Solutions to Quiz 4

Question 1

Using the convention shown in the lectures, a block code has the following generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

(a) What is the parity check matrix (H)?

(b) The codeword 101000 is received. What is the syndrome?

(c) If the codeword above was received, which bit was in error? (*Hint: find the syndromes.*)

Answer

(a)

$$\begin{aligned} H &= \left[P^T \,|\, I_{n-k}\right] \\ &= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

(b) If the received codeword is $c' = c + e$, the syndrome is:

$$\begin{aligned} e &= Hc'^T \\ &= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \end{aligned}$$

(c) The syndromes are found by computing He for each possible value of e.

This is a linear code with weight 3 so the minimum distance is also 3 and we can correct one error. Thus the syndromes are the values of He for e having a single 1 in each bit position. Each syndrome will thus corresponds to one column of H. The syndrome e corresponds to the second column of H so the error must be in the second bit: $e = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$ and the transmitted codeword was $c = c' + e = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$.

To check, we can compute Hc'^T:

$$\begin{aligned} H &= Hc'^T \\ &= \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

Question 2

LTE uses OFDM with a (complex) sampling rate of $f_s = 30.72$ MHz. There are $N = 2048$ samples per OFDM block.

(a) What is the subcarrier spacing?

(b) If a cyclic prefix of 144 samples is used, what is the duration of each OFDM block, including the cyclic prefix? Give your answer in microseconds.

(c) What is the bandwidth of the signal if 1201 subcarriers, including the one at DC (zero frequency), are used?

Answer

(a) To maintain orthogonality, subcarriers are spaced at multiples of the inverse of the OFDM block duration which is $T = N/T_s$ where T_s is

the sampling period. The subcarrier spacing is thus $f_s/N = \boxed{15\,\text{kHz}}$.

(b) The cyclic prefix extends the duration of the block to $2048 + 144$ samples and so the duration is $\frac{N+N_{CP}}{f_s} = \frac{2048+144}{30.72} = \boxed{71.35\,\mu\text{s}}$.

(c) If 1201 contiguous subcarriers, including the one at DC (zero frequency), are used the total bandwidth is approximately[1] $1201 \times 15\,\text{kHz} = \boxed{18.015\,\text{MHz}}$.

[1] The exact value depends on your definition of "bandwidth."