MBPC$ problem can be reformulated as:

$$J(k) = \|E(k)\|^2_{\hat{\mathcal{Q}}_f} + \|\Delta U(k)\|^2_{\hat{\mathcal{R}}_f} \tag{3}$$

where $\hat{\mathcal{Q}}_f = \hat{T}^T\mathcal{Q}_f\hat{T}$ and $\hat{\mathcal{R}}_f = \hat{X}^T\mathcal{R}_f\hat{X}$ and

$$\Delta U(k) = \left[\Delta u(k-N_q)^T \ \ldots \ \Delta u(k+N_g+N_u-1)^T\right]^T$$
$$E(k) = \left[e(k-N_p)^T \ \ldots \ e(k+N_f+N_y)^T\right]^T$$
$$\mathcal{Q}_f = diag[Q(1), \ldots, Q(N_y)]$$
$$\mathcal{R}_f = diag[R(1), \ldots, R(N_u)]$$

and

$$\hat{T} = \begin{bmatrix} a_{-N_p} & \ldots & a_0 & \ldots & a_{N_f} & 0 & 0 & \ldots & 0 \\ 0 & a_{-N_p} & \ldots & a_0 & \ldots & a_{N_f} & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & 0 & a_{-N_p} & \ldots & a_0 & \ldots & a_{N_f} \end{bmatrix}$$

$$\hat{X} = \begin{bmatrix} a_{-N_q} & \ldots & a_0 & \ldots & a_{N_g} & 0 & 0 & \ldots & 0 \\ 0 & a_{-N_q} & \ldots & a_0 & \ldots & a_{N_g} & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \ldots & 0 & a_{-N_q} & \ldots & a_0 & \ldots & a_{N_g} \end{bmatrix}$$

Proof Let $E_f(k) = \left[e_f(k+1)^T \ \ldots \ e_f(k+N_y)^T\right]^T$ and $\Delta U_f(k) = \left[\Delta u_f(k)^T \ \ldots \ \Delta u_f(k+N_u-1)^T\right]^T$. Then the cost-function (2) can be written as:

$$J(k) = \|E_f(k)\|^2_{\mathcal{Q}_f} + \|\Delta U_f(k)\|^2_{\mathcal{R}_f}. \tag{4}$$

The filtered error is:

$$e_f(k+1) = a_{-N_p}e(k-N_p) + \cdots + a_{-1}e(k-1) + \\ +a_0e(k+1) + \cdots + 0e(k+N_f+N_y) \\ \ldots\ldots\ldots\ldots\ldots \\ e_f(k+N_y) = 0e(k-N_p) + \ldots \\ +a_{-N_p}e(k+N_y-N_f-N_q1) + \ldots \\ +a_0e(k+N_y-N_f) + \cdots + a_{N_f}e(k+N_f+N_y)$$

Thus the expression for the filtered error is $E_f(k) = \hat{T}E(k)$. For $\Delta u_f(k), \ldots, \Delta u_f(k+N_u)$ a similar expression can be derived such that $\Delta U_f(k) = \hat{X}\Delta U(k)$.

Therefore with $\hat{\mathcal{Q}}_f = \hat{T}^T\mathcal{Q}_f\hat{T}$ and $\hat{\mathcal{R}}_f = \hat{X}^T\mathcal{R}_f\hat{X}$ the cost function 4 can be written as:

$$J(k) = \|E(k)\|^2_{\hat{\mathcal{Q}}_f} + \|\Delta U(k)\|^2_{\hat{\mathcal{R}}_f}$$

Q.E.D

Remark 2.2. Note that the use of an anti-causal part in the FIR filters is possible when there is *apriori* knowledge of the set-point and of the predicted output over the relevant horizon.

Remark 2.3. Solving the formulation (2) or equivalently (4) is computationally less expensive than formulation (3) since the prediction and control horizon are considerably shorter. Therefore we recommend use of filtering for actual implementation.

Remark 2.4. The positive definite weighting matrices $\hat{\mathcal{Q}}_f$ and $\hat{\mathcal{R}}_f$ no longer have the diagonal block structure as in the case of cost function (4). This shows the way in which the filters ensure extra degree of freedom to the optimisation cost function. Note that with full structured $\hat{\mathcal{Q}}_f$ and $\hat{\mathcal{R}}_f$ the problem can no longer be written in the form:

$$J(k) = \sum_{l=-N_p}^{N_f+N_y} \|(\tilde{y}(k+l) - s(k+l)\|^2_{\hat{Q}(l)} + + \sum_{l=-N_q}^{N_g+N_u} \|\Delta u(k+l-1)\|^2_{\hat{R}(l)}$$

Remark 2.5. The assumption made $F(z) = W_y(z)$, although quite restrictive and unnatural, was made in order to simplify the formulation and show how one can reformulate the original problem.

