Assignment 3 Solutions

1. The Matlab program below computes the SIR distribution for the uplink. The uplink SIR is the ratio of the signal power of the desired mobile user (the one in the center or serving cell) to the sum of the powers of the mobile interferers (the ones in the surrounding cells). All of the student's solutions incorrectly computed the interference power as the sum of the powers that would have been received by the mobile rather than the powers that would have been received by the base.

```
function a = crand1 (n)

% function a = crand1 (n)
%
% return a column vector of n complex values with magnitude <= 1
% that are uniformly distributed in the complex plane

% pre-allocate output array

a=zeros(n,1) ;

% k is indices of the values that need to be generated

k=[1:n].' ;

while length(k) > 0

      % generate values over (-1,1)+j*(-1,1)
      a(k)=2*(rand(size(k))+j*rand(size(k)))-(1+j) ;

      % find indices of values outside unit circle
      k=k(abs(a(k))>1) ;
end

end % crand1
```

```
% asg3.m
% ELEX 7860 Assignment 3 Question 1
% Ed Casas - May 15, 2013

% locations are represented as complex numbers
% all distances are normalized to the cell radius
% center of desired user's cell is at origin (0+0j)
% transmit power assumed =1 for all
% path loss is d^-3

ntrial = 1000 ;

for N=[1 3 7]

    N

    % (optional) center positions of co-channel cells (6x1)

    cp = sqrt(3*N) * exp(i*2*pi*[0:5].'/6) ;

    % desired (same cell) and interfering (co-channel cell) positions
    % (ntrialx1, ntrial*6x1)

    sp = 0.91*crand1(ntrial) ;
    ip = 0.91*crand1(ntrial*6) + repmat(cp,ntrial,1) ;

    % signal and interference (sum over co-channel cells) powers

    sp = abs(sp).^-3 ;
    ip = abs(ip).^-3 ;
    ip = sum(reshape(ip,ntrial,6),2) ;

    % sort trial SIRs in ascending order

    sir = sort(sp./ip) ;

    % show 99, 90 and 50th percentile SIRs in dB

    10*log10(sir(floor(ntrial*[0.01, 0.1, 0.5])))

end
```

The output of the script is:

```
N =  1
ans =

  -2.79875
  -0.49368
   4.45866

N =  3
ans =

    6.4449
    7.8062
   12.0968

N =  7
ans =

   12.601
   13.772
   17.303
```

2. The Q branch output is:

$$Q(t) = \cos(\omega_c t + \phi)(\sin(\omega_c t) + k)$$

and the I branch output is

$$I(t) = \cos(\omega_c t + \phi)(\cos(\omega_c t) + k)$$

Using the identities:

$$\sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

and setting

$$A = \omega_c$$

$$B = \omega_c t + \phi$$

we can find the signals at the two mixer outputs:

$$Q(t) = \frac{1}{2}\left[\sin(2\omega_c + \phi) + \sin(-\phi) + k\cos(\omega_c t + \phi)\right]$$

and

$$I(t) = \frac{1}{2}\left[\cos(2\omega_c + \phi) + \cos(-\phi) + k\cos(\omega_c t + \phi)\right]$$

To recover the baseband value $\cos(-\phi) = \cos(\phi)$ and $\sin(-\phi) = -\sin(\phi)$ we need a low-pass filter that passes the baseband signal but attenuates (filters out) the components at frequency ω_c and $2\omega_c$. An example would be a low-pass filter that passes frequencies below $f_c/2$.

Note that by using $-\sin(\omega_c t)$ as the LO signal for Q we would recover ϕ instead of $-\phi$.

I and Q refer to "in-phase" and "quadrature." On the complex plane the in-phase component (I) is typically shown on the real axis and the quadrature component on the imaginary axis.

3. Solution:

(a) Given $u = 3$ calls/hour, $H = 5$ minutes/call.

$\Rightarrow A_u = u \cdot H = \frac{3}{60} \times 5 = \underline{\underline{0.25 \text{ Erlangs}}}$

(b) Given $C = 1$, $A_u = 0.25$ Erlangs, $GOS = 0.01$.

from the Erlang B chart, we have $A = 0.01$

$\Rightarrow$ number of users, $U = \frac{A}{A_u} = \frac{0.01}{0.25} = 0.04$ users.

But, actually one user could be supported on one channel,

So $U = \underline{\underline{1 \text{ user}}}$

(c) Given $C = 5$, $A_u = 0.25$ Erlangs, $GOS = 0.01$.

$\Rightarrow A = 1.4$ Erlangs $\Rightarrow U = \frac{A}{A_u} = \frac{1.4}{0.25} = 5.6$ users

So $U = \underline{\underline{5 \text{ users}}}$

(d) $U = 5 \times 2 = 10$ users, $\Rightarrow A = U \cdot A_u = 10 \times 0.25 = 2.5$ Erlangs

For $C = 5$, $A = 2.5$ Erlangs $\Rightarrow GOS \doteq \underline{0.07}$

This is not acceptable performance. That means 7 out of 100 calls will be blocked due to channel occupancy during the busiest hour. Generally, $GOS \leq 0.02$ is desired.