Complex Baseband Representation, Power Spectra

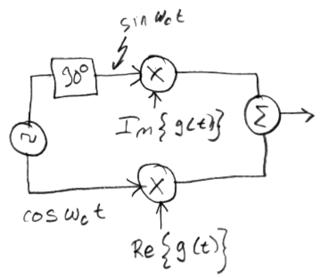
Complex Baseband Representation

The process of modulation can be viewed as shifting the baseband signal to the carrier frequency. Mathematically this can be done by multiplying the baseband signal by a sinusoid:

$$s(t) = g(t)e^{j\omega_c t}$$

where $g(t)$ is the baseband signal and ω_c is the carrier frequency in radians/second. The baseband signal, $g(t)$, represents the envelope of the signal and is, in general, a complex quantity.

When $g(t)$ is complex then the product, $s(t)$, is complex. A realizable modulator implementation only generates and transmits the real component of the product, $\text{Re}\{s(t)\}$:



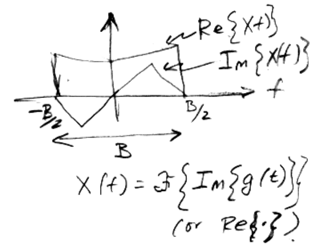
The real and imaginary parts of $g(t)$ can be recovered from $\text{Re}\{s(t)\}$ as long as the carrier frequency is high enough.

This "complex baseband" representation of the modulated signal is often used to describe modulated signals. The complex baseband signal retains all of the information in the modulated signal in the minimum possible bandwidth. For this reason many receivers and transmitters operate on the complex baseband representation of the signal.

Similarly, it is possible to derive the complex baseband equivalent of a bandpass channel. In most cases we will describe signals and channels using their complex baseband representations.

Unlike real (non-complex) baseband signals, complex baseband signals do not have even/odd amplitude/phase symmetry which means that their spectra must be specified for both positive and negative frequencies.

The Nyquist sampling theorem applies to complex baseband signals. We can imagine that the real and imaginary components of a complex signal of bandwidth B extending from $f = -B/2$ to $f = +B/2$ are each sampled separately at a rate of at least B. The resulting complex samples at a rate $\geq B$ are sufficient to reconstruct the continuous complex signal of bandwidth B. Note that the total sampling rate, counting both the real and imaginary samples, is still $2B$.



Exercise 1: A complex baseband modulating signal is $g(t) = e^{j1t} + e^{-j2t}$. What is the spectrum of the baseband signal? What is the equation of the transmitted signal? What is the spectrum of the transmitted signal? What is the minimum Nyquist sampling rate for complex sampling of the baseband signal?

Power Spectral Density

The power spectral density of a signal is defined as the Fourier transform of its autocorrelation function:

$$S(f) = \mathcal{F}(R(\tau))$$

where:

$$R(\tau) = E[s(t)s^*(t-\tau)]$$

This definition is useful since it applies for both periodic and non-periodic signals as well as deterministic and random signals. However, for deterministic signals it's often easier to compute the power spectral density as the magnitude squared of the spectrum of the signal.

Exercise 2: That is the autocorrelation function of white noise? What is the power spectrum? What is the autocorrelation function of a sine wave? What is it's power spectrum? A unit pulse extending over $\pm T/2$?