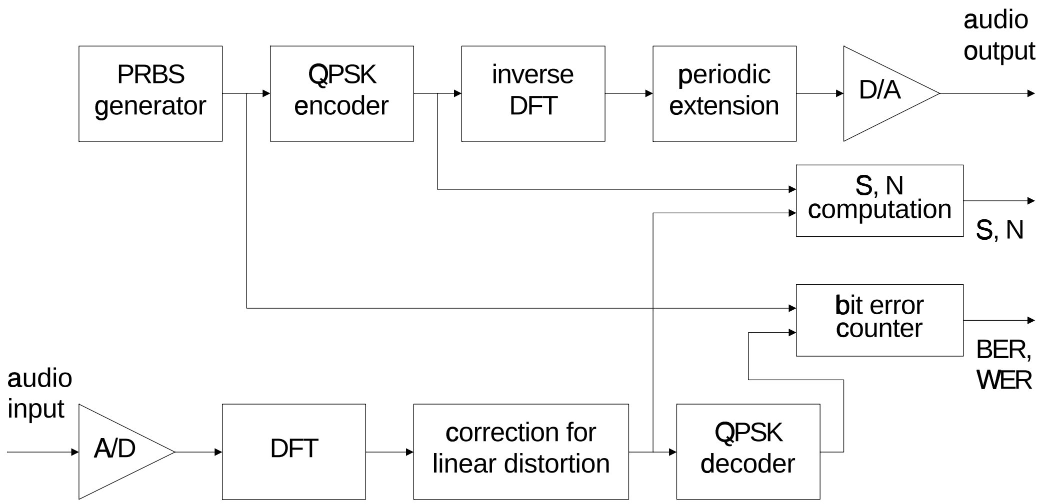

Figure 5.6: Signal processing for experimental measurements.

accurately predict the performance of a real FM receiver and therefore these models cannot be used to accurately predict the performance of OFDM/FM. Some of the effects that the theoretical discriminator model does not take into account are:

- the distortion due to clipping,
- the effect of preemphasis and deemphasis on the noise and signal powers, and
- receiver dynamic range limitations.

The effects on the signal and noise due to preemphasis, deemphasis and clipping, are described in Chapter 2. In addition to these effects, at low IF SNR the receiver's noise output is theoretically much (over 10 dB) higher than the maximum signal output. Receiver dynamic range limitations can prevent the output noise power from reaching its theoretical value. For these reasons it was necessary to use measured SN curves instead of the theoretical SN curves to predict OFDM BER and WER performance.

The following three sub-sections describe how the modulating signal level, the baseband frequency range, and the amount of software preemphasis were chosen. Subsequent sub-sections describe how the bit rate, the IF SNR and E_b/N_0, were computed. The final

5.3 Experimental Software

The simulation program described in Chapter 4 was modified to make the BER and WER measurements over the experimental channel. Instead of using a subroutine that simulates the channel, the OFDM signal samples are written to the D/A and read from the A/D. Two additional processing steps were added for use on the hardware channel:

- periodic extension to provide guard times around each block, and
- correction for phase and amplitude (linear) channel distortion.

These procedures are described in Chapter 7.

Figure 5.6 shows the signal processing steps for the experimental measurements. A block of pseudo-random bits is QPSK-encoded[2] and modulated into a block of OFDM samples. The block of samples is extended to add guard times before and after the block. The samples are sent to the D/A and the resulting analog audio signal modulates the FM transmitter. The recovered baseband (audio) output of the FM receiver is sampled and A/D converted. The received samples are demodulated and the complex data values corrected for the linear distortion effects of the channel. The received complex data values are then decoded into bits and compared to the transmitted data to compute the BER and WER. The program can also compare the transmitted and received data values on each subchannel to measure the signal and noise powers. This was used to measure the SN curves (section 5.4.7).

5.4 Measuring the Baseband SN Characteristics

The numerical methods used to predict the performance of OFDM over a fading channel require that the channel's SN curves be known. Measurements of the experimental channel's SN curves showed that theoretical models for an FM discriminator [3] do not

[2] QPSK is the same as 4-QAM.

distortion-plus-noise power. The distortion-plus-noise power was 40 dB less than the signal power. Other tests are described in Appendix C.

Audio Attenuator

An audio attenuator was used to reduce the D/A output (approximately 140 millivolts rms) to a level suitable for modulating the transmitter (approximately 6 millivolts rms). The attenuator circuit includes a low-pass filter to reduce RF leakage and a switch to turn the transmitter on and off. The attenuator circuit is shown in Figure 5.5.

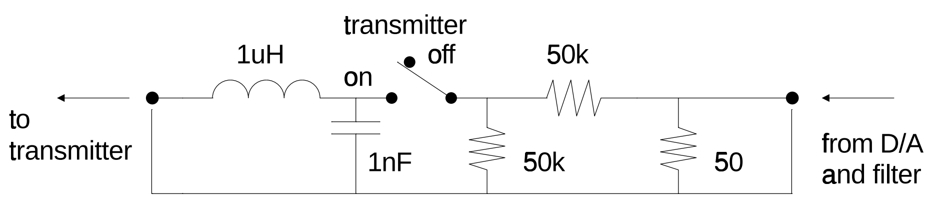

Figure 5.5: Schematic of audio attenuator for transmitter.

Shielding

Shielding is an important consideration because the transmitter output power is much (over 100 dB) greater than the minimum signal power that can be detected by the receiver. Any signal leaking into the receiver directly from the transmitter would alter the level of the received signal level and affect the results. Cast aluminum boxes were used to shield both the transmitter and receiver, and EMI filters were used on power supply leads to avoid RF leakage. Aluminum foil was used as a material gasket to seal the seams around the lids of the aluminum boxes.

A tone was transmitted with no added RF noise and the simulator set for maximum attenuation to test for signal leakage around the channel simulator. The RF attenuator was set to 101.5 dB and the fading simulator set to the -20 dB test level. The received signal was not audible and its level was too low to measure above the noise.

5.2.4 DSP Equipment

Computer

An IBM PC/AT-compatible computer was used for all of the digital signal processing as well as test data generation and BER and WER rate measurement. All of the signal processing computations used IEEE-standard 32-bit floating point.

A/D and D/A Board

The analog interface circuit was designed and built for this project. It contains a 10-bit analog-to-digital converter (A/D), a 12-bit digital-to-analog converter (D/A), a sample-and-hold amplifier, buffer amplifiers, and a timing circuit. It was built on a prototyping board that was plugged into the computer's bus. The circuit is described in Appendix C and the interface software is listed in Appendix E.

Reconstruction and Anti-Aliasing Filters

A Krohn-Hite model 3342 filter was used to reconstruct the analog waveform from the D/A output samples. The reconstruction filter used one eighth-order Butterworth low-pass section with a -3 dB frequency of 4 kHz and one eighth-order Butterworth high-pass section with a -3 dB frequency of 100 Hz. The high-pass section was used to provide AC coupling to the transmitter.

The received signal was passed through a second Krohn-Hite model 3342 low-pass filter to avoid aliasing. This anti-aliasing filter used two eighth-order Butterworth low-pass sections with -3 dB frequencies of 4 kHz.

The filters have a 1 M ohm input impedance, and a 50 ohm output impedance.

The quality of the analog interface was tested by measuring the distortion of a 0.5 to 3.5 kHz OFDM signal sent through the D/A, the filters and the A/D. The SN curve measurement procedure (section 5.4) was used to measure the signal power and the differences between the signal and noise levels.

5.2.3 Receiver

The FM receiver was another Icom model IC-2AT transceiver. The receiver IF filter bandwidth specification is ± 7.5 kHz at −6 dB and ± 15 kHz at −60 dB. The receiver uses a Motorola MC3357 "Low Power Narrow-Band FM IF" IC for most of the IF and AF signal processing functions. The FM detector is a quadrature-type discriminator [50,51]. The receiver frequency was set to 144.150 MHz. The squelch level was set to minimum so that the receiver audio output was always on.

