Assignment 2 Solutions

The following solutions are taken from the solutions manual for the Wireless Communications *texbook by Rappaport (first edition).*

Question 1

5.11 See the MATLAB program p5_11.m and Fig. p5_11.

$P_{e,BPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$, $P_{e,DPSK} = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$

$P_{e,QPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$, $P_{e,FSK,NC} = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$

BPSK, DPSK and QPSK are all linear constant envelope modulation techniques. They can save bandwidth but are poor in power efficiency. Pulse shaping can make the modulation techniques non-constant envelope and even more bandwidth efficient. BPSK and QPSK all need coherent detection which is more complicated than the non-coherent detection. FSK is a nonlinear constant envelope modulation. Using class C amplifier, it is power efficient but occupies a larger bandwidth than linear modulation schemes, even when pulse shaping is used. FSK techniques are not as bandwidth efficient as linear techniques. FSK can use noncoherent detection.

```
%%%%%%%%%%%%%%%%%%%%%%
%                    %
%       p5_11.m      %
%                    %
%  Chapter 5  5.11   %
%                    %
%%%%%%%%%%%%%%%%%%%%%%

EbOverNo_dB = 0:15;     %  Eb/No (dB);
EbOverNo = 10.^(EbOverNo_dB/10);

Pe_BPSK = Q(sqrt(2*EbOverNo));
Pe_DPSK = 0.5*exp(-EbOverNo);
Pe_FSK_NC = 0.5*exp(-0.5*EbOverNo);

figure(1);
semilogy(EbOverNo_dB,Pe_BPSK,'-',EbOverNo_dB,Pe_DPSK,'--',...
     EbOverNo_dB,Pe_FSK_NC,'-.');
grid;
title(['Comparison of the probability of bit error for several',...
       ' digital signaling schemes']);
xlabel('Eb/No  (dB)');
ylabel('Probability of Error');
legend('BPSK, QPSK','DPSK','non-coherent FSK');
```

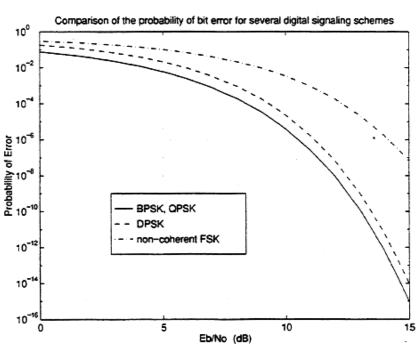

Fig. p5_11

Question 2

5.12 For SNR = 30dB = 1000, B = 200KHz, the maximum possible data rate, $C = B \cdot \log_2\left(1+\frac{S}{N}\right) = 200\times10^3 \times \log_2(1+1000) \doteq \underline{1.99\ Mbps}$

The GSM data rate is 270.833 bps. which is only about $\underline{0.136C}$

Question 3

5.22 We define the RF bandwidth as the band of which everywhere outside, the power spectral density (PSD) is below −40 dB. From Fig. 5.41, we have

For $BT = 0.25$, $B_W = 2\times0.83R_s = \underline{1.66R_s}$

For $BT = 0.5$, $B_W = 2\times1.16R_s = \underline{2.32R_s}$

For $BT = 1$, $B_W = 2\times1.6R_s = \underline{3.2R_s}$

For $BT = 5$, $B_W > 2\times2.5R_s = \underline{5R_s}$

5.22 Cont'd

Since $P_e = Q\left(\sqrt{\frac{2\alpha E_b}{N_0}}\right)$, $\alpha \doteq \begin{cases} 0.68 & \text{for GMSK with } BT=0.25 \\ 0.85 & \text{for simple MSK } (BT=\infty) \end{cases}$

the E_b/N_0 degradation for all these cases will be less than 1dB when compared to the optimum MSK, the larger the BT, the less the degradation.

From the above we can see that when BT decreases, the RF bandwidth becomes small. Although the BER increases, as long as the GMSK irreducible error rate is less than that produced by the mobile channel, there is no penalty in using GMSK.

