ELEX 7860
FINAL EXAM
20130 SOLUTIONS

Q.1 (a) $f = 12\,GHz = 12\times10^{9}$ $\quad \lambda = \frac{c}{f} = 0.025m$

$d = 36{,}000\,km = 36\times10^{6}\,m$

$$P_L = \left(\frac{\lambda}{4\pi d}\right)^2 = \left(\frac{0.025}{4\pi\ 36\times10^{6}}\right)^2 = 3\times10^{-21}$$

$P_{L(dB)} =$ **205 dB**

(b) $EIRP = P_T G_T$

$P_T = 4\,W = 36\,dBm$

$EIRP_{dB} = 36 + 36 =$ **72 dBm**

(c) $\text{link margin} = \left(\frac{S}{N}\right)_{dB} - \left(\frac{S}{N}\right)_{min} = 3\,dB$

$\left(\frac{S}{N}\right)_{min} = 3\,dB$ = minimum required SNR

$S = \frac{(P_T G_T)}{P_L}\overset{\rightarrow EIRP}{G_R}$, $S_{(dB)} = 72 - 205 + G_R$

$N = KTBF = -174 + B_{(dB\cdot Hz)} + F_{dB}$

$= -174 + 10\log(4\times10^{6}) + 1 = -174 + 66 + 1$

$= -107\ dBm$

$$\left(\frac{S}{N}\right)_{dB} = S_{dB} - N_{dB} = (72 - 205 + G_R) - (-107)$$

$$= G_R - 26\ dB$$

$\text{margin} = (G_R - 26) - (3) = 3$

$G_R = 3 + 3 + 26 =$ **32 dBi**

Q1 (ctd)

(d) $A_e = \frac{\lambda^2}{4\pi} G = \frac{(0.025)^2}{4\pi} \cdot \left(10^{\frac{32}{10}}\right)$ ≈ 1600

$= \underline{\underline{0.079\ m^2}}$

common issues: used −173 vs. −174 for kT (no loss of marks)
$-\frac{1}{2}$ − neglected to include 3dB link margin in addition to 3dB SNR
$-\frac{1}{2}$ − arithmetic errors (even though all numbers are integers!)

Q 2

(a) many different paths & lengths ⇒

many different phases ⇒ Rayleigh magnitude

(b) mean signal power = −60 dBm.

$P(r < R) = 1 - e^{-\rho^2}$

$\rho = \frac{R}{R_{rms}}$ is the voltage ratio of threshold R

to rms voltage R

for $R = -80$ dBm. $\rho = 10^{\left(\frac{-80-(-60)}{20}\right)}$ ← voltage! $= 10^{-1} = 0.1$

$P(r < R) = 1 - e^{-(0.1)^2} \approx \underline{\underline{10^{-2}}}$ $(\approx 1\%)$

for $R = -54$ dBm $\rho = 10^{\left(\frac{-54-(-60)}{20}\right)} = 10^{0.3} = 2$

$P(r > R) = 1 - P(r < R) = e^{-(2)^2} = \underline{\underline{0.018}}$ $(\approx 2\%)$

Q.2 (cont'd)

(c) $v = 3\,m/s \Rightarrow f_m = \frac{v}{c} \cdot f_c = \frac{3}{3\times10^8} \cdot 5\times10^9 = 50\,Hz$

$R = -66dB_m \Rightarrow \rho = 10^{\left(\frac{-66-(-60)}{20}\right)} = 10^{-0.3} = 0.5$

$$N_R = \sqrt{2\pi}\, f_m\, \rho e^{-\rho^2}$$

$$= \sqrt{2\pi} \cdot 50 \cdot \tfrac{1}{2} e^{-\left(\frac{1}{2}\right)^2} = \underline{\underline{49\ Hz}} \quad (\text{fades/second})$$

$$\bar{\tau} = \frac{e^{\rho^2}-1}{\rho\, f_m \sqrt{2\pi}} = \frac{e^{\left(\frac{1}{2}\right)^2}-1}{\frac{1}{2}\,50\,\sqrt{2\pi}} = 4.5\times10^{-3} = \underline{\underline{4.5\ ms}}$$

common errors
- using $Q(\cdot)$ or $erf(\cdot)$ to compute probabilities for Rayleigh R.V. (-1)
- thinking -54 is 14dB above -60 dBm