The receiver was placed in a cast aluminum box. A connection was provided to the output of the receiver's IF filter (IC1, pin 5) so that the IF signal level could be measured. The transmitter was disabled to prevent accidental damage to the fading simulator. An EMI power line filter was used on the power supply leads. Figure 5.4 shows the components associated with the receiver.

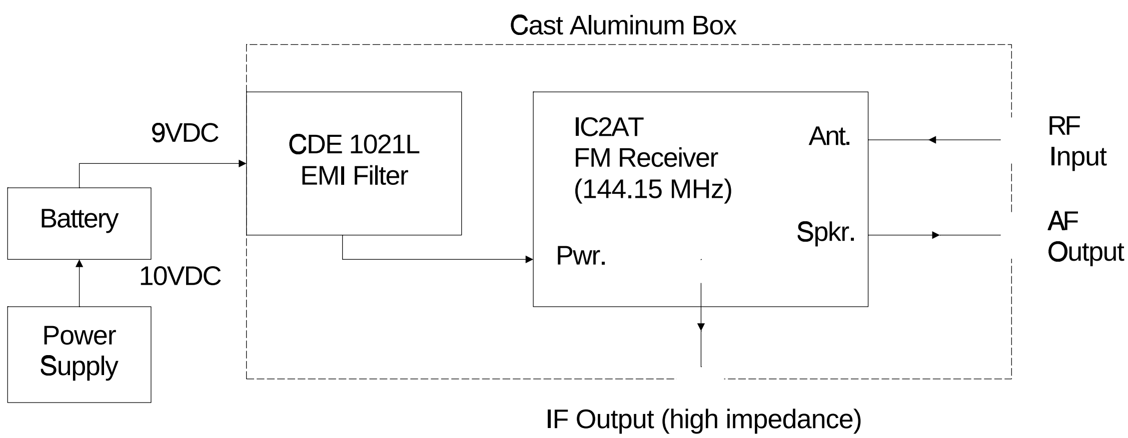

Figure 5.4: Experimental receiver.

the RF noise source. The noise output is the internal device noise of the amplifier. This amplifier has a specified frequency range of 10 to 1000 MHz, gain of 27 $\pm$ 10dB, and noise figure of 10 dB. The spectrum of the noise was measured with a spectrum analyzer set at 1 kHz resolution bandwidth and was found to be constant to within 0.2 dB over the receiver's IF bandwidth. Although the probability distribution of the IF noise was not measured, it should have been Gaussian because the broadband (approximately 1 GHz) noise was passed through an IF filter with a much narrower bandwidth (approximately 15 kHz) [11, section 3.2].

Step RF Attenuator

The RF SNR was set by keeping the noise power fixed and varying the RF signal level. A KAY Elemetrics model 437 step attenuator with a range of 0 to 101.5 dB in 0.5 dB steps was used to vary the RF signal level.

Combiner/Splitter

A Mini-Circuits PSC-2-1 RF power combiner/splitter was used to sum the signal and noise. A second combiner/splitter was used to provide two outputs. One went to the receiver and the other went to the spectrum analyzer. The difference in the levels of the two splitter outputs was measured to be less than 0.2 dB.

Spectrum Analyzer

An HP model 8558B spectrum analyzer was used to measure the relative levels of the signal and the noise. Although the signal power can be measured directly, measurement of the noise power requires several auxiliary measurements and corrections that are described in Appendix D. When the spectrum analyzer was not available it was replaced with a 50 ohm termination to ensure proper matching of the splitter ports. Since the spectrum analyzer was not accurately calibrated, the SNR measurements used only the

5.2.2 Fading Channel Simulator

Figure 5.3 shows the components of the fading channel simulator. The simulator consists of a Rayleigh fading simulator, an RF attenuator, a noise source, and a combiner/splitter.

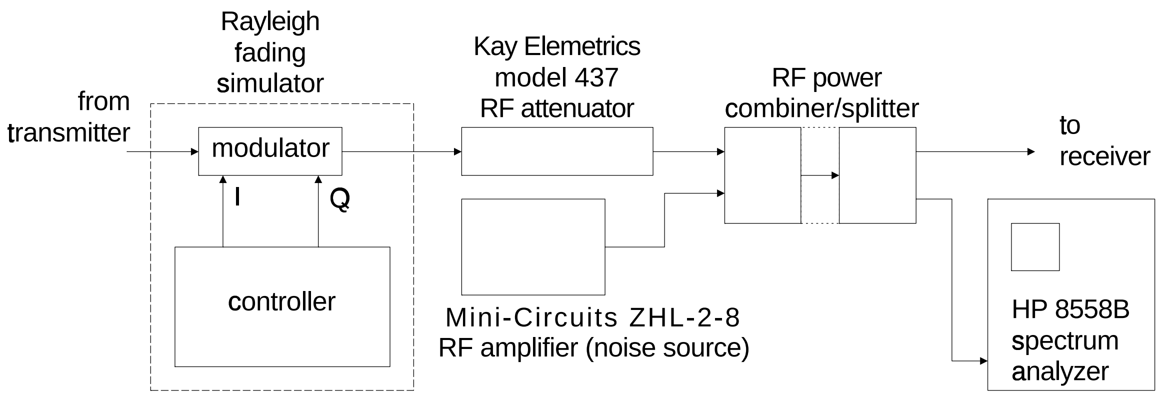

Figure 5.3: Fading channel simulator.

Fading Simulator

A fading simulator was built for these measurements. The fading simulator modulates the RF signal to give it a Rayleigh-distributed envelope and a uniformly-distributed phase. The fading simulator is described in Appendix B and [56].

Noise Source

The signal and noise levels were measured at RF because it was not possible to make accurate measurements at the receiver's IF stage. To ensure that the IF SNR and the RF SNR were equal, the receiver's internal noise was masked by adding a large amount of noise to the RF signal. The additional RF noise resulted in an increase in the IF noise level of at least 11 dB. This level of noise ensured that the noise added by the receiver had a small ($<$ 0.5 dB) effect on the IF SNR.

A Mini-Circuits model ZHL-2-8 RF amplifier with its input terminated was used as

5.2.1 Transmitter

The transmitter was an Icom model IC-2AT narrow-band FM transceiver designed for operation in the 144 to 148 MHz amateur band. Its specifications are similar to those of commercial land mobile radio equipment [52]. The audio processing circuitry contains a +20 dB per decade preemphasis network, a limiter (peak clipping) circuit and a low-pass filter. The maximum peak deviation was measured to be 5 kHz. The carrier frequency was set to 144.150 MHz. The frequency stability is specified as ± 1.5 kHz.

The transmitter was placed in a cast aluminum box and the power leads were passed through an EMI filter to provide RF shielding. A voltage regulator held the transmitter supply voltage at 10 volts DC. An 11 dB 50 ohm attenuator was mounted inside the shielding box to reduce the RF output level to 12.5 dBm[1]. Figure 5.2 shows the components associated with the transmitter.