3. ENSURING ROBUST TRACKING AND DISTURBANCE REJECTION

During the development of $MBPC$ techniques researchers were concerned with ensuring integral action with their control schemes. The aim was to have this property built into the algorithm in order to cope with random steps occurring at random times or, in the stochastic sense, with integrated white noise. These signals represent a class of disturbances encountered in many practical systems, see (D.W.Clarke *et al.*, 1987). Inherent integral action, in conjunction with an appropriate prediction model, written in the control increment $\Delta u(k)$, ensures automatically zero offset in the case of the disturbance types mentioned above (D.W.Clarke and C.Mohtadi, 1989).

The conventional setup of the $MBPC$ formulation in the state space form which include integral action is characterised by:

- The plant model:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ \tilde{y}(k) &= Cx(k) + n(k) = y_x(k) + n(k) \end{aligned}$$

  where $u(k), x(k), \tilde{y}(k), n(k)$ are the input, state, output, output disturbance vectors, respectively, and $y_x(k)$ is the output vector free of disturbance.
- The cost function described in equation (1)
- The LTI state space prediction model:

$$\begin{bmatrix} \Delta x(k+1) \\ y_x(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix} \begin{bmatrix} \Delta x(k) \\ y_x(k) \end{bmatrix} + \hat{B}\Delta u(k)$$
$$y(k) = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} \Delta x(k) \\ \tilde{y}_x(k) \end{bmatrix} + n(k)$$

- The linear inequalities expressing the $MBPC$ constraints:

$$\mathcal{D}\Delta U(k) \leq \mathcal{E} \begin{bmatrix} u(k-1) \\ \hat{x}(k) \\ y(k) \\ c(k) \end{bmatrix}, \mathcal{D} = \begin{bmatrix} L \\ M\Lambda \\ N\mathcal{G} \end{bmatrix}$$
$$\mathcal{E} = \begin{bmatrix} 0 & 0 & 0 & I_\lambda & 0 & 0 \\ -M\mathcal{I} & 0 & 0 & 0 & I_\mu & 0 \\ 0 & -N\mathcal{F} & -N\mathcal{H} & 0 & 0 & I_\nu \end{bmatrix}$$

All the matrices involved in the above equations are defined in (M.Huzmezan and J.M.Maciejowski, 1996).

In this section we develop a scheme to deal with a wider class of disturbance and/or reference signals within $MBPC$ controlled plants. This scheme will generalise the usual integral action of $MBPC$ which deals with piecewise constant signals.

The idea behind this development is the well known *internal model principle* which states that, in order to have a disturbance rejected or a reference tracked, its corresponding model has to be included in the controller. Our approach is to augment the plant model with the disturbance model and then use the augmented model as a prediction model to construct the optimiser cost function.

By assuming that the disturbance and/or reference signal satisfies the difference equation (5) this formulation allows a wide variety of signals such as steps, ramps or sinusoids, to be described:

$$n(k+1) + \alpha_1 n(k) + \alpha_2 n(k-1) = 0 \qquad (5)$$

This equation can be rewritten in the state space representation:

$$\begin{bmatrix} n(k) \\ n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\alpha_2 I & -\alpha_1 I \end{bmatrix} \begin{bmatrix} n(k-1) \\ n(k) \end{bmatrix}$$
$$n(k) = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} n(k-1) \\ n(k) \end{bmatrix}$$

For instance, using the above model a step can be obtained when $n(0) = 0$, $n(1) = a$, $\alpha_1 = -1$, $\alpha_2 = 0$, a ramp when $n(0) = 0$, $n(1) = a$, $\alpha_1 = -2$, $\alpha_2 = 1$, $a \in \mathcal{R}$ or the sinusoid when $n(0) = b\sin(\phi)$, $n(1) = b\sin(aT_s + \phi)$, $\alpha_1 = -2\cos(aT_s)$, $\alpha_2 = 1$, $a \in \mathcal{R}$.

We define the generalised input:

$$\psi(k) = u(k) + \alpha_1 u(k-1) + \alpha_2 u(k-2)$$

the generalised state:

$$\xi(k) = x(k) + \alpha_1 x(k-1) + \alpha_2 x(k-2)$$

and the corresponding output:

$$\begin{aligned} \zeta(k) &= \tilde{y}(k) + \alpha_1 \tilde{y}(k-1) + \alpha_2 \tilde{y}(k-2) \\ &= \tilde{y}_x(k) + \alpha_1 \tilde{y}_x(k-1) + \alpha_2 \tilde{y}_x(k-2) \\ \zeta(k) &= C\xi(k) \end{aligned}$$

Hence, the generalised state space representation of the plant model is:

$$\begin{aligned} \xi(k+1) &= A\xi(k) + B\psi(k) \\ \zeta(k) &= C\xi(k) \end{aligned}$$

together with

$$\eta(k+1) = A_n\eta(k)$$
$$n(k) = C_n\eta(k)$$

where $\eta(k) = \begin{bmatrix} n(k-1) \\ n(k) \end{bmatrix}$ represent the state of the disturbance model.

