# Additional Notes for Lecture 1

*Antennas are characterized in terms of their directivity, gain, and effective aperture.*

## Power Density

Consider an isotropic radiator transmitting power $P_T$. A sphere of radius $r$ has a surface area of $4\pi r^2$. The power density, $U$ on the surface of sphere centered on an isotropic radiator is thus:

$$U = \frac{P_T}{4\pi r^2}$$

## Directivity and Gain

The directivity of an antenna is the ratio of the maximum power density $U_m$ to the average power density $U_0$:

$$D = \frac{U_m}{U_0}$$

The gain of an antenna is the ratio of the maximum power density to the power density of a lossless reference antenna $U_r$, typically an isotropic radiator:

$$G = \frac{U_m}{U_r}$$

This differs from the directivity of the antenna by the efficiency $k$ (the ratio of of total power radiated to power supplied):

$$G = kD$$

## Effective Area

The effective area of an antenna delivering power $W$ to its terminal when receiving a signal with power density $P$ is:

$$A_e = \frac{W}{P}$$

Antenna gain and effective area are clearly related and we expect both to increase and decrease proportionately. It can be shown that the relationship is independent of the antenna pattern and is given by:

$$D = \frac{4\pi}{\lambda^2} A_e$$

Note that the antenna directivity and gain increase as the square of the frequency. This means that for a given antenna size (area) the antenna becomes more directional as the frequency increases. Conversely, for a given antenna gain the effective area (power collected) decreases with the square of the frequency.

This is the reason that the path loss given by the Friis equation appears to increase as the square of the frequency.