ELEX 7860 : Wireless System Design
2022 Winter Term

Solutions to Midterm Exam 1

Revision 2: fixed arithmetic error in Question 2.

Question 1

A rate-1/2, $K = 7$ convolutional code with the structure shown in the lecture notes is used to encode the bit sequence 1, 0, 1. Assuming the encoder begins at an all-zero state, what are the first ~~four~~ three bits output from "Output Data B"?

Hint: First write out the values at the input and each bit of the shift register.

Solution

The industry-standard convolutional encoder, as given in the lecture notes[1], is:

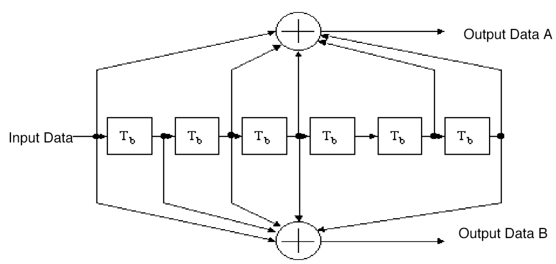


Figure 17-8—Convolutional encoder (k = 7)

The columns in the table below show the input and the values in the first two bits of the shift register for three successive input bits. The other bits in the shift register are zero.

1	0	0
0	1	0
1	0	1

Output Data B is the exclusive-or of the input, the first three bits and the last bit in the shift register. These are $1 \oplus 0 \oplus 0 = 1$, $0 \oplus 1 \oplus 0 = 1$, and $1 \oplus 0 \oplus 1 = 0$. Thus the output will be 1, 1, 0.

[1] From the IEEE 802.11 standard.

Question 2

A communication system using a rate-3/4 FEC code has a BER of 10^{-5} at an SNR of 10 dB. Without coding the BER is 10^{-5} at an SNR of 13 dB. Both systems operate over a 0.5 MHz bandwidth at a bit rate, including any parity bits, of 1 Mb/s.

(a) What is E_b/N_0 without coding in dB?

(b) What is E_b/N_0 with coding in dB?

(c) What is the coding gain at a BER of 10^{-5} in dB?

Hint: In this problem the bandwidth is given explicitly instead of being assumed to be $f_s/2$.

Solution

E_b/N_0can be computed as:

$$E_b/N_0 = \frac{S}{N} \cdot \frac{W}{R}$$

where W is the bandwidth (0.5 MHz) and R is the information bit rate (1 Mb/s without coding and $1 \times 0.75 = 0.75$ Mb/s with rate-3/4 coding).

(a) without coding the SNR is 13 dB and $W/R = 0.5/1 \approx -3$ dB so E_b/N_0without coding is $13-3 = 10$ dB.

(b) with coding the SNR is 10 dB and $W/R = 0.5/0.75 \approx -1.8$ dB so E_b/N_0without coding is $10 - 1.8 = 8.2$ dB.

(c) The coding gain is the reduction in E_b/N_0 with coding at a given BER. In this case it's $10-8.2 = 1.8$ dB.

Question 3

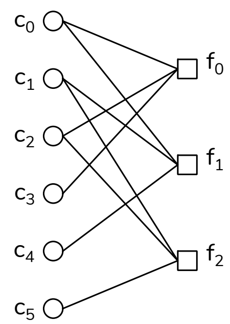
Draw the Tanner graph for a ~~(3,6)~~ (6,3) systematic block code whose parity check matrix is:

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Solution

The Tanner graph is a bipartite graph of the parity check equations. Each equations is the sum of the bits in each row of the parity check matrix that has a 1. In the graph below the nodes on the left are the bits in the received codeword and nodes on the right are the parity bits. The edges (lines) are drawn between the bits in the codeword and the parity bits that include those codeword bits.

$$H = \begin{bmatrix} 1\,0\,1\,1\,0\,0 \\ 1\,1\,0\,0\,1\,0 \\ 0\,1\,1\,0\,0\,1 \end{bmatrix}$$



Question 4

An OFDM system uses symbols of $N = 128$ samples. The sampling rate is 40 MHz and the duration of the cyclic prefix is 800 ns. Only 112 of the subcarriers are used to transmit data using 16-QAM modulation with rate-$1/2$ FEC coding.

(a) What is the duration of an OFDM symbol plus the cyclic prefix in units of microseconds?

(b) How many information bits (i.e. not including parity bits), are transmitted per symbol?

(c) What is the throughput in bits/second taking into account the effect of FEC, number of subcarriers used and the cyclic prefix?

Solution

(a) The duration of the OFDM symbol is the number of samples (N) times the sample period ($1/40$ MHz $=$ 25 ns) $= 128 \cdot 25$ ns $= 3.2$ µs. The duration including the 0.5 µs cyclic prefix is $3.2 + 0.8\mu\text{s} = 4\ \mu s$

(b) 16-QAM encodes $\log_2(16) = 4$ bits on each of the 112 subcarriers for $112 \times 4 = 448$ bits per symbol. But with rate-$1/2$ coding only $448 \cdot \frac{1}{2} = 224$ are information bits.

(c) The throughput is the average number of useful ("information") bits delivered to the user. Taking the above effects into account, this is: $\frac{224\text{ bits}}{4\ \mu\text{s}} = 56$ Mb/s.