Multipath Propagation

Small-Scale Fading

Propagation will typically happen by a combination of paths that might include line of sight, reflection, diffraction and scattering. The received signal will the vector sum of the signals received from all paths.

Over distances of a few wavelengths, typically less than a few meters, the path loss due to these mechanisms will not change very much.

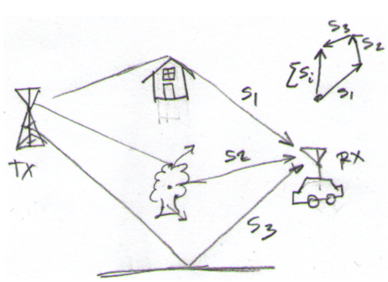
However, the directions of arrival and magnitudes of the signals arriving by each of these paths can be quite different. The result is that the vector sum can change significantly over distances on the order of a wavelength.

This type of fading is known by various names:

- *small-scale fading*: because it changes over distances on the order of a wavelength
- *fast fading*: because it changes quickly relative to fading effects that are a function of distance
- *"multipath" fading*: because this sort of fading only happens when there are multiple components
- *"Rayleigh"* fading: because this is the most common statistical distribution of the envelope that results from this type of fading

Physical Model of the Multipath Channel

Regardless of the propagation mechanism, we can model the received signal as the sum of multiple delayed versions of the transmitted signal.



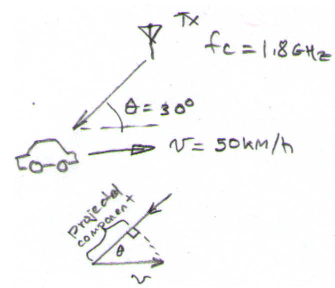
Doppler Shifts

In some cases the path lengths may be changing because the receiver and/or transmitter are moving relative to the objects. This causes the received phase to change at a constant rate which is the same as a frequency shift. This frequency shift is called a "Doppler" shift and the frequency difference is called the Doppler shift.

The Doppler shift is given by:

$$f_D = \frac{v}{\lambda} = \frac{v}{c} f_c$$

where c is velocity of light, v is the rate of change of path length, and f_c is the frequency of the signal with wavelength λ.



Exercise 1: A receiver in a car receives a 1.8 GHz signal while travelling on a road at 50 km/h. The road is at an angle of 30 degrees relative to the direction of arrival of the signal. What is the velocity relative the direction of arrival of the signal? By how much does the path length change each second in meters? In wavelengths? What is the Doppler shift?

Dispersive Fading

If the delays differ by more than a significant fraction of the symbol period then the signal will be distorted by inter-symbol interference (ISI). Otherwise, the signal level will be affected but the waveform will not be significantly distorted.

Another way to look at this distinction is to consider the situation in the frequency domain. Since the phase shifts are a function of frequency as well as time, the vector sum and thus the fading will also be a function of frequency. The sum of the various delays has an effect similar to an FIR digital filter. If the bandwidth of the signal is significantly smaller than the "bandwidth" of the channel then the signal will not be distorted, otherwise it will.

The situation where the fading does not affect the signal's spectrum is called frequency-flat (or just "flat") fading; the other situation is called "frequency-selective" fading. The channel that exhibits the latter type of fading is also a "dispersive" channel because the signal is dispersed (spread) in time.

Measures of Dispersion

The impulse response of the channel will, in general, not have a well-defined shape. We can quantify the amount of dispersion using various metrics including the "mean excess delay" and the "rms delay spread" of the channel impulse response which are the first and second central moments of the impulse response.

When computing either delay spread measure the minimum observed delay should be subtracted out since it has no effect on dispersion.

Similarly, the frequency response of the channel will, in general, not have a well-defined shape and we can define a quantity called the "coherence bandwidth" which can be used to quantify the frequency selectivity of the channel. The coherence bandwidth is the frequency range over which the fading at two frequencies are well-correlated.

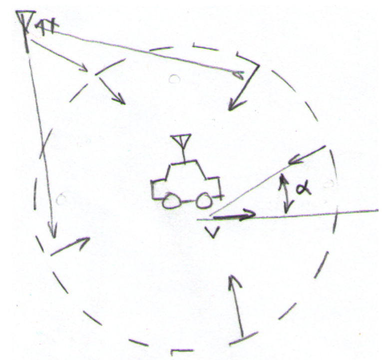
It is also possible to define a "coherence time", related to the Doppler rate, during which the fading on the channel is well-correlated.

Note that the frequency selectivity (dispersiveness) of the channel and how the channel changes with time (the "Rayleigh" fading) are independent of each other. The former is a function of the path lengths (and thus delays) while the latter is a function of velocity of objects. Thus it is possible to have a dispersive channel that does not experience fading and flat-fading (non-dispersive) channel. The nature of the channel and its effect on the signal thus depends on the propagation environment and motion through it.

