ELEX 7860 : Wireless System Design
2013 Winter Session

Assignment 1 Solutions

The following solutions are taken from the solutions manual for the Wireless Communications *texbook by Rappaport (first edition).*

Question 1

3.10 Given $P_t = 10W$, $G_t = 10dB = 10$, $L = 1dB \doteq 1.259$.

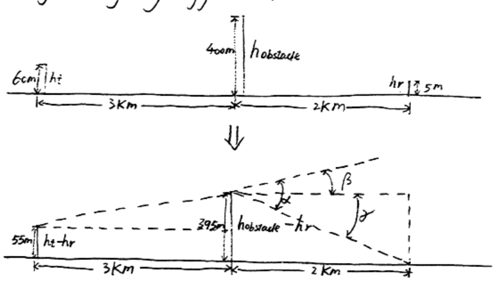
$G_r = 3dB = 2$, $f_c = 900MHz$, $d = 3000 + 2000 = 5000m$.

We have $\lambda_c = \frac{c}{f_c} \doteq 0.333(m)$ and free space received power

$$P_{freespace} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda_c^2}{(4\pi)^2 d^2 \cdot L} = \frac{10 \times 10 \times 2 \times 0.333^2}{(4\pi)^2 \times (5000)^2 \times 1.259}$$

$$\doteq 4.48 \times 10^{-9}(w) \doteq \underline{\underline{-53.5\ dBm}}$$

For the geometry shown below, we can redraw it in another geometry by approximation.



From the figure above we have

3.10 Cont'd

$$\tan\beta = \frac{h_{obstacle} - h_t}{d_1} = \frac{400-60}{3000} \doteq 0.1133 \Rightarrow \beta \doteq 0.11285\ (rad)$$

$$\tan\gamma = \frac{h_{obstacle} - h_r}{d_2} = \frac{400-5}{2000} = 0.1975 \Rightarrow \gamma \doteq 0.1950\ (rad)$$

$$\Rightarrow \alpha = \beta + \gamma = 0.11285 + 0.195 \doteq 0.3078\ (rad)$$

and $v = \alpha \cdot \sqrt{\frac{2 d_1 d_2}{\lambda (d_1 + d_2)}} = 0.3078 \times \sqrt{\frac{2 \times 3000 \times 2000}{0.333 \times (3000 + 2000)}} \doteq 26.12$

Using the approximation equation (3.59.e), we obtain

$$G_d(dB) = 20 \cdot \log_{10}\left(\frac{0.225}{v}\right) \quad v > 2.4$$

$$= 20 \cdot \log_{10}\left(\frac{0.225}{26.12}\right)$$

$$\doteq -41.3\ dB.$$

$$\Rightarrow P_{received} = P_{freespace}(dBm) + G_d$$

$$= -53.5 - 41.3$$

$$= \underline{\underline{-94.8\ dBm}}$$

$\Rightarrow$ loss due to diffraction $L_d = P_{freespace} - P_{received} = \underline{\underline{41.3\ dB}}$

Question 2

3.13 (a) For free space, $P_r = P_0 \left(\frac{d_0}{d}\right)^2$

Given $P_0 = 10^{-6}(W) = -30\ dBm$, $d_0 = 1Km$.

For $d = 2Km$, $P_r = 10^{-6} \cdot \left(\frac{1}{2}\right)^2 = 2.5 \times 10^{-7}(W) \doteq \underline{\underline{-36\ dBm}}$

Similarly, For $d = 5Km$, $P_r \doteq \underline{\underline{-44\ dBm}}$

For $d = 10Km$, $P_r \doteq \underline{\underline{-50\ dBm}}$

For $d = 20Km$, $P_r = \underline{\underline{-56\ dBm}}$

(b) For $n = 3$, $P_r = P_0 \cdot \left(\frac{d_0}{d}\right)^3$

3.13 Cont'd

For $d = 2Km$, $P_r = 10^{-6} \cdot \left(\frac{1}{2}\right)^3 = 1.27 \times 10^{-7}(w) = \underline{\underline{-39\ dBm}}$

