Solutions to Quiz 2

Question 1

You've determined that propagation in a rural area can be modelled by a power law path loss model with exponent of $n = -3$ and a shadow fading standard deviation of $\sigma = 5$ dB.

At a distance of 1 km the SNR is 30 dB.

(a) What is the average SNR at a distance of 3 km?

(b) At this distance, what is the probability that the SNR is less than 10 dB? *Hint: There is a table of the Gaussian CDF on the last page.*

Answer

(a) If the path loss increases as a power, n, of the distance then the path loss at a distance d will be:

$$PL(d) = PL(d_0)\left(\frac{d}{d_0}\right)^n$$

Assuming the noise power is constant then the signal power and path loss SNR will be inversely proportional to the path loss. The SNR in dB at a distance d will be:

$$\begin{aligned} PL(d)_{dB} &= PL(d_0)_{dB} - 10n\log_{10}\left(\frac{d}{d_0}\right) \\ PL(1\,\text{km}) &= 30 - 30\log_{10}(3) \\ &\approx \boxed{15.7\,\text{dB}} \end{aligned}$$

(b) Shadow fading typically has Gaussian distribution if the signal level expressed in dB. When faded to an SNR of 10 dB the signal is $(10-15.7)/5 \approx -1.14$ standard deviations relative to the mean.

From the tables provided, the probability that the signal will be less than this is between 0.115 and 0.159.

If your calculator can compute the Gaussian CDF you could, for example, use the formula in the lecture notes, $\Pr[z > \gamma] = \frac{1}{2}\text{erfc}\left(\frac{\gamma-m}{\sqrt{2}\sigma}\right)$ with m equal to your answer in part (a), $\gamma = 10$, and $\sigma = 5$. Note that this gives the probability that the SNR is greater than γ, the probability that the SNR is less than this is $1 - \Pr[z > \gamma]$. A calculator gives this probability as $\approx \boxed{12.7\%}$.

Question 2

A wireless communication system operates over a Rayleigh fading channel. The system requires an SNR of 10 dB to operate. The system has two receive antennas that experience independent fading. The average SNR on each antenna is 20 dB.

(a) What is the average SNR if maximal-ratio combining diversity is used?

(b) What is the probability that the SNR is too low for the system to operate if selection diversity is used?

(c) What is the probability that the SNR is too low for the system to operate if a single antenna (no diversity) were used and the SNR were the same as in (a)?

Answer

(a) With maximal-ratio combining the combiner output SNR is the sum of the branch SNRs. In this case the branch SNRs are both 20 dB (100 in linear units) so the maximal-ratio combining SNR will be $100 + 100 = 200 \approx \boxed{23\,\text{dB}}$.

(b) With selection diversity the probability that N branches are all faded is P^N where P is the probability that one is faded. If the signal envelope is Rayleigh distributed, it has a CDF $P(r \leq R) = 1 - e^{-\rho^2}$ where $\rho = R/R_{\text{rms}}$ is the fading threshold relative to the mean.

In this question the fading threshold relative to the mean in dB is $\rho = 10 - 20 = -10\,\text{dB} = 10^{-10/20} \approx 0.316$ and $P = 1 - e^{\rho^2} \approx 0.095$. The probability of both branches being faded is thus $\left(1 - e^{\rho^2}\right)^2 \approx \boxed{9.1 \times 10^{-3}}$.

(c) If there was a single antenna whose SNR was 23 dB, the same as in part (a), then $\rho = 10^{(10-23)/20} \approx 0.224$ and the probability that the signal is faded is given by the Rayleigh CDF, $P(r \leq R) = 1 - e^{-\rho^2} \approx \boxed{4.9\,\%}$.

Unfortunately, the question was ambiguously worded and some students interpreted "and the SNR were the same as in (a)" to refer to the SNR before diversity combining. In this case the answer is approximately 0.1 (10%) and this was also marked correct.