Solutions to Quiz 3

Question 1

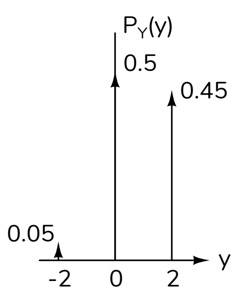
There were two versions of this question. A transmitter outputs a random signal X that is either +1 V (or +2 V) or −1 V (or −2 V) with equal probabilities. The channel adds a random independent noise signal, N, that is −1 V (or −2 V) 10% (or 20)% of the time and 1 V (or 2 V) otherwise. The received signal is $Y = X + N$. Calculate the probability of each possible value of Y. Sketch the marginal probability density function $P_Y(y)$ and the joint probability density function $P_{XY}(x, y)$.

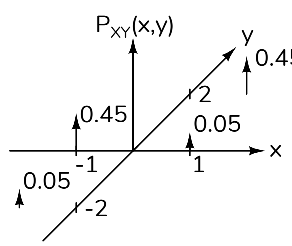
Answer

Since the X and N are independent, $P_{XY}(x, y) = P_X(x)P_N(n)$ and we can calculate the possible values of Y and their probabilities as follows for the first version of the question:

x	n	$p(x)$	$p(n)$	y	$P(x, y)$
+1	+1	0.5	0.9	2	0.45
+1	-1	0.5	0.1	0	0.05
-1	+1	0.5	0.9	0	0.45
-1	-1	0.5	0.1	-2	0.05

from which $p_Y(-2) = 0.05$, $p_Y(0) = 0.5$, $p_Y(2) = 0.45$, and for which $P_Y(y)$ and $P_{XY}(x, y)$ are:

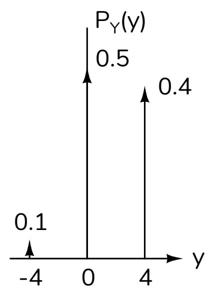


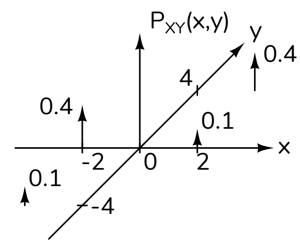


and as follows for the second version of the question:

x	n	$p(x)$	$p(n)$	y	$P(x, y)$
+2	+2	0.5	0.8	4	0.40
+2	-2	0.5	0.2	0	0.10
-2	+2	0.5	0.8	0	0.40
-2	-2	0.5	0.2	-4	0.10

from which $p_Y(-4) = 0.1$, $p_Y(0) = 0.5$, $p_Y(4) = 0.4$, for which $P_Y(y)$ and $P_{XY}(x, y)$ are:

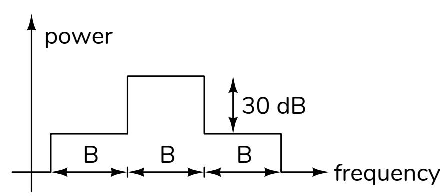




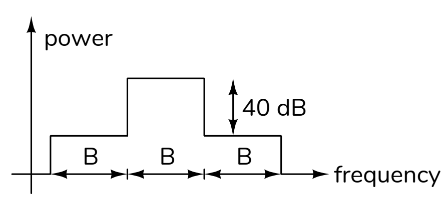
Question 2

The specification for a transmitter requires that the transmitted signal power fall below the "mask" shown at right where B is the bandwidth of the signal at the input to the final amplifier. If the required output power is 25 dBm, what is this amplifier's minimum required output IP3?

There were two versions of the diagram:



and



Answer

The third-order products fall in the adjacent channel and need to be 30 dB (or 40 dB) below the in-channel signal. The third-order products decrease (in dB) 3 times faster than the in-channel signal. If Δ is the reduction of the in-channel signal from OIP3,

$2\Delta = 30$ dB (or 40 dB) is the reduction of the adjacent-channel signal from the in-channel signal and $\Delta = 30/2 = 15$ dB (or $40/2 = 20$ dB). Thus the required OIP $= 25 + 15 = 40$ dBm (or $25 + 20 = 45$ dBm).

Question 3

A receiver has an LNA with a noise figure of 1.8 dB and a gain of 6 dB. This is followed by a mixer with a noise figure of 6 dB and a gain of 3 (or 6) dB. This is followed by amplifier with a noise figure of 10 dB and a gain of 30 dB. What is the noise figure of the cascade of these three devices?

Answer

Converting to linear units, $F_1 = 1.5, G_1 = 4$, $F_2 = 4, G_2 = 2$ (or $G_2 = 4$), and $F_3 = 10, G_3 = 1000$. Using the equation for the cascade noise figure:

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.5 + \frac{4 - 1}{4} + \frac{10 - 1}{4 \times 2(\text{or } 4)}$$

$$\approx 3.38 = 5.3 \text{ dB (or } 2.81 = 4.5 \text{ dB)}.$$