Question 4

$\boxed{5.27}$ (a) For Rayleigh fading channel, $P_{e,BPSK} = \frac{1}{4\Gamma}$

$$\Rightarrow \Gamma = \frac{1}{4 P_{e.BPSK}} = \frac{1}{4\times 10^{-5}} \doteq \underline{\underline{44\ dB}}$$

(b) $P_e = \int_0^\infty P_e(x)\cdot f(x)dx$, for Ricean fading channel, $f(x) = \frac{1+K}{\Gamma}\exp(-\frac{x(1+K)+K\Gamma}{\Gamma}) I_0(\sqrt{\frac{4(1+K)Kx}{\Gamma}})$, for BPSK, $P_e(x) = Q(\sqrt{2x})$. Therefore

$$P_e = \int_0^\infty Q(\sqrt{2x})\cdot \frac{1+K}{\Gamma}\cdot \exp(-\frac{x(1+K)+K\Gamma}{\Gamma})\cdot I_0(\sqrt{\frac{4(1+K)Kx}{\Gamma}})\,dx.$$

This integral is calculated by the MATLAB program p5_27.m and the result is shown in Fig. P5-27.

From Fig. P5-27, we obtain

if $K=6dB$, for $P_e=10^{-5}$, average $E_b/N_0 = \underline{\underline{34dB}}$

if $K=7dB$, for $P_e=10^{-5}$, average $E_b/N_0 = \underline{\underline{30.5\ dB}}$

Question 5

$\boxed{6.7}$ (a) Since $-6dB = \frac{1}{4}$, $Pr[\gamma_i \le \frac{\gamma}{4}] = 1-e^{-\frac{\gamma/4}{\Gamma}} = 0.2$, where γ is the SNR threshold, we have

$$\frac{\gamma}{4\Gamma} = -\ln 0.8 \Rightarrow \frac{\Gamma}{\gamma} = \frac{1}{-4\ln 0.8} \doteq 1.12 \doteq \underline{\underline{0.5\ dB}}$$

Therefore, the mean SNR of the Rayleigh fading signal is 0.5 dB above the SNR threshold. Using equation (6.59), we have

(b) P_2 (6dB below the mean SNR threshold) $= 0.2^2 = \underline{\underline{0.04}}$

(c) P_3 (6dB below the mean SNR threshold) $= 0.2^3 = \underline{\underline{0.008}}$

(d) P_4 (6dB below the mean SNR threshold) $= 0.2^4 = \underline{\underline{0.0016}}$

(e) From the above we can see that for a M branch selection diversity receiver, the probability that the

6.7 Cont'd

SNR will be 6dB below the mean SNR threshold is $\underline{\underline{0.2^M}}$

Question 6

$\boxed{6.11}$ (a) Based on the definition of y, (it should be more suitable to call y the compliment of the system reliability), we have

$$1-y = \exp[-P^{-1}(x)/\gamma_0] \Rightarrow \gamma_0 = \underline{\underline{\frac{-P^{-1}(x)}{\ln(1-y)}}}$$

(b) $y = \underline{\underline{[1-e^{-\frac{P^{-1}(x)}{\gamma_0}}]^M}}$

6.11 Cont'd

(c) For BPSK, $P_e(\gamma) = Q(\sqrt{2\gamma})$. Given $x=10^{-3}$, we have

$$\gamma_0 = \frac{-\frac{[Q^{-1}(x)]^2}{2}}{\ln(1-y)} \doteq \frac{-\frac{3.1^2}{2}}{\ln(1-10^{-3})} \doteq 4802.6 \doteq \underline{\underline{36.8\ dB}}$$

(d) In this case, $\gamma_0 = \frac{-P^{-1}(x)}{\ln(1-y^{\frac{1}{M}})}$, thus

$$\gamma_0 = \frac{-\frac{3.1^2}{2}}{\ln[1-(10^{-3})^{\frac{1}{4}}]} = 24.54 \doteq \underline{\underline{13.9\ dB}}$$