Solutions to Quiz 3

Question 1

A binary symmetric channel operates at a rate of 1 Mbps with a bit error rate of 0.1 (10%). Is it possible to transmit information over this channel, without errors, at a rate of 500 kbps? Explain.

Answer

The capacity of a channel is the maximum rate at which information can be transmitted over a channel with an arbitrarily low error rate. Substituting $p = 0.1$ in the equation for the capacity of the binary symmetric channel we obtain:

$$\begin{aligned} C &= 1 + p\log_2 p + (1-p)\log_2(1-p) \\ &= 1 + 0.1\log_2(0.1) + (1-0.1)\log_2(1-0.1) \\ &\approx 0.5310\,\text{bits/bit} \end{aligned}$$

If the rate over the channel is 1 Mb/s, the maximum error-free rate would be $\approx 0.5310 \times 1\,\text{Mb/s} = 531\,\text{kbps}$. Since this exceeds 500 kbps, $\boxed{\text{yes it would be possible}}$ to transmit data error-free over this channel at 500 kbps.

Question 2

Assuming an AWGN channel with an SNR of 0 dB, what is the minimum bandwidth that would enable error-free transmission at a rate of 10 kb/s?

Answer

The capacity of the AWGN channel is:

$$C = B\log_2\left(1 + \frac{S}{N}\right)$$

Setting the capacity $C = 10\,\text{kb/s}$, S/N to 1 (0 dB), and solving for B:

$$\begin{aligned} B &= \frac{10,000}{\log_2(1+1)} \\ &= \boxed{10\,\text{kHz}} \end{aligned}$$

Question 3

A code contains the following four 8-bit codewords:

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00000000
11111111
00001111
11110000
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(a) What is the minimum distance of this code?

(b) What is the maximum number of errors in each codeword that are guaranteed to be detected?

(c) What is the maximum number of errors in each codeword that are guaranteed to be corrected?

Answer

(a) The distances between the codewords above are given in the following table:

0	8	4	4
	0	4	4
		0	8

so the minimum distance of this code is $\boxed{4}$.

(b) The maximum number of errors in each codeword that are guaranteed to be detected is $\lfloor \frac{d-1}{2} \rfloor = \lfloor 3/2 \rfloor = \boxed{1 \text{ error}}$.

(c) The maximum number of errors in each codeword that are guaranteed to be corrected is $d - 1 = 4 - 1 = \boxed{3 \text{ errors}}$.

Question 4

What are n and k for a Hamming code with a code rate of 0.968? *Hint: $n - k$ is less than 10.*

Answer

Any code has a rate k/n. A Hamming code has $n = 2^{(n-k)} - 1$. n and k must to be integers. Since we're told they're smaller than 10, we can try decreasing values:

- for $n - k = 9$ $n = 511$ and the code rate is (511-9)/511 = 0.982
- for $n - k = 8$ $n = 255$ and the code rate is (255-8)/255 = 0.968

So for this code $\boxed{n = 255, k = 247}$.