

Quadrature Amplitude Modulation

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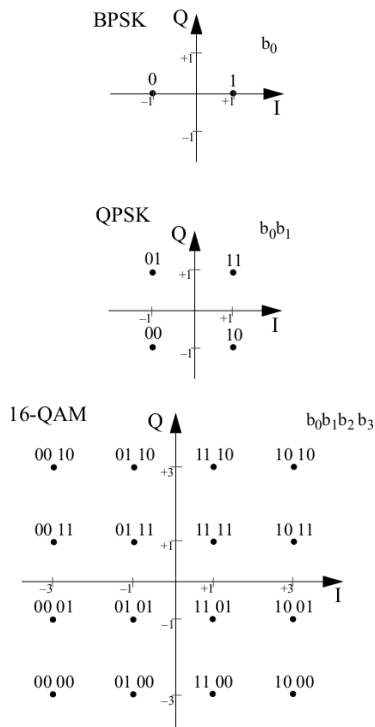
QAM

The simplest type of digital modulation involves transmitting a sequence of waveforms (“symbols”) $s_i(t)$ of equal duration T where each waveform is chosen independently from a set of M . This allows us to transmit up to $b = \log_2(M)$ bits per symbol.

A common sets of such symbols are those where the real and imaginary parts of the complex baseband signal are each modulated in amplitude. This known as Quadrature Amplitude Modulation (QAM).

Constellation Diagrams

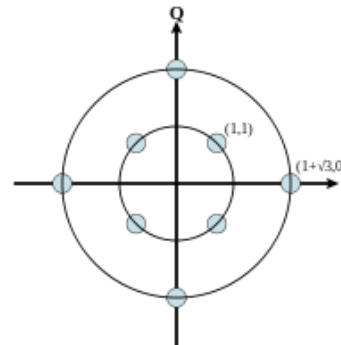
Each of the possible symbols is a combination of a real and a complex value and can be plotted as a point in the complex plane. Each point represents one of the M possible symbols. This plot is called a constellation. For example, the plots below show the constellations used by the 802.11n WLANs as given in the IEEE standard:



QAM modulation with M symbols is known as M -QAM, for example 16-QAM, 256-QAM, etc. The constellations for $M = 2$ and $M = 4$ are the same as those for BPSK and QPSK modulation as explained below. Values of M from 2 up to 1024 (10 bits per symbol) or even more are used. Higher values of M are used on channels with low levels of noise and distortion.

Constellation sizes that are even powers of 2 ($M = 2, 4, 16, 64, \dots$) are typically used to make the constellation the same in both axes and simplify implementation.

However, non-square constellations are also used for low values of M or where maximum power efficiency is desired. For example, here is an example of a non-square constellation for $M = 8$ ($b = 3$ bits/symbol) (from Wikipedia):

