Inter-Symbol Interference

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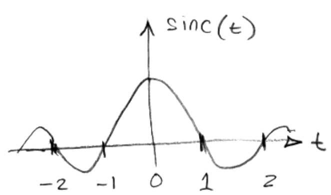
What limits the speed at which we can transmit symbols over a bandwidth-limited channel?

We know that the output of a channel is the convolution of the channel's impulse response and the input to the channel. The impulse response of the channel thus extends the duration of each transmitted symbol in time and into subsequently-transmitted symbols. If the amplitude of the impulse response has a significant amplitude at lags on the order of the bit duration then there is a possibility that each symbol will interfere with subsequent symbols. This interference is called inter-symbol interference (ISI).

Nyquist no-ISI Criteria

There are impulse responses that are zero at periodic time intervals. A typical example is the sinc function:

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



If we can force the channel's impulse response to be zero at multiples of the symbol period then the ISI from previous impulses at multiples of the symbol period will be zero and we will have no ISI at these times. This condition is called the Nyquist no-ISI condition.

Note that the channel impulse response is the response of the channel to impulses, not to arbitrary waveforms. But practical systems transmit waveforms other than impulses. Therefore the symbol's waveform must be considered a result of an (im)pulse-shaping filter that is part of the overall impulse response that must meet the Nyquist no-ISI condition.

In order to have no ISI at the receiver, we must treat this pulse-shaping filter, and any filtering done at the transmitter, the channel and the receiver all together as part of the channel.

Exercise 1: What is the impulse response of a pulse-shaping filter when the symbols consist of pulses of duration T? What is this pulse-shaping filter's transfer function?

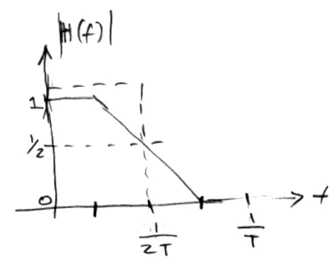
Working in the time domain to evaluate the overall impulse response would require convolving the various impulses responses. It is usually more convenient to work in the frequency domain where we can simply multiply the transfer functions.

It is possible to derive the corresponding no-ISI condition for the channel's transfer function. The Nyquist no-ISI condition is that the channel frequency response have odd symmetry around half of the symbol frequency. For positive frequencies the condition is:

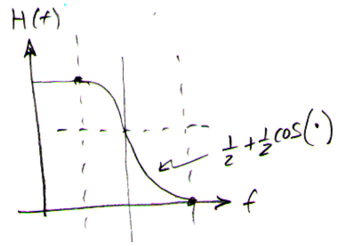
$$H(\frac{1}{2T}+f)+H(\frac{1}{2T}-f) = 1 \text{ for } 0 < f < \tfrac{1}{2T}$$

Exercise 2: What is the transfer function of a channel with infinite bandwidth? Does this channel meet the Nyquist no-ISI condition?

Just as there are many impulse responses that meet the no-ISI condition, there are also many no-ISI transfer functions. For example, the following two straight-line transfer functions meet the no-ISI condition:



The dashed line is a "brick-wall" filter whose response is 1 below half of the symbol rate and zero above that. Although such a filter would have the minimum overall bandwidth, it is not physically realizable and has other problems as described below. The filter with the linear transfer function is also difficult to implement. A more practical transfer function is the so-called raised-cosine function which is a half-cycle of a cosine function centered around half of the symbol rate.



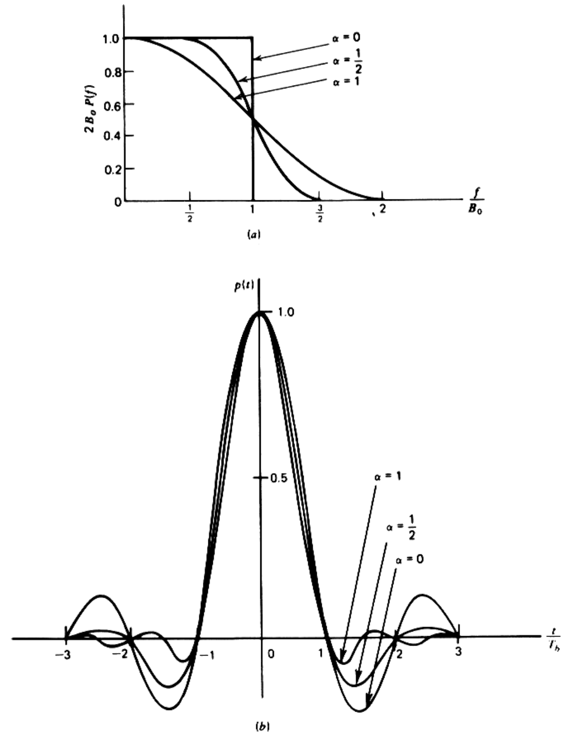
Exercise 3: What is the argument of the cosine function in the diagram above?

Note that it is symmetry about $1/2T$ that ensures there will be no ISI rather than the exact filter shape. Thus we are free to implement other transfer functions, possibly arbitrary ones, if they make the implementation easier.

Excess Bandwidth

However, the width of the transfer function in the transition between passband and stopband does have an impact on the shape of the impulse response and on the sensitivity of the receiver to errors in the timing of the sampling point.

