Solutions to Midterm Exam 2

Question 1

What is the effective area of an antenna that has a directivity of 8 (linear units) and an efficiency of 50% at a frequency of 24 GHz? Give your answer in square millimetres.

Solution

The ratio of gain to directivity:

$$\frac{G}{D} = k$$

is the antenna's efficiency, k. Given the directivity and efficiency we can solve for the gain: $G = kD = \frac{1}{2} \cdot 8 = 4$.

The wavelength is $\lambda = c/f = 300 \times 10^6 / 24 \times 10^9 = 12.5$ mm

The gain is related to the effective area and wavelength by:

$$G = \frac{4\pi A_e}{\lambda^2}$$

from which we can solve for A_e:

$$A_e = \frac{G\lambda^2}{4\pi} = \frac{4\lambda^2}{4\pi} = \frac{12.5^2}{\pi} = 49.7 \text{ mm}^2$$

Question 2

A communication link between earth and the moon (at a distance of 384×10^6 m) operates at a frequency of 1.5 GHz and uses antenna gains of 0 dBi and 60 dBi on each end of the link. If the transmit power is 10 W, what is the received power? Give your answer in dBm.

Solution

The Friis equation gives the received power:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

In this problem $P_T = 10$ W, $G_1 = 10^{0/10} = 1$, $G_2 = 10^{60/10} = 1 \times 10^6$, $\lambda = \frac{300\times10^6}{1.5\times10^9} = 0.2$ m and $d = 384 \times 10^6$ m. Thus:

$$P_R = 10 \cdot 1 \cdot 1\times10^6 \left(\frac{0.2}{4\pi \cdot 384 \times 10^6}\right)^2 \approx 17.2\times10^{-15} \text{ W}$$

which is $10 \log_{10}(17.2 \times 10^{-15}) + 30 = -107.7$ dBm.

Question 3

What is the effective area of the larger of the two antennas in the previous question? If it were circular, what would be the diameter?

Solution

The antenna with the higher gain would have the higher effective area (and thus be more likely to be "larger"). Using the equation above for effective area:

$$A_e = \frac{G\lambda^2}{4\pi} = \frac{10^6(0.2)^2}{4\pi} \approx 3.18 \times 10^3 \text{ m}^3$$

The area of a circle is give by $\pi(d/2)^2$, so $d \approx 2 \cdot \sqrt{3.18 \times 10^3/\pi} = 63.7$ m.

Question 4

Measurements of NLOS path loss in a neighbourhood show that for distances between 100 m and 1000 m the mean is approximated by a power law with a path loss exponent of 2.5 . If the mean path loss at 100 m is 50 dB, what is the expected mean path loss at a distance of 400 m?

Solution

Using the equation:

$$PL(d)_{\text{dB}} = PL(d_0) + 10n \log\left(\frac{d}{d_0}\right)$$

with $d = 400\ m$, $d_0 = 100\ m$, $PL(d_0) = 50$ dB and $n = 2.5$ we find

$$PL(400) = 50 + 25 \log\left(\frac{400}{100}\right) \approx 65 \text{ dB}$$