Centralized Power Control
for
Cellular Radio Systems

Ed. Casas

June 9, 1997

Abstract

Various power control algorithms have been proposed to optimize carrier-to-interference ratios in cellular systems. This talk compares two types of CPC algorithms: CIR balancing and linear optimization.

Outline

- Cellular Systems and Frequency Reuse
- What is Power Control?
- Why Power Control?
- What is Optimum Power Control?
- Previous Work

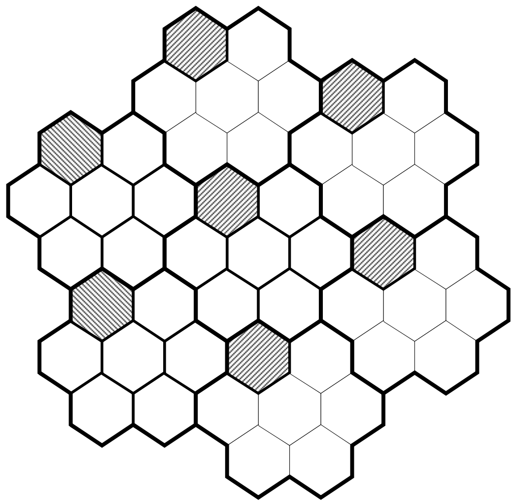
- Example System
- Notation
- CIR Balancing
- Eigenvalue Solution
- Linear Optimization
- Linear Programming Solution
- Possible Research Topics
- Summary

Cellular Systems and Frequency Reuse

- early radio telephone systems used high antennas and high power to serve a whole city from a single base station and each channel could only be used once in each city
- current cellular systems use lower antennas and lower transmit powers to allow each channel to be re-used many times within the same city
- frequency re-use increases number of calls that can be accommodated in the same city
- although there are some variations due to terrain, user density and available cell sites, cellular systems tend to use simple, geometric patterns to establish frequency re-use

Example of Frequency Re-Use

For example, hexagonal cells may be grouped in clusters of 7 cells and each frequency is used in one cell in each cluster:



What is Power Control?

- both the base and mobile transmitter powers can be adjusted dynamically over a wide range
- typical cellular systems adjust their transmitter powers based on received signal strength
- this method adjusts for differences in path loss as users move closer or further from their base stations
- there is no attempt to simultaneously optimize transmitter powers for all users

Why Power Control?

- in a cellular system the quality of a call is usually determined by the carrier-to-interference ratio (CIR)

- traditional re-use distances are selected to maintain an acceptable CIR under worst-case situations (e.g. all users on the edges of their cells) with simple power control

- proposed "optimum" power control schemes adjust transmitted powers dynamically so as to meet CIR requirements

- this results in:
  - reduced power consumption
  - reduced intra-system interference to improve call quality

  - reduced out-of-system interference to help meet regulatory requirements

- in the case of DSSS CDMA systems or where the frequency re-use distances can be reduced dynamically (Dynamic Channel Assignment) power control can also help to increase system capacity

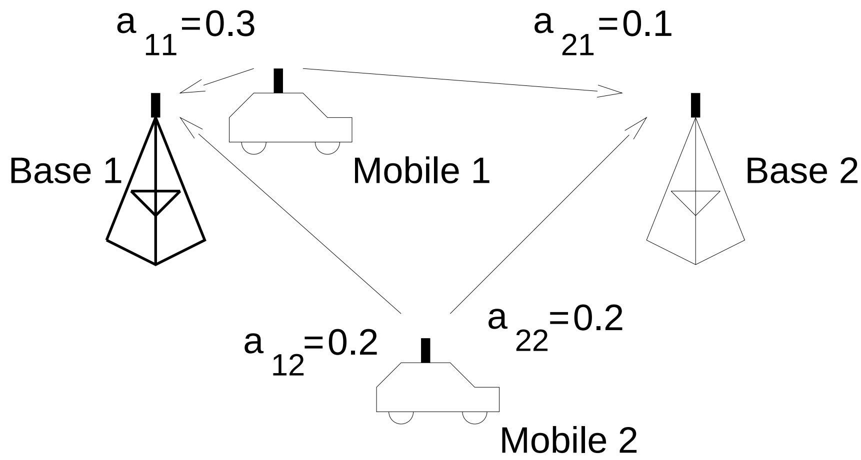
What is Optimum Power Control?