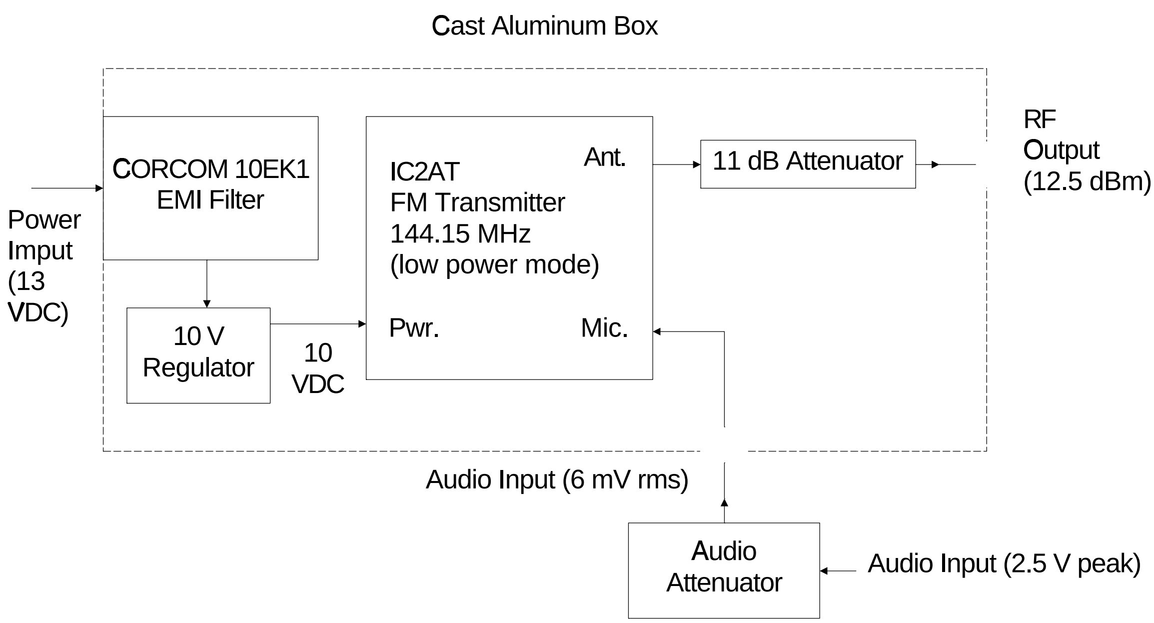

Figure 5.2: Experimental transmitter.

[1] 0dBm ≡ 1mW.

Chapter 5

Experimental Measurements

5.1 Introduction

This chapter describes measurements on an experimental OFDM/FM system. Sections 5.2 and 5.3 describe the experimental hardware and software. Section 5.4 describes how the SN curves of the FM channel were measured and compares the results to those of a theoretical FM channel. The next section describes how the experimental system's BER and WER performances were measured and compares the results with those obtained using the numerical techniques described in Chapter 4. The final section describes measurements to determine the effect of random FM.

5.2 Experimental Hardware

This section describes the equipment used in the measurements: the FM transmitter, the fading channel simulator, the FM receiver, and the digital signal processing equipment. Figure 5.1 shows a block diagram of the overall experimental setup.

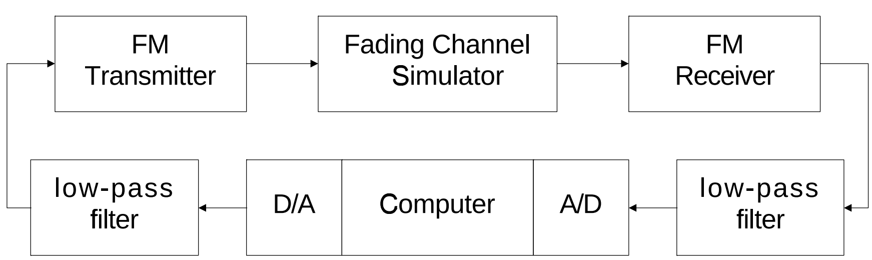

Figure 5.1: Block diagram of experimental setup.

4.5.7 OFDM Modulation/Demodulation

A fast (radix 8/4/2) FFT from the IEEE DSP subroutine library [54] is used to modulate the data values into signal samples.

The inverse FFT converts each block of $N/2$ complex values into N samples. Since each block can transmit N bits and the duration of the block is N/f_s where f_s is the sampling rate, the nominal bit rate is f_s. However, if the channel is bandlimited, not all of the subchannels may be usable and this will reduce the bit rate. Any guard times left between blocks to avoid interference between blocks (see Chapter 6) will further reduce the overall bit rate.

4.5.8 IF SNR to Baseband Signal and Noise Power Conversion

The conversion from IF SNR (r) to baseband signal power (s) and noise power (n) is done with look-up tables. The tables contains values for IF SNRs from -50 to $+60$ dB in 0.1 dB (2.3 percent) increments. The tables can contain theoretical values (for example, equation 2.9) or measured values. Linear interpolation is done between measured points.

The conversion routines were tested by converting a range of IF SNRs and plotting the results to obtain the SN curves.

4.5.9 Simulated Fading Channel

The channel simulator simply scales the inverse FFT output sample and the Gaussian RNG output sample by the s and n values in conversion table and then sums the two scaled samples. The appropriate s and n values are selected by the fading signal level (r).

4.5.5 FEC

An FEC routine was used to test the effect of various FEC codes. The codes are idealized block codes that can correct up to m errors in n bits. They are idealized codes because an actual code may attempt error correction in some words with more than m errors. This attempt may succeed or may introduce additional errors. However, steps can be taken to ensure that these events do not affect the results (see Section 6.3.2).

The FEC routines were tested by generating bit sequences with a known error pattern and measuring the number of errors after FEC processing.

4.5.6 QAM Encoding/Decoding

A QAM data encoder/decoder and an OFDM modulator/demodulator are required for the baseband simulation. The QAM data encoding is done by using two bits per subchannel – one bit for the imaginary (quadrature) component and one bit for the real (in-phase) component. A 0-bit is converted to a value of -1, and a 1-bit to $+1$. Unused subchannels are set to zero. The decoding is done by comparing the received values to a threshold of zero. Figure 4.3 shows the QAM encoding constellation and the decision thresholds used for decoding.

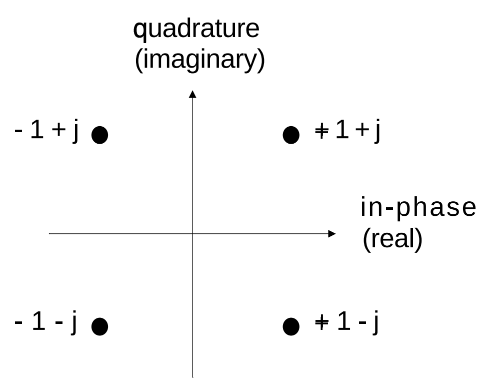

Figure 4.3: QAM Encoding. The real and imaginary axes are the decision region boundaries.

4.5.3 Noise Generators

Two random number generators (RNGs) are used to generate the noise. The Gaussian generator converts two uniformly distributed random variables into two normally distributed ones [54]. The uniform RNG is a linear modulo-congruential type with the multiplier, increment and modulus taken from [58] and agreeing with the criteria in [59]. The recursion equation for this generator is

$$x_{i+1} = ax_i + c \bmod m \tag{4.5}$$

where $a = 8 \times 8385161 + 5 = 67081293$, $c = 2 \times 7090885 + 1 = 14181771$, and $m = 2^{26} = 67108864$. The period of this generator is m. The RNG was implemented using double precision floating point numbers to ensure that full integer precision was maintained [60].

The modulo congruential uniform random number generator was tested by starting at a seed of zero and measuring the period before the state returned to zero. The period was found to be 67108864. The generator was also tested by comparing the sequence generated using the floating point numbers against a sequence computed with infinite-precision arithmetic.

A simple impulsive noise generator was also implemented. Each sample had the same probability of containing an impulse and all noise impulses had the same amplitude.

4.5.4 Bit and Word Error Rate Measurement

This routine compares the transmitted data and the received data and computes bit and word error rates. The bit and word error measurement routines were tested by generating known error patterns and checking the resulting bit and word error counts. Routines are also provided to compute the 95 percent confidence interval statistics for the means [55].

The durations of the baseband simulation and Monte-Carlo integration computations were made as long as practical to reduce the statistical uncertainty of the results. The 0.95 confidence intervals for the desired values were computed to estimate this uncertainty. The overall simulation was divided into a number of separate trials (usually twelve). The variance of the results (the BERs or WERs) in the different trials was used to compute the confidence intervals [55].

The following sections describe the various software routines and how they were tested. Testing can help find errors although it does not guarantee correctness. The tests were done on both the Sun 3/50 and the IBM PC/AT.

