Solutions to Quiz 2

Question 1

You are walking in a shopping mall at a speed of 3.6 km/hour while connected to a base station at a frequency of 3 GHz. Assuming Clarke's model applies, how many times *per minute* does the received signal level drop 10 dB below the mean?

Answer

Your velocity is $3600 \text{ m/hr} / 3600 \text{ s/hr} = 1$ m/s. The maximum Doppler rate is

$$f_m = \frac{v}{c} f_c = \frac{1}{3 \times 10^8} 3 \times 10^9 = 10 \text{ Hz}$$

A threshold 10 dB below the mean in linear units (V/V) is $\rho = 10^{-10/20} = 1/\sqrt{10}$. The level crossing rate is:

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} = \sqrt{2\pi} \cdot 10 \cdot \frac{1}{\sqrt{10}} e^{-\frac{1}{10}} =\approx 7.17 \text{ Hz}$$

which is $\approx$ 430 crossings per minute

Question 2

A multipath channel has three paths with lengths $d_0 = 300$ m, $d_1 = 600$ m and $d_2 = 900$ m. The received signal level on each path is inversely proportional to the square of the path length: $P_i = \frac{k}{d^2}$ where k is unknown. What are the excess delays, the normalized power delay profile, the mean excess delay and the RMS delay spread? *Hint: assume* $k = 300^2$.

Answer

The path delays in microseconds are computed by dividing the path lengths by the velocity of propagation:

$$t_i = d_i / c$$

The excess delays are obtained by subtracting the minimum delay:

$$\tau_i = t_i - t_{min}$$

The normalized power delay profile is obtained by dividing the power on each path by the total power:

$$p(\tau) = \frac{P(\tau)}{\sum P(\tau)}$$

The mean excess delay is:

$$\bar{\tau} = \sum p(\tau)\tau =$$

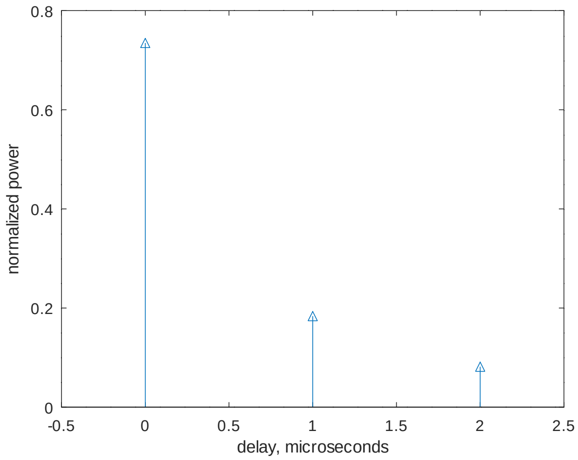
and the "RMS delay spread" of the channel is:

$$\sigma = \sqrt{\sum p(\tau)(\tau - \bar{\tau})^2}$$

The numerical results and a plot of the power delay profile can be computed using a program such as Matlab (Octave, actually):

```
octave:1> d=[300 600 900]
d =

   300   600   900

octave:2> t=d./300
t =

   1   2   3

octave:3> tau=t-min(t)
tau =

   0   1   2

octave:4> P=300^2./d.^2
P =

   1.0000   0.2500   0.1111

octave:5> p=P/sum(P)
p =

   0.734694   0.183673   0.081633

octave:6> taubar=sum(p.*tau)
taubar = 0.3469
octave:7> sigma=sqrt(sum(p.*(tau-taubar).^2))
sigma = 0.6244
octave:8> stem(tau,p,"marker","^")
octave:9> axis([-0.5 2.5])
octave:10> xlabel('delay, microseconds')
octave:11> ylabel('normalized power')
```

Thus the excess delays are 0, 1 and 2 μs, the normalized power delay profile is plotted below:



the mean excess delay is $\approx 0.347\ \mu$s and the RMS delay spread is $\approx 0.624\ \mu$s.