Solutions to Quiz 1

A block code has the following parity check matrix:

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Question 1

The generator matrix has dimensions $(n-k) \times n$ and this matrix has dimensions 3×7 thus $n-k = 3$, $n = 7$ and $k = n-(n-k) = 7-3 = 4$.

Question 2

For a systematic code with the parity bits transmitted first the parity check matrix is

$$H = \left[P^T | I_{n-k} \right]$$

and the generator matrix is:

$$G = \left[I_k | P \right]$$

so in this case the generator matrix is:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Question 3

If the data to be transmitted is d the codeword transmitted is $c = dG$. For $d = [1\,1\,1\,1]$ and G as above the transmitted codeword is $d = [1\,1\,1\,1\,1\,1\,1]$.

Question 4

If the codeword $c = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$ is received the syndrome is $Hc^T = [0\,0\,0]$. This codeword does not contain an error since the syndrome is all zeros.

If the codeword $c = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$ is received the syndrome is $Hc^T = [1\,0\,1]$. This codeword does contain an error since the syndrome is not all zeros.

Calculations

Matlab (Octave, actually) can check the calculations:

```
H = [ 1 0 1 1 1 0 0 ;
      1 1 1 0 0 1 0 ;
      0 1 1 1 0 0 1 ]

G = [eye(4),H(:,1:4)']

   1   0   0   0   1   1   0
   0   1   0   0   0   1   1
   0   0   1   0   1   1   1
   0   0   0   1   1   0   1

d = [ 1 1 1 1 ]

mod(d*G,2)

   1   1   1   1   1   1   1

c=[ 1 0 1 1 1 0 0 ]

mod(H*c',2)

   0
   0
   0

c = [ 1 0 0 1 1 1 0 ]

mod(H*c',2)

   1
   0
   1
```