4.5.1 Fading Waveform Generator

The Rayleigh fading envelope of the received signal is generated using the method described in [3,56] and in Appendix B.

The routine can also simulate switched diversity between independently-fading input signals. The diversity routines simulate the phase transients produced by switching between antennas by setting a number of samples to zero at each switching time (see section 6.2).

The fading generator was tested by measuring the level crossing rate (LCR) and the cumulative probability distribution function (CPDF) of the generated waveform.

4.5.2 Data Generator

The transmitted data bits were generated with a pseudo-random bit sequence (PRBS) generator implemented as a shift register with feedback. The generator polynomial is taken from [57] and generates a maximal length sequence of period $2^{23} - 1$. The period was measured and found to be correct.

processing simulates the effect of FEC coding. The final step in the simulation is to compare the transmitted and received bit sequences and to compute the BER and WER.

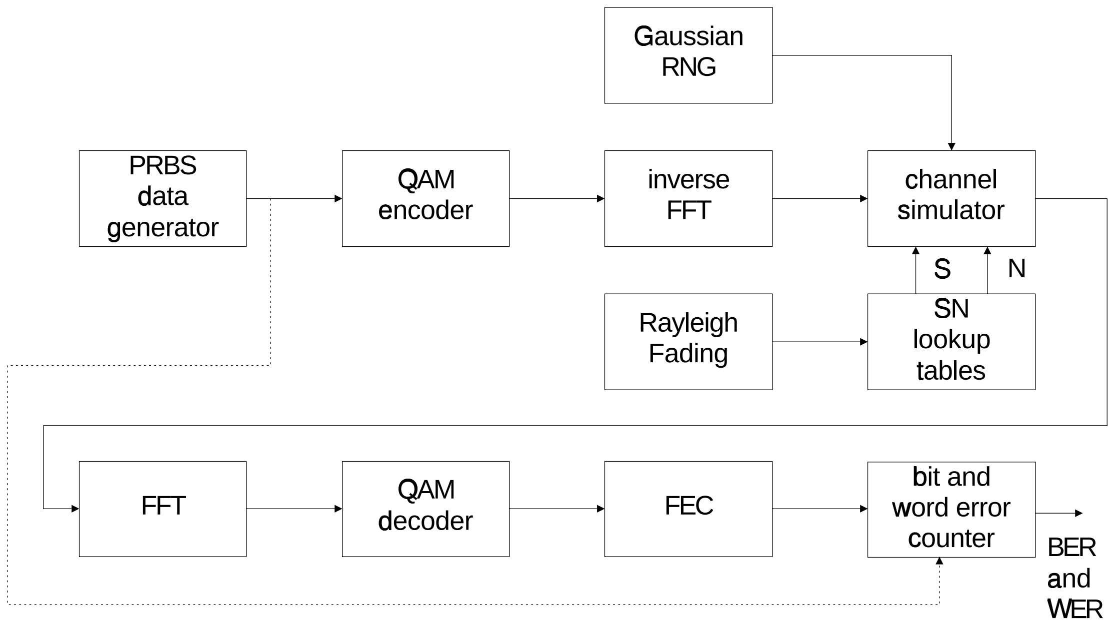

Figure 4.2: Simulation using the equivalent baseband channel.

A more detailed simulation that included the IF signal processing stages could have studied other effects but would have required much more computation.

A listing of the simulation program is given in Appendix E.

4.5 Description of Software

The computer programs were written in FORTRAN 77 because it was the only compiler on the computers available at the start of the research project and efficient FORTRAN FFT routines were available [54]. The *sed* stream editor was used to generate different versions to run on an IBM PC/AT (MS-DOS) and a Sun 3/50 (BSD 4.2 UNIX).

The programs re-used the same fading, noise, and data waveforms when computing the BER statistics for different block sizes. This reduces the computation time by a factor of about 2 and reduces the uncertainty in the measurement of the effect of the block size.

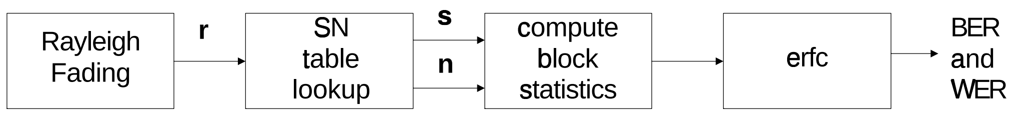

Figure 4.1: The numerical integration method.

computed using equation 3.5. It is more efficient than the more detailed simulations described in the following section because it does not involve generating random data or noise or computing DFTs.

A listing of this program is given in Appendix E.

4.4 Simulation of the Equivalent Baseband Channel

The OFDM system can be studied in more detail by generating a sampled baseband OFDM waveform and passing it through the equivalent baseband channel. This involves performing most of the baseband signal processing for an OFDM system. This method allows testing of signal processing procedures that cannot be included in the equivalent baseband channel model or whose effect is too complex to analyze (see Chapter 6). This approach is considerably slower[1] since it involves generating pseudo-random data and noise as well as computing the FFTs required for OFDM modulation and demodulation.

Figure 4.2 shows the steps involved in the baseband simulation. A block of bits from a pseudo-random bit sequence (PRBS) generator is QAM-encoded into a block of complex values. The inverse FFT generates the baseband OFDM signal. The Rayleigh fading generator produces the signal envelope. The SN lookup table and the channel simulator implement the equivalent baseband channel (shown in Figure 3.1) and combine the baseband signal and Gaussian noise. The FFT demodulates the OFDM signal and the QAM decoder recovers the binary data. Optional forward error correction (FEC)

[1] A typical simulation (about 3 million samples for each of four SNRs and three block sizes) required about 8.5 hours CPU ("user") time on a Sun 3/50 compared to about 1.5 hours for the numerical integration procedure.

Since the signal level is constant over the block $\langle \boldsymbol{s} \rangle^2 = \langle \boldsymbol{s}^2 \rangle$ and equation 3.3 simplifies to

$$\mathrm{BER}_{\mathrm{block}}(\alpha, \beta, \gamma) = Q\left(\frac{\alpha}{\sqrt{\gamma}}\right). \tag{4.2}$$

When the block is very long, each block will have the same statistics and therefore each block will have the same BER. The overall BER will be the block BER for the average block statistics. That is, if

$$\overline{\alpha} = \int_0^\infty s(r)p(r)dr, \quad \overline{\beta} = \int_0^\infty s^2(r)p(r)dr, \quad \text{and} \quad \overline{\gamma} = \int_0^\infty n^2(r)p(r)dr, \tag{4.3}$$

then

$$\mathrm{BER}_{\mathrm{long\ block}} = \mathrm{BER}_{\mathrm{block}}(\overline{\alpha}, \overline{\beta}, \overline{\gamma}). \tag{4.4}$$

The values of $s(r)$ and $n(r)$ were taken from the measured SN curves (see Chapter 5). The SN values were interpolated from the measured values over the range of -60 to $+50$ dB in 0.1 dB steps. A listing of the program that computes the bounds is given in Appendix E.

4.3 Monte-Carlo Integration of the Block BER Equations

The second numerical method uses a Monte-Carlo numerical integration technique to evaluate the integral obtained from the BER analysis in the previous chapter (equation 3.4). The method involves generating random fading waveforms and using the α, β and γ statistics to compute the expected overall BER.

Figure 4.1 shows the steps in this procedure. A Monte-Carlo procedure is used in which signal level vectors $\boldsymbol{r}$ are chosen at random from a sequence which is Rayleigh distributed with the appropriate Doppler rate and block duration. From each $\boldsymbol{r}$ the values of α, β and γ and the resulting $\mathrm{BER}_{\mathrm{block}}$ are calculated. These $\mathrm{BER}_{\mathrm{block}}$ values are then averaged to obtain the overall BER of the system.

