Solutions to Quiz 1

Question 1

(a) You measure the power of a Rayleigh fading signal three times and find the power levels are −16 dBm, −10 dBm, and −13 dBm. What would be a reasonable estimate of the mean power of the signal? Give your answer in dBm.

Answers

The sample mean and sample median are reasonable estimators of the mean of the underlying distribution. An estimate of the mean power using the sample mean is:

$$\begin{aligned}\widehat{R^2_{\text{rms}}} &= \frac{1}{N}\sum_i P_i \\ &= \frac{1}{3}\left(10^{-16/10} + 10^{-10/10} + 10^{-13/10}\right) \\ &\approx 58.4\,\mu\text{W} \approx \boxed{-12.3\,\text{dBm}}\end{aligned}$$

and an estimate using the sample median is:

$$\widehat{R^2_{\text{rms}}} = \boxed{-13\,\text{dBm}}$$

(b) For signal described above, what fraction of the time would you expect the signal level to be less than −33 dBm?

Answer

The CDF of the Rayleigh distribution can be written as a function of $\rho^2 = R^2/R^2_{\text{rms}}$. Using $R^2 = -33$ dBm and $R^2_{\text{rms}} = \widehat{R^2_{\text{rms}}} = -13$ dBm,

$$\rho^2 = 10^{(-33--13)/10} = 1 \times 10^{-2}$$

and the probability that the signal amplitude is less than −33 dBm, is

$$\begin{aligned}P(r \le R) &= 1 - e^{-\rho^2} \\ &\approx 1 - e^{-0.01} \approx \boxed{1.0\%}.\end{aligned}$$

Question 2

A receiver is moving in a propagation environment that approximates Clarke's model for fading. The noise level is constant and the average signal-to-noise ratio (SNR) is 23 dB. The receiver outputs a noise pulse each time the SNR drops below 3 dB. You find that these pulses happen at an average rate of 10 per second. If the carrier frequency is 100 MHz, how fast is the receiver moving? Give your answer in km/h.

Answers

If the average SNR is 23 dB and the threshold is 3 dB SNR then the threshold is $\rho = 3 - 23 = -20$ dB and $\rho^2 = 10^{-20/10} = 1 \times 10^{-2}$.

From the equations for the level crossing rate, N_R, and maximum Doppler rate f_m we can solve for the velocity, v:

$$\begin{aligned}v &= \frac{c}{f_c}\frac{N_R}{\sqrt{2\pi}\rho e^{-\rho^2}} \\ &= \frac{300 \times 10^6}{100 \times 10^6} \cdot \frac{10}{\sqrt{2\pi} \cdot 1 \times 10^{-1} \cdot e^{-(0.1)^2}} \\ &\approx 121\text{ m/s} \approx \boxed{435\,\text{km/h}}\end{aligned}$$

Question 3

You are setting up a point-to-point (line of sight) WLAN link operating over a distance of 1 km at a frequency of 2.4 GHZ. The WLAN transceivers require a received signal level of −50 dBm to ensure adequate performance. At each end of the link you are using antennas with gains of 16 dBi. How much power do you need to transmit to achieve the required performance?

Answers

Given the received power $P_R = 10^{-50/10} = 1 \times 10^{-5}$ mW, the antenna gains, $G_T = G_R = 10^{16/10} \approx 40$, distance $d = 1 \times 10^3$ m, and wavelength $\lambda = \frac{c}{f} =$

${}^{3\times10^8\,\mathrm{m/s}}\!/\!{}_{2.4\times10^9\,\mathrm{Hz}} \approx 0.125\,\mathrm{m}$, we can solve the Friis equation for the transmit power:

$$
\begin{aligned}
P_T &= \frac{P_R}{G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2} \\
&\approx \frac{1 \times 10^{-5}}{40 \cdot 40 \cdot \left(\frac{0.125}{4\pi \cdot 1\times10^3}\right)^2} \\
&\approx 63.8\,\mathrm{mW} \approx \boxed{18\,\mathrm{dBm}}
\end{aligned}
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