Error Rates

Error Rates in AWGN

This section summarizes the bit error rates for some common modulation methods. In the following error rate equations the signal is of the form:

$$s(t) = \sqrt{\frac{2E_s}{T_s}} \cos(f(t))$$

where E_s is the energy per symbol and T_s is the symbol period. For modulations that transmit b bits per symbol $E_b = E_s/b$.

For BPSK, QPSK or MSK with coherent demodulation:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

For differential BPSK:

$$P_{e,DBPSK} = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

For Non-Coherent FSK:

$$P_{e,NCFSK} = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$

For GSMK:

$$P_{e,GMSK} = Q\left(\sqrt{\frac{2\gamma E_b}{N_0}}\right)$$

where γ depends on BT (0.68 for BT=0.25).

For M-ary QAM:

$$P_{e,M-QAM} \approx 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_{min}}{N_0}}\right)$$

where E_{min} is the energy of the signals with the lowest energy (corresponding to the points closest to the origin in the constellation).

Error Rates over Slow-Fading Channels

When the channel changes slowly relative to the symbol duration we can use a "quasi-static" approximation that the average error rate is the average of the AWGN error rates. Since different SNRs have different probabilities we compute a weighted average error rate:

$$P_{e,avg} = \int_0^\infty P_e(X)p(X)dX$$

where $P_e(X)$ is the probability of error for $E_b/N_0 = X$, and $p(X)$ is the value of the probability density function of E_b/N_0.

When the fading is Rayleigh, then the probability distribution of the power has a Chi-squared distribution with two degrees of freedom (since the I and Q components are independent and normally distributed):

$$p(X) = \frac{1}{\Gamma}\exp\left(-\frac{X}{\Gamma}\right) X \geq 0$$

where Γ is the average value of X (average value of E_b/N_0).

The average error rates in slow Rayleigh fading for some common modulations:

$$P_{e,PSK} = \frac{1}{2}\left(1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right) \approx \frac{1}{4\Gamma}$$

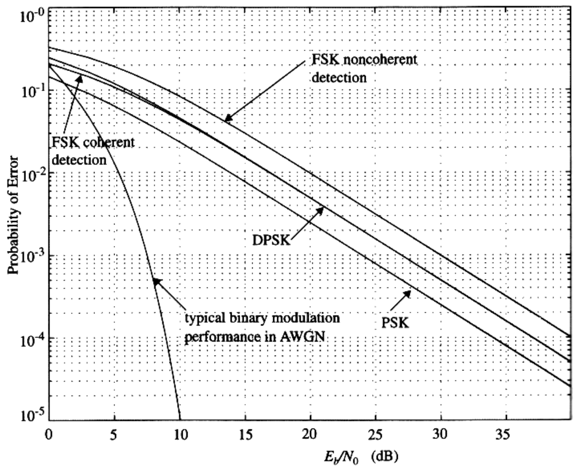
$$P_{e,FSK} = \frac{1}{2}\left(1 - \sqrt{\frac{\Gamma}{2+\Gamma}}\right) \approx \frac{1}{2\Gamma}$$

$$P_{e,DPSK} = \frac{1}{2(1+\Gamma)} \approx \frac{1}{2\Gamma}$$

$$P_{e,NCFSK} = \frac{1}{2+\Gamma} \approx \frac{1}{\Gamma}$$

$$P_{e,GMSK} = \frac{1}{2}\left(1 - \sqrt{\frac{\delta\Gamma}{\delta\Gamma+1}}\right) \approx \frac{1}{4\delta\Gamma}$$

The approximations are valid when Γ is much greater than 1 and the GMSK BT-dependent parameter δ is defined above.



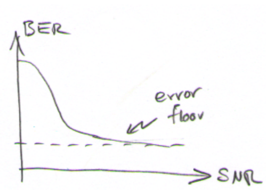
The important result is that the error rates on fading channels have an inverse-linear dependence on the average SNR unlike for AWGN channels where the error rate drops exponentially with SNR.

Since errors typically happen during fades it is reasonable to expect that it is the probability distribution of the fading that determines the error rate.

The slow decrease of the error rate with increasing SNR makes it impractical to achieve low error rates on fading channels simply by increasing the average SNR. Instead, mechanisms must be used to reduce the impact of fading. The most important of these is diversity.

Irreducible Error Floors

Dispersive (or frequency-selective) channels result in inter-symbol interference. The level of this self-interference depends on the channel impulse response rather than the channel SNR. As the channel SNR increases the interference due to ISI becomes more significant compared to the channel noise. At high SNRs the error rate becomes independent of the channel SNR since all of the errors are caused by ISI rather than by additive noise. The result is that the BER vs SNR curve reaches an "irreducible error floor" where the BER is constant and independent of the SNR:



Other mechanisms can create error floors. In general, anything that introduces noise or distortion whose level is independent of the channel SNR can result in an error floor. Typical examples include interference from co-channel users and phase noise caused by fading.