This is an efficient way to evaluate the BER for any channel that can be accurately modelled by the equivalent baseband channel model. The word error rate can also be

Chapter 4

Numerical Evaluation of Bit and Word Error Rates

4.1 Introduction

This chapter describes three numerical methods that were developed to evaluate the BER and WER performance of OFDM. The first three sections describe the three numerical methods: a bound computation for very short and very long block lengths, a Monte-Carlo integration, and a baseband signal processing simulation. The final section describes the software routines used.

A comparison of results obtained with the three methods can be used as a check of the analysis and also as a check of the software. The results obtained with these numerical methods are presented along with the experimental results in Chapter 5.

4.2 BER Bounds for Large and Small Blocks

This section gives bounds on the BER of OFDM using blocks that are very short or very long relative to the average fade duration. This is the fastest method but only provides bounds on the BER performance.

When the block is very short, the signal level will be constant during the block. Thus $\langle s \rangle = s$, $\langle s^2 \rangle = s^2$, and $\langle n^2 \rangle = n^2$. The bit error rate for each block, $\mathrm{BER}_{\mathrm{block}}$, as a function of the IF SNR can be averaged over the Rayleigh signal level distribution (equation 2.1) to find the average overall BER. That is,

$$\mathrm{BER}_{\text{short block}} = \int_0^\infty \mathrm{BER}_{\mathrm{block}}(s(r), s^2(r), n^2(r))\, p(r)\, dr. \tag{4.1}$$

The assumption of random errors would simplify the prediction of the error rate of words contained within an OFDM block. Experimental results (see Chapter 5) showed that the independent error assumption is a good approximation for the system that was studied. The prediction of the error rate for words spanning more than one block cannot be done using this method because $\text{BER}_{\text{block}}$ changes from block to block.

3.5.3 Example of SSB Receiver SN Curves

Figure 3.3 is an example of an SSB receiver SN curve. It shows the linear SN curves and the effect of AGC.

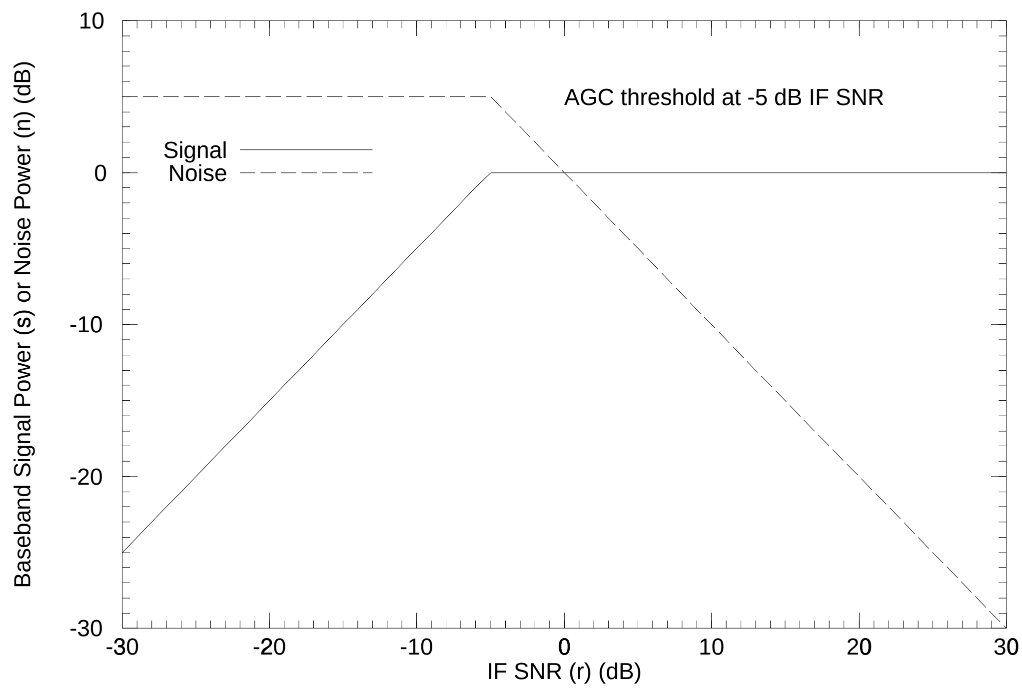

Figure 3.3: Example of SSB receiver SN curves.

3.6 Word Error Rate Analysis

In many data communications systems, any error within a related group of bits (a *word*) invalidates all of the bits. In such systems the performance measure of interest is the word error rate, WER. If bit errors occur randomly with a probability p, the probability of i errors in N bits is:

$$\binom{N}{i} p^i (1-p)^{N-i}. \tag{3.5}$$

3.5 Modelling the OFDM/SSB Channel

SSB transmission is equivalent to translating the OFDM signal spectrum from baseband to the RF frequency. If carrier phase and carrier frequency synchronization are perfect, the effect of fading on the baseband signal is the same as the effect on the RF signal. In this case the analysis of the SSB channel is simplified because the SN curves are linear and the baseband noise has the same spectrum and distribution as the RF noise.

3.5.1 Frequency and Phase Synchronization

Coherent demodulation of the SSB signal requires the use of a pilot carrier. Multipath fading makes accurate carrier phase and frequency synchronization difficult because the signal undergoes rapid phase and amplitude changes. The demodulator must continuously track the phase of the pilot signal. The effect of constant frequency and phase offsets on an OFDM modem for telephone channels was analyzed in [38]. The OFDM/SSB system studied in [1] uses two pilot carriers to reduce the effects of frequency-selective fading on carrier synchronization.

Perfect frequency and phase recovery must be assumed when using the equivalent baseband model for OFDM/SSB because the model does not include frequency or phase shifts.

3.5.2 AGC

SSB receivers use automatic gain control (AGC) circuits to maintain a constant output level as the received signal fades. A limit must be set on the maximum gain of the AGC to avoid excessively large noise outputs during fades.

The effect of such an AGC can be incorporated into the SN curves. To simulate the changing gain of the AGC, the signal level curve is kept constant above the AGC limit and the noise level curve is set to the amount required to achieve the correct SNR. The model assumes an ideal AGC circuit that is not affected by noise and has no delay.

The model assumes an ideal squelch circuit that is not affected by noise and has no hysteresis or delay.

3.4.6 FM Receiver SN Curves

Figure 3.2 is an example of an FM SN curve to demonstrate the above effects. This example shows the theoretical non-linear SN curves of an FM discriminator with a threshold at an IF SNR of approximately 10 dB. The effect of baseband SNR limiting due to random FM or clipping noise has been added so that the maximum baseband SNR is 18 dB. This example also shows the effect of squelch with threshold at -10 dB IF SNR.

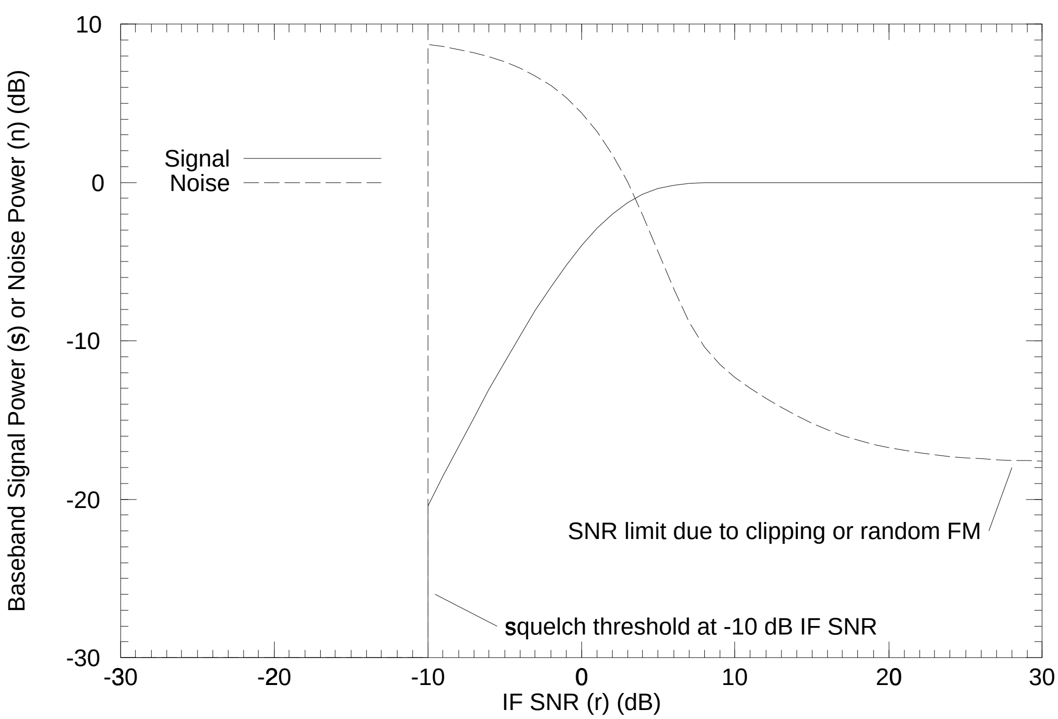

Figure 3.2: Example of an FM SN curve.