For $d = 5Km$, $P_r = \underline{\underline{-51\ dBm}}$

For $d = 10Km$, $P_r = \underline{\underline{-60\ dBm}}$

For $d = 20Km$, $P_r = \underline{\underline{-69\ dBm}}$

(c) For $n = 4$, $P_r = P_0 \left(\frac{d_0}{d}\right)^4$

For $d = 2Km$, $P_r = \underline{\underline{-42\ dBm}}$ For $d = 5Km$, $P_r = \underline{\underline{-58\ dBm}}$

For $d = 10Km$, $P_r = \underline{\underline{-70\ dBm}}$ For $d = 20Km$, $P_r = \underline{\underline{-82\ dBm}}$

(d) For two ray ground reflection model using the exact expression

$$P_r(d_0) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 d_0^2} \Rightarrow P_t = \frac{P_r(d_0) \cdot (4\pi)^2 \cdot d_0^2}{G_t \cdot G_r \cdot \lambda^2}$$

Given $P_r = 10^{-6}W$, $d_0 = 1Km$, $G_t = G_r = 0dB = 1$, $\lambda = \frac{c}{f_c} = 0.1667m$,

$$\Rightarrow P_t = \frac{10^{-6} \times (4\pi)^2 \times (1000)^2}{1 \times 1 \times 0.1667^2} \doteq 5.679 \times 10^3 (W)$$

From problem 3.6, the exact expression is

$$P_r(d) = \frac{P_t \cdot G_t G_r \lambda^2}{(4\pi)^2 \cdot d^2} \cdot 4 \cdot \sin^2\left(\frac{\theta_\Delta}{2}\right), \quad \text{where } \theta_\Delta = \frac{2\pi}{\lambda} \cdot \frac{2 h_t h_r}{d}$$

For $d = 2Km$, $\theta_\Delta = \frac{2\pi}{0.1667} \times \frac{2 \times 40 \times 3}{2000} = 4.5216$ rads

$$\Rightarrow P_r = \frac{5.679 \times 10^3 \times 1 \times 1 \times (0.1667)^2}{(4\pi)^2 \times (2000)^2} \times 4 \times \sin^2\left(\frac{4.5216}{2}\right)$$

$$\doteq 5.97 \times 10^{-7}(w) \doteq \underline{\underline{-32.25\ dBm}}$$

Similarly, For $d = 5Km$, $\theta_\Delta \doteq 1.809$ rads

$\Rightarrow P_r \doteq 9.88 \times 10^{-8}(w) \doteq \underline{\underline{-40\ dBm}}$

For $d = 10Km$, $\theta_\Delta = 0.904$ rads

$\Rightarrow P_r = 7.64 \times 10^{-9}(w) \doteq \underline{\underline{-51.17\ dBm}}$

3.13 Cont'd

For $d = 20Km$, $\theta_\Delta \doteq 0.452$ rads

$\Rightarrow P_r = 5.02 \times 10^{-10} (W) \doteq \underline{-63\ dBm}$

(e) For Extended Hata model,

$L_{50}(urban) = 46.3 + 33.9 \log_{10} f_c - 13.82 \log_{10} h_t - a(h_r) + (44.9 - 6.55 \log_{10} h_t) \log_{10} d + C_M$

Where $C_M = 3dB$ for large city

$a(h_r) = 3.2(\log_{10} 11.75 h_{re})^2 - 4.97\ dB$ for $f_c \geq 400MHz$

$\Rightarrow a(h_r) = 3.2[\log_{10}(11.75 \times 3)]^2 - 4.97 \doteq 2.69\ dB$

For $d = 2km$, $L_{50}(2Km) = 46.3 + 33.9 \log_{10} 1800 - 13.82 \log_{10} 40 - 2.69 + (44.9 - 6.55 \log_{10} 40) \log_{10} 2 + 3$