Q.3 (a) $P(\text{all } N \text{ bits correct}) = \left[P(1 \text{ bit correct})\right]^N = \left[1-BER\right]^N = 1-PER$

$$BER = 1-\left(1-PER\right)^{\frac{1}{N}} = 1-(1-0.01)^{\frac{1}{125\times8}} = \underline{\underline{10^{-5}}}$$

(b) stationary $\Rightarrow$ error rate in AWGN

$$Pe,_{DBPSK} = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

$$E_b = \frac{S}{R} \quad \text{(R} \rightarrow 1\,Mb/s\text{)} \qquad N_0 = \frac{N}{W} \quad (W \rightarrow 1\,MHz)$$

$$\frac{E_b}{N_0} = \frac{S}{N} \cdot \frac{W}{R} = \frac{S}{N}$$

$$P_e = \frac{1}{2} \exp\left(-\frac{S}{N}\right) = 10^{-5}$$

$$\frac{S}{N} = -\ln\left(2 \times 10^{-5}\right) = 11 \approx \underline{10\,dB}$$

common errors:
- using eqn. for fading channels (if stopped ⇒ no fading) (usually)
- using eqn. for coherent BPSK (close but enough)
- using PER for computing SNR $-\frac{1}{2}$
- using 125 instead of 125x8 as exponent $-\frac{1}{2}$
- using fading curves $-\frac{1}{2}$

Q.4

total offered load = $100 \times \frac{15}{60} \times 0.5 = 12.5$ Erlang (100: tenants; 15: hours/call; 0.5: call/hour)

GoS ≤ 1%

blocked calls cleared, exponential duration, "random" arrivals ⇒ Erlang-B blocking probability

from graph of Erlang-B operating curves at 1% blocking & 12.5 Erlang load need ≈ <u>20 channels</u>

$$\frac{20}{24} \simeq \frac{5}{6} \approx \underline{83\%} \text{ of a T1.}$$

- common errors: - not giving answer as fraction of T1 (24)
  - misreading the graph

Q5 cascode

(a) Calculations need to be done in linear units: $X_{lin} = 10^{\frac{X\,dB}{10}}$

specs in linear units

	gain (P.U.) (G)	NF (P.U.) (F)	IP3 (mW) (IIP)
LNA ①	10	2	100
mixer ②	$\frac{1}{4}$	4 (NF is loss, not gain)	1000
IF Ampl. ③	1000	4	1000

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} =$$

$$= 2 + \frac{4-1}{10} + \frac{4-1}{10 \cdot \frac{1}{4}} = 2 + \frac{3}{10} + \frac{3}{2.5}$$

$$= 3.5 = \underline{5.4\ dB}$$

$$\frac{1}{IIP3} = \frac{1}{I_1} + \frac{G_1}{I_2} + \frac{G_1 G_2}{I_3}$$

$$= \frac{1}{100} + \frac{10}{1000} + \frac{10 \cdot \frac{1}{4}}{1000} = 0.01 + 0.01 + 0.0025$$

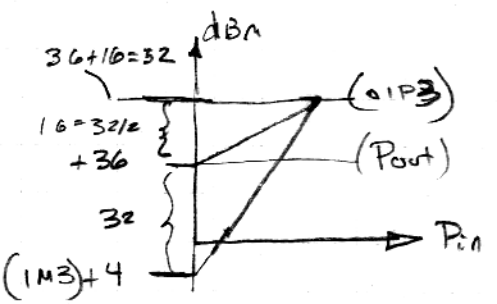
$$= .0225$$

$$IIP3 = 44.4 = \underline{16.5\ dBm}$$

(b) $P_{out} = 4W = 36\ dBm$

$IM3 = 36 - 32 = 4\ dBm$

$OIP3 = 36 + \frac{32}{2} = \underline{52\ dBm}$



common errors:
- converting to dB ($-\frac{1}{2}$)
- using 0.25 as mixer NF ($-\frac{1}{2}$)

Q. 6

(a) 4 codewords ⇒ $\log_2(4) = 2$ bits transmitted per codeword

(b) n = # of bits / codeword = 5 (given)
K = # of data bits = 2 (above).

this is a (5, 2) code.

(c) y receive 11000

differences are:

00110 11000 ✓✓✓✓	d=4
01101 11000 10101	d=3
10010 11000 01010	d=2
11001 11000 00001	d=1

minimum distance (← d=1)

would choose codeword w/ minimum distance:
11001

Since distance is 1, 1 bit error would be corrected.

- common errors:
  - not showing computation of distances (in spite of question saying "show your work") (−1)
  - wrong # of data & parity bits (−1)
  - not specifying n, K values (−1)