the noise could be corrected by changing the transmitted power spectrum. The noise spectrum was therefore assumed to be independent of the IF SNR.

Experimental measurements showed that subchannels at lower frequencies had more noise. This caused subchannels at lower frequencies to have higher error rates. This effect was corrected by transmitting more power in the noisier subchannels. Measurements of the error rates in the different subchannels showed that the error rates could be made approximately equal by using a -10 dB per decade preemphasis at the transmitter in addition to the standard $+20$ dB per decade.

3.4.3 Random FM Noise

The spectrum of the random FM is independent of the SNR and decreases as $(1/f)$ for frequencies above the Doppler rate [48]. Random FM could have been included in the model by adding noise whose level was a function of the Doppler rate. However, this was not necessary since the level of the random FM noise was not significant at any of the Doppler rates measured.

3.4.4 Clipping Noise

Clipping at the transmitter (section 2.4.1) was modelled as an additional source of additive white Gaussian noise. Although the clipping is not independent of the signal, it forms a relatively small portion of the total noise power. The modulating signal level will normally be set so that clipping distortion does not have a measurable effect on the BER. This was the case during the experimental measurements.

3.4.5 Squelch

Squelch can be incorporated into the SN curves by making the values of s and n equal to zero when the IF SNR level is below the squelch threshold.

where $\alpha = \langle s \rangle$, $\beta = \langle s^2 \rangle$, and $\gamma = \langle n^2 \rangle$. This result applies for the Gray-coded signalling constellation shown in figure 4.3.

3.3.3 Evaluating the BER

The resulting overall BER can be obtained by using the joint distribution of α, β, and γ,

$$\mathrm{BER} = \int_0^\infty \int_0^\infty \int_0^\infty \mathrm{BER}_{\mathrm{block}}(\alpha, \beta, \gamma) p(\alpha, \beta, \gamma)\, d\gamma\, d\beta\, d\alpha. \tag{3.4}$$

To obtain estimates of the overall BER, the joint distribution of α, β, and γ is required. Unfortunately, no closed form expression is available for this joint distribution, $p(\alpha, \beta, \gamma)$. In Chapter 4 a Monte-Carlo numerical integration procedure is described that can be used to evaluate this expression.

3.4 Modelling the OFDM/FM Channel

The following sections explain how the OFDM/FM channel is modelled.

3.4.1 Baseband Noise Distribution

Although the probability distribution of the noise depends on the SNR, the noise after the DFT can be assumed to be Gaussian because the DFT in the OFDM demodulation process converts any noise distribution to a distribution that is approximately Gaussian.

3.4.2 Baseband Noise Spectrum

As described in Chapter 2, the spectrum of the discriminator noise output depends on the SNR. The noise distribution among the subchannels thus depends on the IF SNR. Fading causes different subchannels to receive different average noise powers from block to block. It was assumed that differences in the spectral distribution of the noise power from block to block would be small and that any overall long-term uneven spectral distribution of

3.3 BER Analysis

An expression for the BER of a single OFDM block transmitted over the equivalent baseband channel is obtained in this section.

3.3.1 Mean and Variance of the Received Value

In Appendix A the ensemble average of the m'th received data value Y_m over the set of all possible data vectors with the m'th data value fixed at $X_m = D$ ($D = \pm 1 \pm j$) and over all possible noise vectors $\boldsymbol{w}$ is found conditioned on a given fixed signal level $\boldsymbol{r}$ (and the corresponding $\boldsymbol{s}$ and $\boldsymbol{n}$) as:

$$\mathrm{E}\left[\, Y_m \mid X_m = D, X_{-m} = D^*,\, \boldsymbol{r} \,\right] \approx D \left\langle s \right\rangle . \tag{3.1}$$

The conditional variance (given the same conditions as for the mean) of a received data value Y_m, is found to be:

$$\mathrm{var}(\, Y_m \mid X_m = D, X_{-m} = D^*,\, \boldsymbol{r} \,) \approx 2 \left(\left\langle s^2 \right\rangle - \left\langle s \right\rangle^2 + \left\langle n^2 \right\rangle \right) . \tag{3.2}$$

3.3.2 Bit Error Rate Within a Block

Since the interference between the subchannels is caused by a large number of independent subchannels, the Central Limit Theorem implies that the distribution of this interference is approximately Gaussian. This is confirmed by measurements of the interference distribution (Chapter 5). Since the effect of the channel noise is also Gaussian (because of the linearity of the DFT) and independent of the interchannel interference (because $\boldsymbol{r}$ is a constant), the sum of the noise and interference will also be Gaussian and their powers will add.

Using signal-space arguments [11] it is then possible to obtain the BER within a block:

$$\mathrm{BER_{block}}(\alpha, \beta, \gamma) = \frac{1}{2}\mathrm{erfc}\left(\frac{\alpha}{\sqrt{2(\beta - \alpha^2 + \gamma)}} \right) = Q\left(\frac{\alpha}{\sqrt{(\beta - \alpha^2 + \gamma)}} \right) \tag{3.3}$$

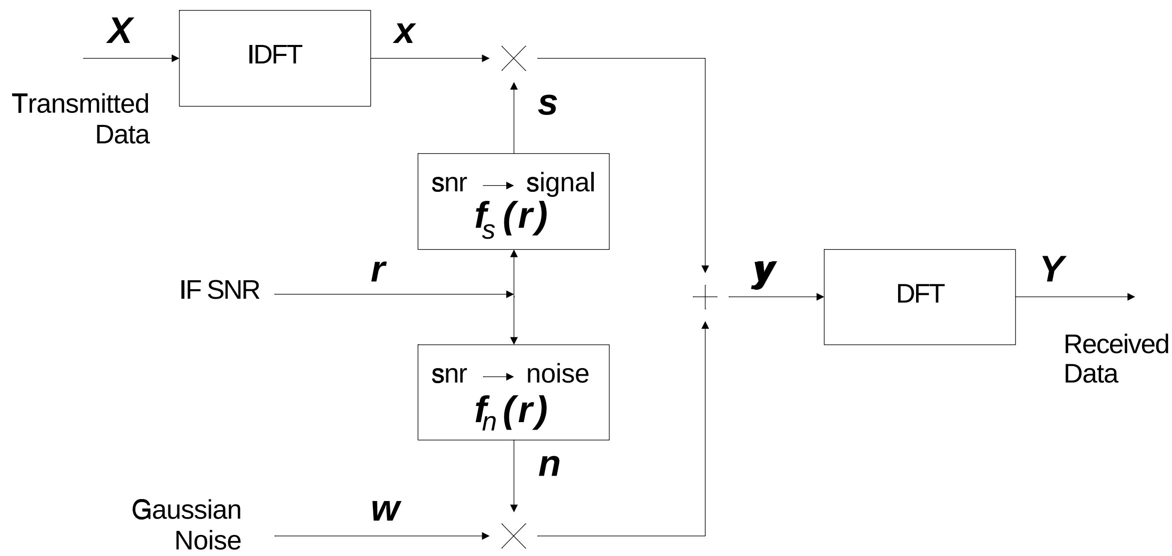

Figure 3.1: The equivalent baseband channel model.

noise gain $\boldsymbol{n}$.