This state space representation, augmented with the noise model (5) and using the vector $y_x(k)$ of outputs free of disturbance, can be written as:

$$\begin{bmatrix} \xi(k+1) \\ \eta(k+1) \\ y_x(k) \\ y_x(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & A_n & 0 & 0 \\ 0 & 0 & 0 & I \\ CA & 0 & -\alpha_2 I & -\alpha_1 I \end{bmatrix} \begin{bmatrix} \xi(k) \\ \eta(k) \\ y_x(k-1) \\ y_x(k) \end{bmatrix}$$
$$+ \begin{bmatrix} B \\ 0 \\ 0 \\ CB \end{bmatrix} \psi(k)$$

$$\tilde{y}(k) = \begin{bmatrix} 0 & C_n & 0 & I \end{bmatrix} \begin{bmatrix} \xi(k) \\ \eta(k) \\ y_x(k-1) \\ y_x(k) \end{bmatrix}$$

Actually this is the only way we can connect the output $\tilde{y}(k)$ with the generalised state vector. Let $z(k) = \left[\xi(k)^T \;\; \eta(k)^T \;\; y_x(k-1)^T \;\; y_x(k)^T\right]^T$ then the model written in a compact form is:

$$z(k+1) = \hat{A}z(k) + \hat{B}\psi(k)$$
$$\tilde{y}(k) = \hat{C}z(k) \qquad (6)$$

In fact this is going to be the model used to build the prediction across the N_y horizon and finally the cost function to be optimised.

In this process the state corresponding to the compact form will be partly measured and/or partly estimated depending on the type of formulation employed (e.g. with or without state measurement available). The difference between the measured and the estimated output will be used to start the predictor which will provide the predicted values of $\tilde{y}$ across the N_y horizon. This is actually how feedback information comes into play.

The cost function of the generalised formulation is:

$$J(k) = \sum_{l=1}^{N_y} \|(\hat{C}z(k+l) - r(k+l)\|^2_{Q(l)} +$$
$$+ \sum_{l=1}^{N_u} \|\psi(k+l-1)\|^2_{R(l)}$$

Here the reader should note the differences compared with the conventional cost function (1). The model used for prediction was defined by (6).

Associated with the cost function and the prediction model we introduce the following constraints upon the variables involved in the control process:

- the inputs levels: $u_l(l) \le u(l) \le u_u(l)$ where $k \le l \le k + N_u - 1$
- the generalised input levels: $\psi_l(l) \le \psi(l) \le \psi_u(l)$ where $k \le l \le k + N_u - 1$
- the output (in general other linear combinations of states): $y_l(l) \le \tilde{y}(l) \le y_u(l)$ where $k + N_1 \le l \le k + N_2$

These constraints can be cast into the following linear inequality by stacking together scalar inequalities over the control and prediction horizon, respectively :

$$\mathcal{D}\Psi(k) \le \mathcal{E} \begin{bmatrix} u(k-2) \\ u(k-1) \\ z(k) \\ y(k) \\ c(k) \end{bmatrix}$$

Now the constrained quadratic programming problem to be solved on-line is completely defined.

In terms of implementation Figure 3 defines the structure used and the corresponding signals.

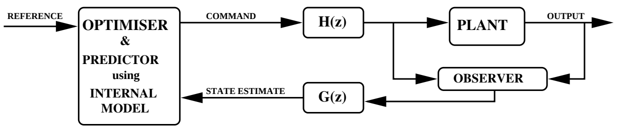

Fig. 3. The structure of the generalised $MBPC$ controller

The filters $G(z)$ and $H(z)$ have the following form
$G(z) = 1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}$ and
$H(z) = (1 + \alpha_1 z^{-1} + \alpha_2 z^{-2})^{-1}$
where $\xi(k) = G(z)x(k)$ and $u(k) = H(z)\psi(k)$.

The proposed formulation results in the disturbance being rejected asymptotically if the $MBPC$ controller stabilises the system (6) which means that $z(k)$ tends to zero in an asymptotic manner.

Therefore assuming $r(k+l) = 0$ for $l = 1 \ldots N_y$ then $\tilde{y}(k)$ tends asymptotically to zero which ensures the required property.

As a result we require that the original plant, in its discrete time representation, should have zeros differently located than the poles of the filter $H(z)$ in order to avoid their cancelling.

For example, in the case of ramp rejection/following since the plant operates on $\Delta\Delta u(k)$ rather than $u(k)$ (by plant here we refer the model used for prediction), we require that the original plant should have no zeros located at $z = 1$ which otherwise will cancel the ramp model $((1 - z^{-1})^2)$ introduced by this way of operation.

Note that higher order disturbance models than (5) can be handled in an analogous manner. This gives a way to of handling more than one type of disturbance simultaneously.

4. CONCLUSIONS

The contribution of this paper consists of a different approach towards two conventional issues in $MBPC$ schemes, filtering robust tracking and/or disturbance rejection.

We have proved how filtering via FIR filters appearing in the $MBPC$ cost-function can be casted into a conventional $MBPC$ setup of a larger dimension with non-diagonal weights.

Robust tracking or disturbance rejection can now be achieved due to a new formulation which provides rejection and/or tracking for a wider class of reference and/or disturbance signals via an appropriate prediction model constructed using the *internal model principle.* Therefore, including the disturbance model together with the prediction model ensures offset-free control even when there is a significant mismatch between the model and the actual plant.

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