Antennas and Free-Space Propagation

Exercise 1: If the effective area of an antenna is $1\ \mathrm{m}^2$, what is the path loss, in dB, at a distance of 100 m? At the distance to a geo-stationary satellite ($\approx 36,000$ km)? How does it increase (in dB) with distance?

$$A_e = 1\,\mathrm{m}^2 \qquad d = 100\,\mathrm{m}$$

$$\frac{P_T}{P_R} = \frac{4\pi d^2}{A_e} = \frac{4\pi\, 10^4}{1} = 126{,}000$$

$$= 51\,\mathrm{dB}$$

$$= \frac{4\pi\,(36\times 10^6)^2}{1} = 162\ \mathrm{dB}.$$

$$10\log\left(\frac{d_2}{d_1}\right)^2 =$$

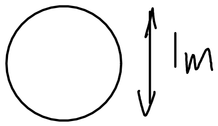
20 dB / decade increase in distance.

Exercise 2: What is the directivity of an isotropic radiator?

$$U_m \qquad U_0$$

$$\frac{U_m}{U_0} = 1$$

Exercise 3: For some types of antennas, such as reflectors, the effective aperture can be approximated by the physical area of the antenna[1]. What are the approximate effective aperture and directivity of a 1-m diameter satellite dish antenna receiving signals at $\approx$ 15 GHz ("Ku-band")?



$$f = 15 \times 10^9 \text{ Hz}$$

$$\lambda = \frac{3 \times 10^8}{150 \times 10^8} = 2 \text{ cm.}$$

$$A_e \approx \pi \left(\frac{1}{2}\right)^2 = 0.75 \text{ m}^2$$

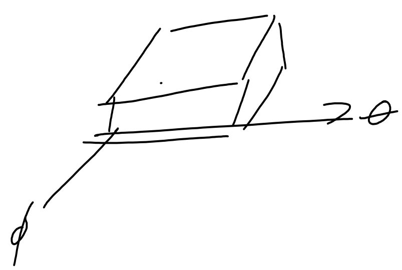
$$\checkmark \ \frac{A_e}{D} = \frac{\lambda^2}{4\pi} \qquad D = \frac{0.75 \cdot 4\pi}{(0.02)^2} = 2 \times 10^4$$

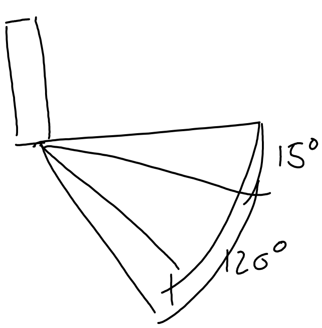
$$\simeq 43 \text{ dB}$$

Exercise 4: What is the maximum value of k?

if $U_0 = U_m$ $\quad k = 1$

Exercise 5: Another useful approximation relates the gain of an antenna to it's beamwidth. Since a sphere has a surface "solid angle" of 4π steradians ($\approx$ 41253 square degrees), we can approximate the gain by dividing this by the solid angle covered by an ideal (rectangular, "brick-wall") antenna pattern. What is the approximate directivity of an antenna with beamwidths of 15×120 degrees? If the antenna's efficiency is $k = 70\%$, what is the gain?





$k = 0.7$

$$G \approx \frac{41\,253}{15 \times 120} \cdot 0.7$$

$$\sim 16 \qquad \simeq 12 \text{ dB}$$

Exercise 6: A point-to-point link uses a transmit power of 1 Watt, transmit and receive antennas with gains of 20dB and operates at 3 GHz. How much power is received by a receiver 300m away?

$P_T = 1\,W \qquad G_T = G_R = 20\,dB = 10^{\frac{20}{10}} = 100$

$d = 300\,m$

$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \qquad \lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9}$

$= 0.1\,m$

$= 1 \cdot 10^2 \cdot 10^2 \left(\frac{0.1}{4\pi \cdot 300}\right)^2$

$= \cancel{10^4} \frac{10^{-2}}{(4\pi)^2\, 9 \times \cancel{10^4}} = 7 \times 10^{-6}$

$= \frac{V^2}{R}$

$\simeq 18\,mV$ @ $50\,\Omega$

Exercise 7: What is the far-field distance for an cell phone antenna operating at 3 GHz that has a physical size of $1 \times 1 \times 3$ cm? For a 100 m diameter antenna?

$3\,GHz \rightarrow \lambda = 0.1\,m$

for

$L = 0.03\,m \qquad d = \frac{L^2}{\lambda} = \frac{(.03)^2}{0.1} = 9\,mm$??

for

$L = 100\,m \qquad d = \frac{L^2}{\lambda} = \frac{10^4}{.1} = 100\,km$

Exercise 8: If we kept the *effective aperture* (not gain) constant at one end of a link (transmitter or receiver), how would the path loss change as a function of frequency? What if we kept it constant at both ends? Is this a feasible approach for mobile systems?

at one end:

$$\frac{P_R}{P_T} = G_T \frac{4\pi A_{er}}{\cancel{\lambda^2}} \left(\frac{\cancel{\lambda}}{4\pi d}\right)^2 \rightarrow$$

if A_e is fixed at one end (Rx) then path loss independent of λ

at both ends

$$= \frac{4\pi A_{et}}{\lambda^2} \frac{4\pi A_{er}}{\cancel{\lambda^2}} \left(\frac{\cancel{\lambda}}{4\pi d}\right)^2$$

Path loss decreases with frequency