Both $\boldsymbol{s}$ and $\boldsymbol{n}$ are functions of $\boldsymbol{r}$, the IF SNR. Since the IF noise level is assumed fixed, $\boldsymbol{r}$ is proportional to the IF signal level. The functions $\boldsymbol{s} = f_{\boldsymbol{s}}(\boldsymbol{r})$ and $\boldsymbol{n} = f_{\boldsymbol{n}}(\boldsymbol{r})$, define the SN curves of the receiver. Different modulation methods can be modelled by changing these functions. Theoretical or measured SN curves can be used.

This model assumes that the receiver SN curves are memoryless. This allows the average signal and noise powers to be obtained by averaging the instantaneous signal and noise powers. This quasi-static approximation is valid when the Doppler rate is less than the baseband bandwidth [48,53], as is the case with land mobile radio applications. However, the quasi-static model cannot include time-dependent effects such as a delay in the response of an AGC amplifier.

Chapter 3

Modelling and Error Rate Analysis

3.1 Introduction

This chapter proposes a simple model for the mobile radio channel, the *equivalent baseband channel* (EBC) model, and uses it to obtain an expression for the BER of OFDM. The model is shown to apply to the OFDM/FM channel and to the OFDM/SSB channel with perfect carrier synchronization. The independent error assumption is used to predict the word error rate.

3.2 The Equivalent Baseband Channel Model

The model used to analyze the effects of fading on the OFDM signal and to predict the performance of OFDM over mobile radio channels must include all significant effects and yet be kept simple enough for analysis and efficient numerical work. To do this, the EBC model converts the effects of fading at IF (or RF) into equivalent effects at baseband. Two effects are considered: time-varying channel gain (fading), and additive noise.

The notation $\boldsymbol{a}$ is used to indicate a vector $\{a_0, a_1, \ldots, a_{N-1}\}$ and $\langle \boldsymbol{a} \rangle$ is used to denote the time average of the variable $\boldsymbol{a}$ over N samples ($\langle \boldsymbol{a} \rangle = \frac{1}{N}(a_0 + a_1 + \ldots + a_{N-1})$). The word *sample* will be used for quantities in the time domain, while the term *value* will be used for quantities in the frequency domain.

Figure 3.1 is a diagram of the equivalent baseband channel model.

The received samples, $\boldsymbol{y}$, are the sum of received data and noise samples. The received data samples are the transmitted data samples scaled by the channel gain, $\boldsymbol{s}$. The received noise samples are zero-mean Gaussian random variables scaled by the channel

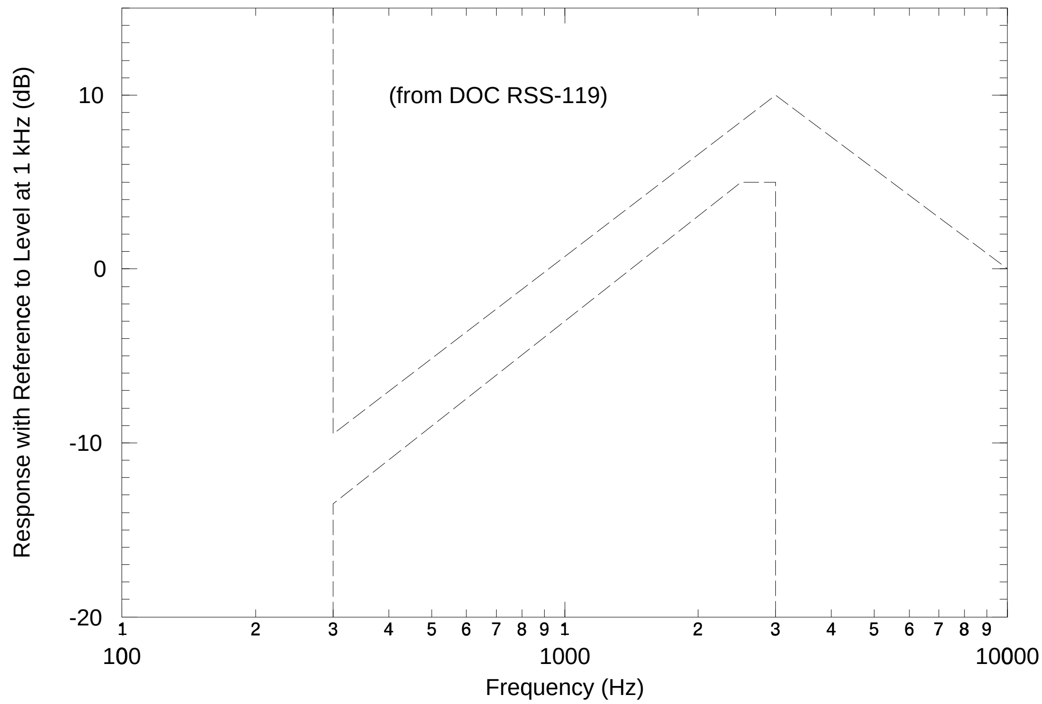

Figure 2.7: Preemphasis filter specification. From [52].

Unlike the signal, the noise is only affected by the deemphasis filter. If the noise has a parabolic spectrum (such as at a high IF SNR), then deemphasis can reduce the total baseband noise power and can flatten the noise spectrum. When the IF SNR is low and the discriminator output noise has a flat noise spectrum, deemphasis concentrates the noise at low frequencies and makes the click noise distribution less impulsive.

at a higher IF SNR. For example, broadcast FM receivers use B of about 180 kHz and W about 15 kHz to improve the baseband SNR [49].

The type of FM demodulator also affects the receiver SN curves. Discriminators and Phase Locked Loops (PLLs) are the two common types of FM demodulators. PLL demodulators have lower thresholds than discriminators [49]. There are also different types of discriminators (quadrature, filter, and pulse-counting) [50,51].

2.4.5 Random FM

Multi-path propagation and the vehicle's motion result in random variations of the received signal phase. These phase changes are detected by the discriminator and cause *random FM* noise. The power of the random FM noise is independent of the received signal level and thus random FM sets a limit on the achievable baseband SNR [48].

2.4.6 Capture Effect

The FM demodulator tends to demodulate only to the strongest signal in the IF passband [3]. This is known as the *capture effect.*

2.4.7 Preemphasis and Deemphasis

The use of preemphasis and deemphasis filtering affects the baseband noise spectrum and the baseband noise power [3,49]. At high IF SNR the noise has a parabolic (f^2) noise spectrum so that there is more noise at higher frequencies.

A preemphasis filter with an f^2 response (+20 dB per decade) is used at the transmitter to increase the power of the high frequency components. Figure 2.7 shows the specifications for the f^2 response of the preemphasis filter [52]. A deemphasis filter (-20 dB/decade) is used at the receiver to remove the effect of preemphasis on the baseband signal.

where c is the peak to rms voltage ratio of the modulating signal.

Figure 2.6 shows this theoretical baseband output signal power (s) and noise power (n) as a function of IF SNR for a discriminator-type FM demodulator without deemphasis. This type of curve will be referred to as a receiver SN curve.