Flat-Fading Model

Clarke developed a simple model whose predictions agree reasonably well with the statistics of flat-fading NLOS channels.

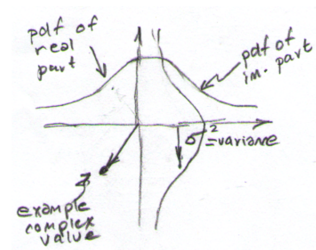
The model consists of a large number of signal paths arriving from directions that are uniformly distributed in a circle around a moving receiver. Each path has equal loss (amplitude) but random phase (0 to 2π). Since the sources are equidistant, all delays are equal and the channel is flat (not dispersive).



Envelope Distribution

The probability distribution of the amplitude of the signal can be obtained by decomposing the (complex) vector sum of the different paths into real and imaginary components. According to the Central Limit Theorem the real and imaginary components will then be normally distributed since each is the sum of a large number of independent random variables.

The probability density function of the signal envelope (magnitude of a complex r.v.) whose real and imaginary components are normally distributed is Rayleigh. The following diagram tries to show the pdfs of the real and imaginary components and a sample value drawn from the distribution:

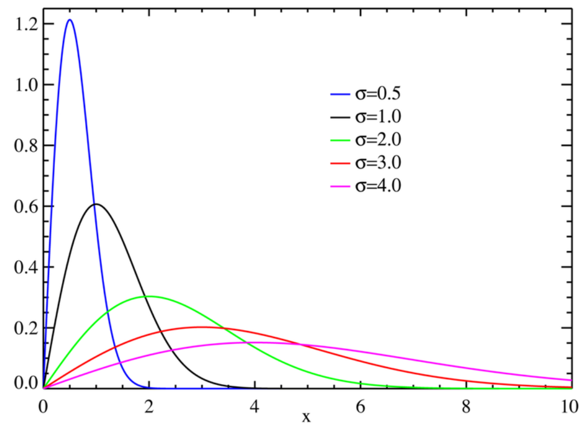


The Rayleigh pdf has the form:

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) & r \geq 0 \\ 0 & r < 0 \end{cases}$$

The Rayleigh distribution has only one parameter, σ^2, which is the variance of each component (real or imaginary) of the signal. The power of the complex, zero-mean signal is twice this, $2\sigma^2$, and the rms value of the signal is $\sqrt{2}\sigma$.

The following plot of the Rayleigh pdf is from the Wikipedia article:



The corresponding cumulative distribution of a Rayleigh random variable is:

$$\Pr(r \leq R) = \int_0^R p(r)dr = 1 - e^{-\rho^2}$$

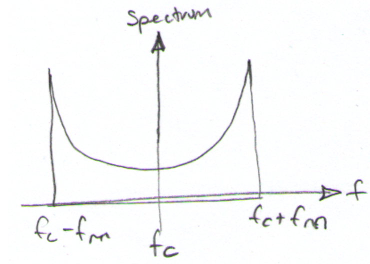
where $\rho = R/R_{rms}$ is normalized to the signal's rms level, $R_{rms} = \sqrt{2}\sigma$.

Exercise 2: What fraction of the time is a Rayleigh-distributed signal 10dB below the mean? 20dB? 30dB? This is a useful result to remember.

The mean of the Rayleigh-distributed *envelope*, a real, positive-valued random variable, can be derived to be 1.2533σ and its variance can be derived to be $0.4292\sigma^2$.

Doppler Spectrum

The signal will be spread in frequency due to the Doppler shifts of the (infinite number of) components. Each component will have a Doppler shift proportional to the cosine of the angle α relative to the direction of motion. Assuming an omnidirectional antenna the Doppler spectrum has a "bathtub" shape extending over the range $f_c \pm f_m$ where f_m is the maximum Doppler shift ($f_m = f_c v/c$):



Level Crossing Rate and Mean Fade Duration

From the power spectrum it is possible to derive two useful time-domain statistics. The level crossing rate is the rate at which the received signal level crosses a threshold ρ in one direction. The level crossing rate is:

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

and the average fade duration is:

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

Exercise 3: How often will the signal drop 10dB below the mean if the carrier frequency is 1.8 GHz and the velocity is 100 km/h? On average, how long will each of these fades last?

Ricean Distribution

When there are both line of sight (LOS) and NLOS components the received signal is the sum of a fixed component and a Rayleigh-distributed component. The ratio of the powers of the direct and Rayleigh components is given by the parameter called the "Ricean K factor":

$$K(dB) = 10 \log \frac{A^2}{2\sigma^2}$$

where A is the amplitude of the direct (LOS) component.