OFDM

OFDM Waveform

The OFDM waveform is generated using the inverse Fourier Transform (DFT):

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}$$

where k is the frequency (subcarrier) index, n is the time (sample) index and the X_k are the complex values modulated onto each subcarrier. The symbol period is NT_s where T_s is the sample period.

Exercise 1: What is the frequency of subcarrier k? What is the subcarrier frequency spacing?

OFDM multiplexes multiple carriers (called subcarriers) in the frequency domain. Unlike conventional FDMA techniques, the subcarriers are orthogonal to each other when considered over the symbol period.

The individual subcarrier values can be recovered at the receiver by using a DFT on the received sample values:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

Note that the only differences between a forward and inverse DFT are the level normalization factor of $\frac{1}{N}$ and the sign of the complex exponential. Thus the same hardware can be used for both operations.

The DFT can be computed efficiently using the FFT algorithm. The FFT requires $O(N\log_2 N)$ operations instead of $O(N^2)$ and is significantly more efficient for large values of N.

Exercise 2: How much more efficient is the FFT than the DFT for $N = 64$?

The OFDM receiver does not need filters to separate the subcarriers because the subcarriers are orthogonal over the symbol period.

OFDM is also known as Discrete Multi-Tone (DMT) or Orthogonal QAM (OQAM). It is also known as COFDM as explained below.

Modulation

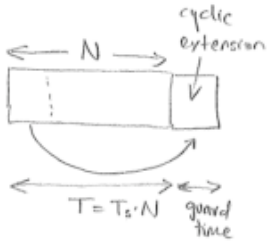
Any type of modulation can be used on the individual subcarriers as long as the modulating value is fixed for the symbol duration. Typically QAM modulation (QPSK, 16-QAM, etc) is used. Differential PSK is also possible with the differential coding either in frequency (between adjacent subcarriers in the same symbol) or time (between the same subcarrier in adjacent symbols).

Exercise 3: In order for differential OFDM to work well what are the requirements on the channel frequency and time response?

For channels where the gain varies widely it can be more efficient to use different constellations at different subcarriers.

Cyclic Extension

To avoid inter-symbol interference due to dispersive channels, a typical OFDM transmitter extends each symbol periodically. This extension is called the guard time. The initial portion of the frame is repeated at the end of the frame. Since the signal is periodic there is no discontinuity due to this extension.



Exercise 4: How long should the guard time be relative to the delay spread of the channel? How long should the symbol period have to be relative to the guard time?

In addition, each symbol can be windowed in the time domain to reduce the level of adjacent channel signal power.

Coding and Interleaving

Dispersive channels are frequency selective and will cause some subcarriers to be attenuated more than others. This will cause errors to happen in bursts near the faded subcarriers.

Convolutional FEC coding is usually applied to the data that is modulated onto the subcarriers. Since convolutional codes perform best with random errors, interleavers are used to randomize the locations of the errors.

A simple type of interleaver is a block interleaver. A block interleaver writes bits row-wise into a two-dimensional matrix and reads them out column-wise.

Exercise 5: What is the order of bits read out of a 3x3 interleaver?

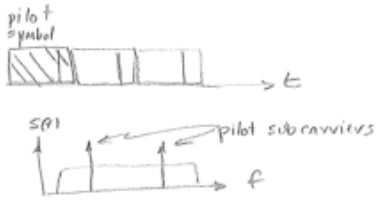
Note that it is the bits that are interleaved, not the subcarriers. This approach of interleaving bits independently of which subcarrier they came from is called bit-interleaved coded modulation (BICM).

The combination of OFDM and coding is called coded OFDM (COFDM).

Receiver Synchronization

The receiver needs to recover symbol timing so that it can decide which samples to process. It also needs to estimate the phase and amplitude shift introduced by the channel on each subcarrier so this can be undone and the original data recovered.

This is done by transmitting pilot symbols periodically (for broadcast systems) or at the start of the frame (for packet-based systems).



The pilot symbols typically have good time-domain correlation properties that the receiver can search for using a correlator. The pilot symbols also have known amplitude and phase values for each subcarrier that can be used to estimate the amplitude and phase of the channel at each subcarrier frequency.

In addition, each symbol typically has pilot tones (subcarriers) that have known amplitude and phase and that can be used to track timing errors.

Amplitude Distribution

The OFDM signal is a sum of a large number of sinusoids with random phases and (possibly) amplitudes. The resulting amplitude distribution is therefore Rayleigh. The peak-to-average power ratio (PAPR) for the OFDM waveform is much higher than for single-carrier modulation such as QAM. Thus the power amplifiers must have a higher dynamic range which in turn leads to higher power consumption.

Added May 7, 2013:

Subcarrier Power and Bit Allocation

Not all subcarriers need to be transmitted at the same power or with the same modulation. Some subcarriers may not be transmitted at all. One reason is hardware limitations – for example the transmitter may not be able to transmit subcarriers at DC the reconstruction/anti-aliasing filters may cause too much attenuation at the highest subcarrier frequencies. Another reason to avoid transmitting on some subcarriers is to ensure the signal meets the power spectral density mask.

In some cases the channel has a higher SNR at some frequencies due to lower loss, less noise or higher allowed transmit power at those frequencies. Some OFDM systems, such as ADSL, transmit higher-order constellations on those subcarriers.

Exercise 6: An OFDM system uses $N = 64$ but only subcarriers $k = 3 \ldots 50$ are used. The sampling rate is 10 MHz. Each subcarrier uses 16-QAM modulation with a rate-3/4 convolutional FEC code. A cyclic extension of 16 samples is used. What is the symbol period? What is the symbol duration including the guard time? Over what frequency range do the subcarriers extend? How many data bits are transmitted per symbol? What is the data rate?