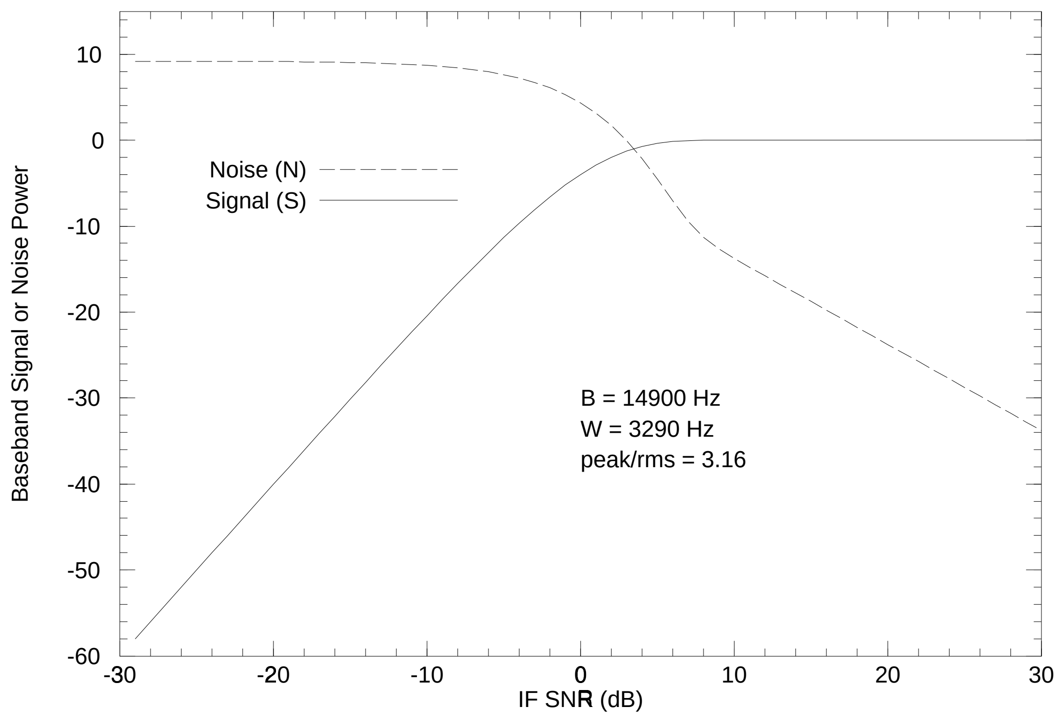

Figure 2.6: The theoretical SN curves of a narrowband FM receiver: receiver output signal power (s) and noise power (n) as function of IF SNR (ρ).

Above a threshold value of IF SNR the output level is constant and the noise power decreases. Below this threshold the output signal level drops and the noise output level becomes constant.

The ratio of the IF bandwidth to the baseband bandwidth affects the location of the threshold and the shape of the SN curves. The values used in Figure 2.6 (B =14.9 kHz, W =3.3 kHz) are those of the narrowband receiver described in Chapter 5. For a larger ratio of B/W there is more SNR improvement above threshold but the threshold occurs

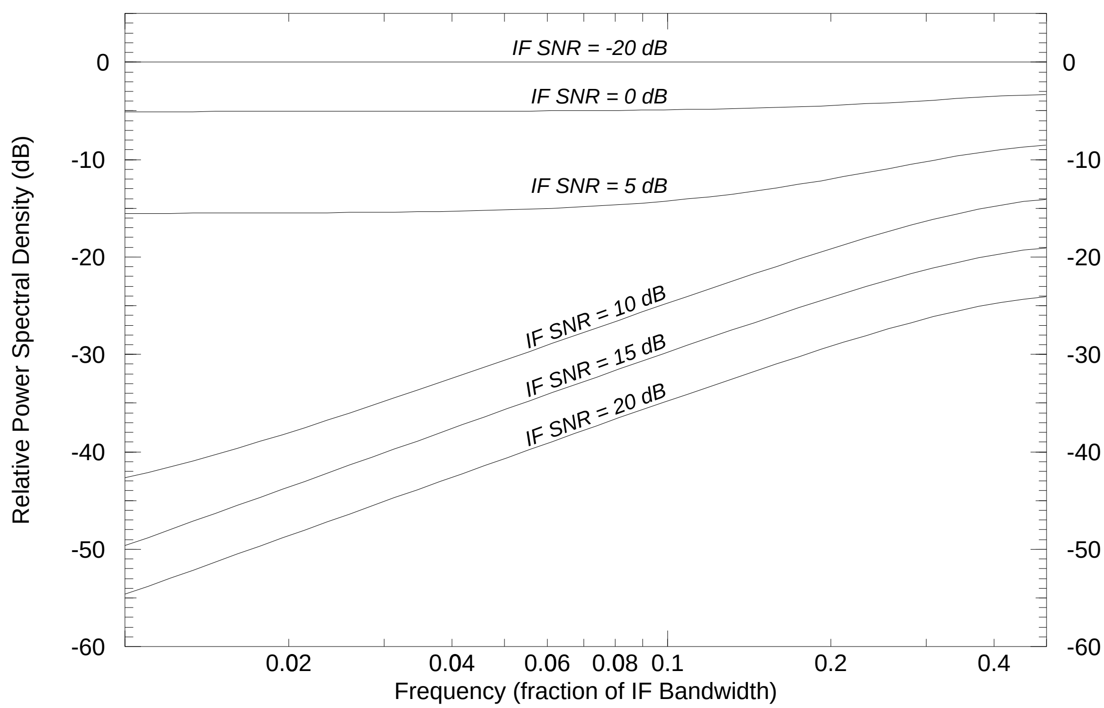

Figure 2.5: Baseband noise spectra for different IF SNRs.

2.4.4 Receiver SN Curves

The baseband noise spectrum can be integrated over the baseband filter bandwidth to obtain the total baseband noise power [3].

The resulting FM baseband noise power level is:

$$N(\rho) = \frac{a(1-e^{-\rho})}{\rho} + \frac{8\pi BWe^{-\rho}}{\sqrt{2(\rho+2.35)}} \tag{2.7}$$

where

$$a = \frac{4\pi^2W^3}{B}\sum_{n=0}^{\infty}\frac{(-\pi)^n}{n!(2n+3)}\left(\frac{W}{B}\right)^{2n} \tag{2.8}$$

and W is the baseband filter noise bandwidth.

The FM baseband signal level is:

$$S_o(\rho) = \frac{\pi^2}{c}(B-2W)^2(1-e^{-\rho})^2, \tag{2.9}$$

Figure 2.4 is a block diagram of an FM receiver. The received RF signal is amplified and converted down to IF (Intermediate Frequency). The IF filter has a noise bandwidth, B, which is about 15 kHz for narrowband FM systems. The limiter removes amplitude variations. The discriminator demodulates the FM signal to produce the baseband signal. The baseband signal is then passed through a -20 dB/decade deemphasis filter (see Section 2.4.7). The noise bandwidth[2] of this filter, W, is about 3 kHz. A squelch circuit shuts off the output when the signal level drops below some threshold.

2.4.3 Discriminator Output Noise Spectrum and Probability Distribution

The IF SNR affects the spectrum and the probability distribution of the baseband noise.

When the IF signal level is much higher than the noise level, the noise added to the FM signal causes small phase changes. The discriminator, a differentiator, acts as a high-pass filter. In this case the baseband noise has a Gaussian probability distribution and a power spectrum that increases 6 dB per octave.

When the IF signal level is comparable to the noise level or lower than it, the noise will often be large enough to cause extra zero crossings of the limited signal. These appear as sudden changes of frequency and are demodulated as noise impulses. In this case the baseband *click noise* has a more impulsive distribution and a flat (white) noise spectrum.

The power spectrum of the noise can be approximated by [3,48]:

$$\mathcal{W}(f) = \frac{[2\pi f(1-e^{-\rho})]^2}{B\rho} e^{-\frac{\pi f^2}{B^2}} + \frac{8\pi B e^{-\rho}}{\sqrt{2(\rho+2.35)}} \tag{2.6}$$

where ρ is the IF SNR and B is the IF filter noise bandwidth. The first term increases as f^2 and dominates at high SNR. The second term is independent of f and dominates at low SNR. Figure 2.5 shows how the noise spectrum changes with IF SNR.

[2] Noise bandwidth is defined in Appendix D.