## Error Detection and Correction

**Exercise 1**: A block code has two valid codewords, 101 and 010. The receiver receives the codeword 110. What is the Hamming distance between the received codeword and each of the valid codewords? What codeword should the receiver decide was sent? What bit was most likely in error? Is it possible that the other codeword was sent?

**Exercise 2**: What is the minimum distance for the code in the previous exercise? How many errors can be detected if you use this code? How many can be corrected? What are $n$, $k$, and the code rate ($k/n$)?

**Exercise 3**: How many errors can an $N$-fold repetition code detect? Correct? What is the code rate?

**Exercise 4**: What are $n$, $k$, and the code rate for a Hamming code with $m = 3$? $m = 6$? $m = 2$?

**Exercise 5**: What is the generator matrix for the (5,3) code that computes two parity bits as: $p_0 = d_0 \oplus d_1$ and $p_1 = d_1 \oplus d_2$ where $d_i$ is the $i$'th data bit?

**Exercise 6**: What is the parity check matrix for the code above? If data vector $[101]$ is to be transmitted, what is the codeword? If there are no errors, what is the result of multiplying the received codeword by $H$? If the channel introduces an error into the second bit?

**Exercise 7**: How many possible correctable error patterns are there for a $(31, 26)$ Hamming code? How many possible received bit patterns?

**Exercise 8**: What are the possible syndromes for the code above? What was the syndrome when the second bit was in error?

**Exercise 9**: Does a syndrome of zero correspond to an error? If each syndrome corresponds to a different error vector, what is the largest value of $n$ for which a $n-k$-bit syndrome can correct a single error? What are $n$ and $k$ for Hamming code with $m = 3$?

**Exercise 10**: What is the block size for a RS code using symbols from GF(64) in bit? In symbols?

**Exercise 11**: How many parity symbols would we need if we wanted to correct 8 8-bit symbol errors? What are $(n, k)$ for this code?

**Exercise 12**: A block FEC code uses values from GF(4). The 4 possible elements are represented using the letters A through D. The valid code words are: ABC, DAB, CDA, and BCD.

What is the minimum distance of this code? How many errors can be detected? Corrected?

If the codeword ADA is received, was an error made? Can it be corrected? If so, what codeword should the decoder decide was transmitted?

If each codeword represents two bits, how many bit errors were corrected?

Repeat if the codeword received was AAA.

**Exercise 13**: Assuming one bit at a time is input into the encoder in the diagram above, what are $k$, $n$, $K$ and the code rate?

**Exercise 14**: Consider the convolutional encoder above. If the only the bits corresponding to the outputs A, A and B, and B are transmitted corresponding to every three input bits, what is the code rate of this punctured code?

**Exercise 15**: Give the numbering of the bits coming out of a 4x4 interleaver. If bits 8, 9 and 10 of the interleaved sequence have errors, where would the errors appear in the de-interleaved sequence? If the receiver could correct up to one error per 4-bit word, would it be able to correct all the errors without interleaving? With interleaving?

**Exercise 16**: If errors on the channel happened in bursts and you were using a RS code using 8-bit words, would you want to interleave bits or bytes?

**Exercise 17**: If the likelihood of a bit being 1 is 0.5 what is the log-likelihood? What if the probability of a bit being 0 is 0.25? What is the product of the two probabilities? The ratio? The sum and difference of the two log-likelihoods?

**Exercise 18**: What are the log-likelihood ratios for: A punctured bit? An erasure? A shortened bit equal to 0? If 0 and 1 are equally likely to have been transmitted?

**Exercise 19**: What is k for a rate 0.9 code with n=10000? If d = n-k+1 how many errors could this code correct? [This is actually an upper bound on d.]

**Exercise 20**: What are the units of Energy? Power? Bit Period? How can we compute the energy transmitted during one bit period from the transmit power and bit duration?

**Exercise 21**: A system needs to operate at an error rate of $10^{-3}$. Without FEC it is necessary to transmit at 1W at a rate of 1 Mb/s. When a rate-1/2 code is used together with a data rate of 2 Mb/s the power required to achieve the target BER decreases to 500mW. What is the channel bit rate in each case? What is the information rate in each case? What is $E_b$ in each case? What is the coding gain?