$= 145.18\ dB$

Since $L_{50}(1Km) = 134.8\ dB$

$\Rightarrow P_r = P_0(dBm) - [L_{50}(20km) - L_{50}(1Km)]$

$= -30 - [145.18 - 134.8] = \underline{-40.38\ dBm}$

Similarly, for $d = 5\ Km$, $P_r = \underline{-54.07\ dBm}$

for $d = 10\ Km$, $P_r = \underline{-64.4\ dBm}$

for $d = 20\ Km$, $P_r = \underline{-74.8\ dBm}$

See the MATLAB program p3_13.m and Fig. p3-13.

```
%%%%%%%%%%%%%%%%%%%%%%
%                    %
%      p3_13.m       %
%                    %
%  Chapter 3  3.13   %
%                    %
%%%%%%%%%%%%%%%%%%%%%%

fc = 1800*10^6;                       % carrier frequency
lambda = 3*10^8/fc;                   % wave length of carrier frequency (m)
ht = 40;                              % height of transmitter (m)
hr = 3;                               % height of receiver (m)
d0 = 1000;                            % reference distance (m)
p0 = 10^(-6);                         % received power of reference distance (w)
p0_dBm = 10*log10(p0/10^(-3));        % power in dBm
E0 = p0*480*pi^2 / lambda^2;          % E-field in d0
step = 50;                            % step length of distance (m)
d = (1000:step:20000);                % slected distance to plot (m)
window = (-10:10)*lambda;             % average window (m)

start = 1;
while start == 1,
   model = menu('Please choose a path loss model','Free space model','n = 3',
                'n = 4','Two-ray ground reflection model','Hata model');

   pd = [];
   if model == 1                      %  free space model
      for i = 1:length(d)
          d_window = d(i) + window;
          pr = p0*(d0./d_window).^2;
          pd = [pd mean(pr)];
      end;

      pd_dBm = 10*log10(pd*1000);
      figure(1);
      semilogx(d/1000, pd_dBm,'.');
   end;

   if model == 2                      %  n = 3
      for i = 1:length(d)
          d_window = d(i) + window;
          pr = p0*(d0./d_window).^3;
          pd = [pd mean(pr)];
      end;

      pd_dBm = 10*log10(pd*1000);
      figure(1);
      semilogx(d/1000, pd_dBm,'-');
   end;

   if model == 3                      %  n = 4
      for i = 1:length(d)
          d_window = d(i) + window;
          pr = p0*(d0./d_window).^4;
          pd = [pd mean(pr)];
      end;

      pd_dBm = 10*log10(pd*1000);
      figure(1);
      semilogx(d/1000, pd_dBm,':');
   end;

   if model == 4                      %  Two-ray ground reflection model
      for i = 1:length(d)
          d_window = d(i) + window;
          sita_delta = 4*pi*ht*hr./(lambda*d_window); %phase difference between
                                                      %two rays
          pr = p0*(d0./d_window).^2 .*4.*(sin(sita_delta/2)).^2;
                                      %  exact expression of received power
          pd = [pd mean(pr)];
      end;

      pd_dBm = 10*log10(pd*1000);
      figure(1);
      semilogx(d/1000, pd_dBm,'-.');
   end;

   if model == 5                      %  Hata model
      a_hr = 3.2*log10(11.75*hr)^2 - 4.97;   % antenna correction factor
      for i = 1:length(d)
          d_window = d(i) + window;
          loss = (44.9 - 6.55*log10(ht))*log10(d_window/1000);
                                             % path loss between d and 1km(dB)
          pr_dBm = p0_dBm - loss;            % received power (dBm)
          pr = 10.^((pr_dBm - 30)/10);       % received power (w)
          pd = [pd mean(pr)];
      end;

      pd_dBm = 10*log10(pd*1000);
      figure(1);
      semilogx(d/1000, pd_dBm,'--');
   end;

   hold on;
   grid on;
   title('Received Power of Different Model');
   xlabel('d(Km)');
   ylabel('Pr(d)  (dBm)');
   legend('.','Free space','-','n = 3',':','n = 4','-.','Two ray','--','Hata');

   start = menu('Continue or quit?', 'Continue','Quit');
end;

hold off;
axis([1 30 -85 -20]);
```

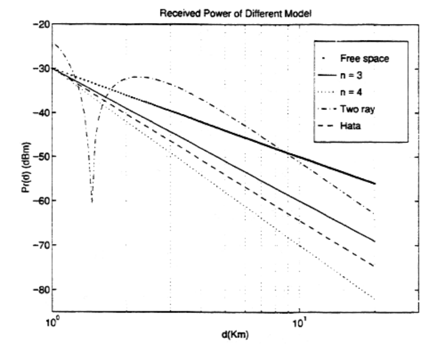


Fig. p3_13

Question 3

$\boxed{3.16}$ Given noise figure $F = 8dB \doteq 6.3$, receiver bandwidth $B_W = 30KHz$,