- two different optimality criteria have been proposed in the literature:
  - equal CIR for all users (“CIR Balancing”)
  - minimize total power subject to a set of constraints (minimum CIRs, thermal noise, min/max power levels, etc) expressed as linear functions of transmitter powers (“linear optimization”)
- power control algorithms can be distributed (using information for one link only) or centralized (using information for all links)
- I will only consider centralized power control schemes because:
  - these have the potential for the best performance
  - computing and backhaul costs are decreasing while demands on spectrum are increasing

Previous Work

- original work on linear optimization by Bock & Ebstein (1964)
- original work on CIR balancing by Aein (1973)
- recently renewed interest mainly in CIR balancing for DSSS CDMA systems

Example System



Notation

- we define the fraction of the signal transmitted from mobile j that is received by the receiver for mobile i as a_{ij}

- these "attenuations" can be written as a matrix $\boldsymbol{A}$ where row i corresponds to attenuations from all mobiles to the receiver for mobile i. For the example 2-link case:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

- for future convenience we also define a matrix $\boldsymbol{X}$ by first scaling each row so that the attenuation for the desired mobile is 1 ($x_{ii} = 1$) and then setting $x_{ii} = 0$. For example:

$$\boldsymbol{X} = \begin{bmatrix} 0 & a_{12}/a_{11} \\ a_{21}/a_{22} & 0 \end{bmatrix} = \begin{bmatrix} 0 & x_{12} \\ x_{21} & 0 \end{bmatrix}$$

- define power transmitted by mobile i as p_i or as vector $\boldsymbol{P}$:

$$\boldsymbol{P} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

- now $\boldsymbol{XP}$ gives the normalized received interference power:

$$\boldsymbol{XP} = \begin{bmatrix} x_{12}p_2 \\ x_{21}p_1 \end{bmatrix}$$

Example

For the example system:

$$\boldsymbol{A} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}$$

$$\boldsymbol{X} = \begin{bmatrix} 0 & 0.666 \\ 0.5 & 0 \end{bmatrix}$$

CIR Balancing

- the condition that all users must see the same CIR can then be written as:

$$\boldsymbol{X}\boldsymbol{P} = \lambda \boldsymbol{P}$$

where λ is the interference-to-carrier power (CIR^{-1})

- this is an eigenvalue problem
- the feasible CIRs will be given by the values of λ (eigenvalues) and the corresponding values of $\boldsymbol{P}$ will be the transmitter powers (eigenvectors)
- the transmitter powers can be scaled without affecting the CIRs

Eigenvalue Solution

- we can solve simple problems by finding roots of characteristic equation
- for more complex problems can use numerical methods
- for the example $\boldsymbol{X}$ given above

$$\lambda = \pm 0.577$$

and the only feasible solution, $\lambda = 0.577$, results in a CIR of 1.8 (2.5 dB) and

$$\boldsymbol{P} = \begin{bmatrix} 0.756 \\ 0.654 \end{bmatrix}$$

Linear Optimization for CPC

- a more general approach to CPC can be obtained using the optimization technique called Linear Programming (LP)

- LP minimizes a cost function subject to any number of constraints which are linear functions of positive quantities (in our case, transmitter powers)

- the simplest choice is to use the total transmitted power as the cost function to be minimized:

$$\text{cost} = \sum_i p_i$$

- in addition, we can specify any number of constraints that are linear functions of the transmitter power.

- for example, using the same notation as above, we can make sure each user obtains a minimum CIR of λ^{-1} by using the constraints:

$$\boldsymbol{XP} < \lambda \boldsymbol{P} \quad \text{or} \quad (\boldsymbol{X} - \lambda \boldsymbol{I})\boldsymbol{P} < 0$$

  where λ is now a constant determined by system requirements rather than an unknown

Linear Programming Solution

- there is an efficient algorithm, the simplex algorithm, for solving linear programming problems
- need to set additional constraints (e.g. minimum transmit power or thermal noise) to obtain non-trivial solutions
- for example, setting $\lambda = 0.8$ (CIR = 1.25 or 1 dB) and adding the constraint that all powers must be 1 or greater, we obtain the following constraint equation:

$$\begin{bmatrix} -0.8 & 0.666 \\ 0.5 & -0.8 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \boldsymbol{P} <= \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \end{bmatrix}$$

- the minimum cost solution found using the simplex method is

$$\boldsymbol{P} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- the resulting CIRs are 1.5 and 2
- this solution meets the constraints but the CIRs are not equal

Some Possible Research Topics

- most cellular systems include both distributed and centralized control elements. It would be useful to come up with PC algorithms with combined centralized and distributed features.

- although dynamic channel assignment (DCA) and PC are closely related, most algorithms concentrate on aspect or the other. It would be useful to look for algorithms that do both jobs concurrently and efficiently.

Summary

- power control can improve the performance (call quality, power consumption, and possibly capacity) of cellular radio systems

- the first CPC method, CIR balancing, uses an equal-CIR criteria and requires solving an eigenvalue problem

- the second CPC method, linear optimization, is more flexible and involves using linear programming

- there are several interesting topics related to PC that could be investigated