This parameter, α, is called the "excess bandwidth. The following diagram[1] shows how the excess bandwidth parameter for a raised-cosine transfer function affects the impulse response.



Larger values of excess bandwidth result in smaller values of the impulse response which in turn reduces the amount of ISI near the sampling point. This makes the receiver less sensitive to errors in the timing of the sampling point.

Exercise 4: What is the possible range of values of α?

Equalization

To avoid ISI, the total channel response, including the pulse-shaping filter, the transmit filters, the channel and the receiver filter(s) have to meet the Nyquist no-ISI condition.

When the channel is unlikely to meet the no-ISI conditions, the transmitter and receiver typically use filters, known as "equalizers", to ensure the no-ISI condition is met.

Transmitter and receiver filters typically have other functions beside equalization. For example, the transmit filter may limit the bandwidth of the transmitted signal to limit interference to adjacent channels. The receiver filter may filter out noise and interference from adjacent channels and thus improve the SIR and SNR. The communication system designer would design the transmitter and receiver fil-

[1]From Simon Haykin, "An Introduction to Analog and Digital Communication", 1989.

ters to meet both the filtering and equalization requirements.

A common situation is a flat channel where interference is not an issue. In this case a reasonable approach is to put half of the filtering at the transmitter and half at the receiver. In order to achieve an overall raised cosine transfer function, each side has to use a "root raised cosine" (RRC) transfer function. The product of the two filters is thus the desired raised-cosine function which meets the no-ISI condition.

Exercise 5: Could equalization be done at the transmitter only? Why or why not?

Adaptive Equalizers

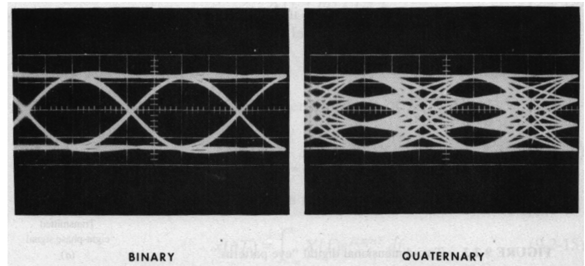
In many communication systems the transfer function of the channel cannot be predicted ahead of time. One example is a modem used over the public switched telephone network (PSTN). Each phone call will result in a channel that includes different "loops" and thus different frequency responses. Another example is multipath propagation in wireless networks. The channel impulse response changes as the receiver, transmitter or objects in the environment move around.

To compensate for the time-varying channel impulse response the receiver can be designed to adjust the receiver equalizer filter response. Various algorithms can be used. Some require a known "training sequence" from which the channel impulse response or equalizer filter parameters can be estimated. "Blind" equalizers can adjust the equalizer filters without knowing the data that is being received (for example, by maximizing the eye opening).

Eye Diagrams

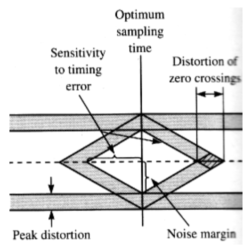
An eye diagram is superimposed plots of duration T of the received waveform (for random data). The eye diagram graphically demonstrates the effect of ISI. Some examples of eye diagrams produced by an appropriately-triggered oscilloscope[2]:

[2] From John G Proakis, "Digital Communications", 3rd Ed., 1983.



The vertical opening at the sampling time, called the "eye opening", represents the amount of ISI at the sampling point.

The horizontal opening indicates how sensitive the receiver would be to errors in sampling point timing[3]:



OFDM

And alternative to adaptive equalization is to use a technique called Orthogonal Frequency Division Multiplexing (OFDM). OFDM transmits many (typically 64 or more) symbols in parallel at different "subcarrier" frequencies. This reduces the symbol period by an amount equal to the number of parallel streams without reducing the overall bit rate. OFDM systems insert a "guard time" between symbols that is longer than the duration of the impulse response of the channel. This minimizes interference between symbols. OFDM has become more popular than adaptive equalization because it is simpler to implement and more robust. OFDM is used by most ADSL, WLAN and 4G cellular standards.

ISI and Shannon Capacity Bounds

Note that the symbol rate limitations resulting from ISI do not by themselves limit the achievable bit rate or the capacity of the channel. For example, once the ISI is controlled we can use an arbitrarily large symbol

[3] Proakis, op. cit.

set (any value of M) and transmit any number of bits per symbol.

The limitation on symbol rate is also different than the constraint on channel capacity defined by the Shannon bound which takes into account the signal to noise ratio as well as the bandwidth.

Sequence Estimation

Using an equalizer to eliminate ISI at the sampling point and then making a decision based on that sample is not the only way to design a receiver for band-limited channels. In fact, there is no guarantee that such a receiver is optimum in the sense that it has the lowest possible error rate.

For example, another approach is for the receiver to predict the waveforms that would be received for all of possible combinations of the preceding n-symbol sequences for the (known) channel impulse response. The receiver then choose the sequence whose waveform most closely matches the received waveform. This is an application of the "sequence estimation' approach that is used for various purposes. The complexity of a sequence estimator grows as n^M where n is the duration of the impulse response and M is the number of possible symbols. This complexity can be very high.

Exercise 6: A receiver estimates each received symbol based on the waveform received over the six (subsequent) symbol periods. 4-ary symbols are used. How many possible waveforms must the receiver consider for each received bit?