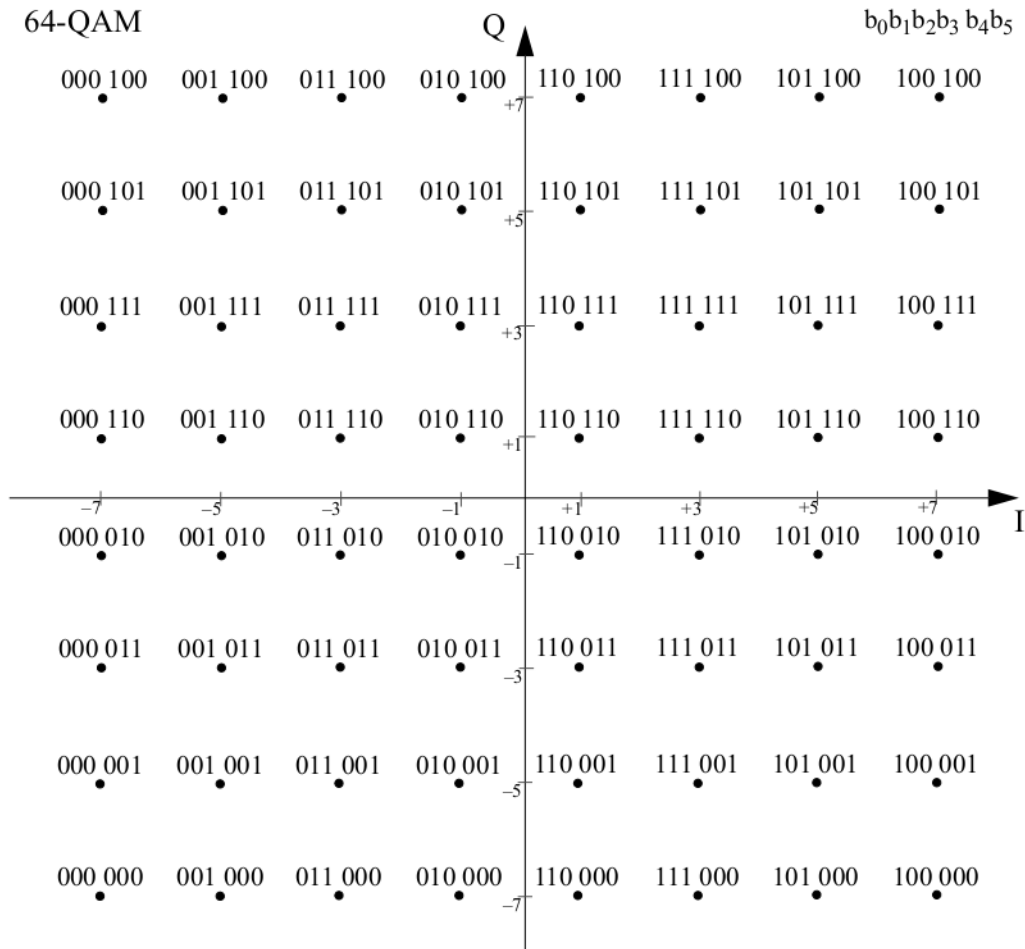


Exercise 1: How many different phases are there for the signal points in the BPSK and QPSK constellations? What might the constellation diagram for 8-PSK look like?

Symbol and Bit Energies

If the amplitude of a symbol is S_i , the power is $|S_i|^2/2$. This is proportional to the square of the distance from the origin to the constellation point. Note that each symbol can have a different power.

The energy of a given symbol is the power times the symbol duration:



Receiver

The receiver decomposes the received signal into real and imaginary components to obtain a complex (2D) received value.

The receiver must also estimate the gain and phase shift introduced by the channel and correct for it. This is part of a process called synchronization.

Ideally, the result is that the received value would match one of the transmitted symbols. However, noise added by the channel, distortion caused by the transmitter or receiver, or synchronization errors will cause the received value to differ from the transmitted value.

Typically the noise is uniformly distributed in angle and all symbols are equally probable. In this case an optimum receiver should choose the constellation point nearest the received value to minimize the symbol error rate.

$$E_{S_i} = T \frac{|S_i|^2}{2}$$

where $|\cdot|$ is the magnitude of a complex value and T is the symbol duration.

Since each symbol conveys b bits, the energy per bit is:

$$E_b = \frac{E_S}{b}$$

The average symbol or bit energy for a given constellation can be computed as the average over all symbols.

Exercise 2: What is the average bit energy for the QPSK and 16-QAM constellations shown above assuming the symbol rate is 1 Msymbol/second? What is the ratio of peak power to average power for each constellation?

We can visualize the receiver as using “decision thresholds” drawn around each constellation point that define when that symbol is selected for a received value.

Exercise 3: Draw the decision thresholds on the QPSK and 16-QAM constellations.

Synchronization can be particularly difficult on wireless channels with multipath propagation because the phase and amplitude are constantly changing. Known “pilot” symbols embedded within packet headers are typically used for synchronization.

Errors

Errors happen when additive noise or distortion cause the received value to be closer to a different constellation point than to the point that was transmitted.

The symbol error rate can be approximated by computing the probability that the noise voltage exceeds the distance to the decision threshold. This happens if either the real or imaginary part of the additive noise causes the received signal component to cross a decision threshold. For typical normally-distributed noise (AWGN), this can be computed using the error ($\text{erf}()$ or $\text{erfc}()$) or Q (“Marcum’s Q ”) functions.

Constellations are usually *gray coded*. This means that the bits corresponding to adjacent points in the constellation differ by only one bit. Since errors between adjacent points are the most likely, gray coding minimizes the bit error rate.

Constant Envelope Constellations and PSK

RF amplifiers typically operate most efficiently at high output levels where the amplifier operates non-linearly and cannot accurately follow changes in signal power (magnitude).

Since QAM signaling modulates both the amplitude and phase of the signal it requires a linear amplifier.

The exception is constellations where all of the points have equal magnitude *and* the amplitude is maintained constant during the time that the carrier transitions between symbols. In this case only the

phase of the signal changes. This type of modulation is called Phase Shift Keying (PSK). QAM with constant envelope for $M = 2$ is called BPSK (binary PSK), for $M = 4$ is called QPSK (Quadrature PSK) and for $M = 8$ is called 8-PSK. We will cover the time-domain issues in a future lecture.

Exercise 4: What is the difference between 4-QAM and QPSK?