$\Rightarrow$ noise floor $= K \cdot B_W \cdot F \cdot T_0$, where K is Boltzman constant, $T_0 = 290°K$

$\Rightarrow$ noise floor $= 1.38 \times 10^{-23} \times 30 \times 10^3 \times 6.3 \times 290$

$\doteq 7.56 \times 10^{-16} (W) \doteq -121.2\ (dBm)$

$\Rightarrow$ threshold γ = noise floor (dBm) + SNR (dB)

$= -121.2 + 20 = -101.2\ (dBm)$

Since $Pr[P_r(d_{max}) > \gamma] = Q\left(\frac{\gamma - \overline{P_r(d_{max})}}{\sigma}\right) = 0.95$, we have

$\frac{\gamma - \overline{P_r(d_{max})}}{\sigma} \doteq -1.645$

$\Rightarrow \overline{P_r(d_{max})} = \gamma + 1.645\sigma = -101.2 + 1.645 \times 8 = -88.04 (dBm)$

Given $P_t = 15\ W$, $\lambda = \frac{c}{f} \doteq 0.1667\ m$, $G_t = 12dB \doteq 15.85$, $G_r = 3\ dB \doteq 2$

$\Rightarrow P_r(d_0) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_0^2}$

3·16 Cont'd

$$= \frac{15 \times 15.85 \times 2 \times 0.1667^2}{(4\pi)^2 \times (1000)^2} \doteq 8.373 \times 10^{-8} (W) \doteq -40.77 \, dBm$$

Since $\overline{P_r(d_{max})} = P_r(d_0)\ (dBm) - 10 \cdot n \log_{10}\left(\frac{d_{max}}{d_0}\right)$, we have

$$10 \times 4 \log_{10}\left(\frac{d_{max}}{d_0}\right) = P_r(d_0) - \overline{P_r(d_{max})} = -40.77 - (-88.04)$$

$$\Rightarrow \log_{10} \frac{d_{max}}{d_0} \doteq 1.182$$

$$\Rightarrow d_{max} \doteq \underline{\underline{15.2\ (Km)}}$$

Question 4

$\boxed{4.8}$ From Fig. P4.7, we have

$$v(t) = \begin{cases} 3t & 0 \le t < 10 \\ 30 & 10 \le t < 90 \\ 300 - 3t & 90 \le t < 100 \end{cases}$$

$$\lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{900 \times 10^6} \doteq 0.33\ (m)$$

For $\rho = 0.1$, $T = 100$ Second, we have

$$N_R = \frac{1}{T} \int_0^T \sqrt{2\pi} f_m \cdot \rho \cdot e^{-\rho^2} dt$$

4.8 Cont'd

$$= \frac{1}{T} \int_0^T \sqrt{2\pi} \cdot \frac{v(t)}{\lambda} \cdot \rho \cdot e^{-\rho^2} dt$$

$$= \frac{1}{T} \cdot \frac{\sqrt{2\pi}}{\lambda} \cdot \rho \cdot e^{-\rho^2} \cdot \int_0^T v(t)\, dt$$

$$= \frac{1}{100} \times \frac{\sqrt{2\pi}}{0.33} \times 0.1 \times e^{-0.1^2} \cdot \left[2 \times \int_0^{10} 3t\, dt + \int_{10}^{90} 30\, dt \right]$$

$$= \frac{1}{100} \times \frac{\sqrt{2\pi}}{0.33} \times 0.1 \times e^{-0.1^2} \times 2700$$

$$\doteq \underline{\underline{20.1\ (\text{Crossings}/s)}}$$

$$\overline{\tau} = \frac{1}{N_R} \cdot P_r[r \le R] = \frac{1 - e^{-\rho^2}}{N_R} = \frac{1 - e^{-0.01}}{20.1} \doteq 4.95 \times 10^{-4} (s)$$

$$\underline{\underline{= 0.495\ (ms)}}$$