RF Design - Noise

Noise Temperature

Noise is an unpredictable (random) voltage that is superimposed on a signal. There are various sources of noise. For example, the random motion of electrons in any conductor results in "thermal" noise. These noise sources have a broad spectrum, typically constant over the band of interest.

Because the noise is broad-band we typically measure the noise power spectral density (PSD) and reference it to the thermal noise PSD that would be generated by a resistor at a given temperature. This PSD is kT where k is Boltzmann's constant, and T is the temperature in Kelvin. When passed through a filter with a (brick-wall or integrated) bandwidth B, the noise power is:

$$N = kTB \ .$$

Using the equation above we can convert an RF noise power N to a "Noise Temperature," T. Unlike the noise power, the noise temperature is independent of the bandwidth.

Noise Figure

Noise figure[1] is a measure of the noise performance of a device that is useful for calculations involving several devices operating in cascade (series). It is the ratio of the input SNR to the output SNR:

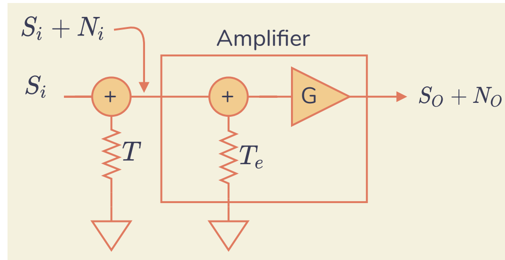
$$F = \frac{S_i/N_i}{S_o/N_o} = \frac{S_i/S_o}{N_o/N_i} \ .$$

For example, for an attenuator the thermal noise level at the output is the same as at the input and $N_o = N_i$. If the output signal power is $S_o = LS_i$ then noise figure will be $1/L$.

Exercise 1: What is the noise figure of a 6 dB attenuator?

A practical amplifier not only amplifies the input but adds additional noise. We can convert the noise power added by the amplifier to an equivalent temperature, T_e. Thus the apparent noise added by the amplifier is kT_eB.

Consider an amplifier with gain G:



The amplifier outputs a signal power $S_o = GS_i$. If the noise input power is $N_i = kTB$ then the noise output power is $N_o = Gk(T + T_e)B$. The noise figure is thus:

$$F = \frac{G \cdot k(T + T_e)B}{G \cdot kTB} = \frac{T + T_e}{T} \ .$$

Since F is a function of T, a standard reference input temperature, $T = T_0 = 290$ K is used when specifying a device's noise figure. $kT_0 = 4 \times 10^{-21}$W/Hz $= -174$ dBm/Hz for $T_0 = 290$ K.

Exercise 2: What are the minimum possible values of T_e and F?

Exercise 3: The datasheet for a low-noise amplifier (LNA) specifies a noise figure of 2 dB. What is the noise temperature T_e?

Note that the input noise temperature T varies depending on what is connected to the input of the amplifier. The value $T_0 = 290$ K is typical for an antenna pointed at the ground. But T may be much lower for an antenna pointed into deep space, and much higher for a directional antenna pointed at the sun.

Cryogenically-cooled low-noise amplifiers used for radio astronomy and satellite ground stations can have noise temperatures of a few K. However, conventional low-noise amplifiers (LNAs) can have noise temperatures higher than their physical temperature.

The SNR ratio definition for noise figure can be used for other two-port devices such as mixers (ignoring the LO terminal). Although one-port devices such as oscillators generate noise, they do not have noise figures.

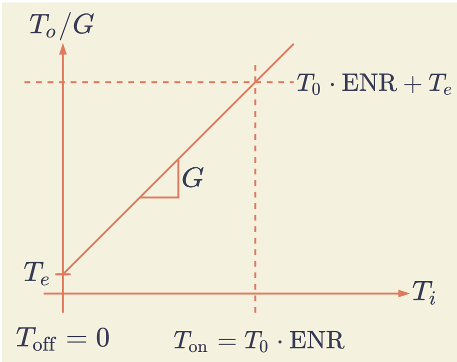
Exercise 4: An LNA with a noise figure of 0.3 dB receives a signal with an SNR of 6 dB. What is the output SNR?

Measuring Noise Figure

A spectrum analyzer (or special-purpose receiver called a "Noise Figure Meter") and a calibrated noise

[1] Often called "noise factor" when specified in linear units.

source can be used to measure an amplifier's noise figure. The figure below shows the output noise power (temperature) as a function of the input noise power (temperature):



The noise source outputs a noise level defined by its "excess noise ratio" (ENR) $= T_{\mathrm{on}}/T_0$. Since the noise source and amplifier-generated noise are independent, their powers will add and it's possible to solve for T_e and F.

The "Y-factor" method uses the ratio of two output powers with the source on and off: $Y = {}^{(T_0\mathrm{ENR}+T_e)}/_{T_e}$. We can solve for $T_e = {}^{T_0\mathrm{ENR}}/_{(Y-1)}$ and using $F = 1 + {}^{T_e}/_{T_0}$ we find $F_{\mathrm{dB}} = \mathrm{ENR}_{\mathrm{dB}} - 10\log_{10}(Y-1)$.

Corrections can be made for a source temperature when off, $T_{\mathrm{off}} \neq 0$ and for the noise added by the spectrum analyzer. These are measured by connecting the noise source directly to the spectrum analyzer and measuring the spectrum analyzer noise figure and T_{off}. Modern instruments can make automated noise figure measurements by turning the external noise source on and off and then computing and applying these corrections.

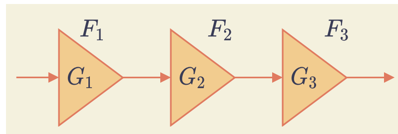
Exercise 5: A noise source with an ENR of 15 dB is connected to an LNA. The noise PSD at the output of the LNA is measured as −152 dBm/Hz and with the noise source on and −165.2 dBm/Hz with it off. Assuming the spectrum analyzer adds negligible noise and the "off" noise source temperature is 0 K, what are T_e and F? Do not confuse mW and dBm.

Cascade Noise Figure

When multiple devices (amplifiers, attenuators, mixers, etc) are connected in series (cascade) both the signal and noise generated by each stage are amplified by subsequent stages. However, while the signal is amplified by each stage, the noise contributed by one stage is only amplified by the succeeding stages. Thus the impact of noise added by later stages has less impact on the overall SNR and thus has less impact on the overall noise figure.

We can use the same output/input SNR-ratio definition of noise figure for the cascaded system as for a single device.

Friis, the Bell Labs researcher who derived the free-space path loss formula, also derived a formula for the noise figure of a cascade of devices. It is possible to show that when several devices (amplifiers, attenuators, mixers, etc) with gains $G_1, G_2, \ldots$ and noise figures $F_1, F_2, \ldots$ (both in linear units) are connected in series (cascade):



the overall (or "system" or "cascade") noise figure is given by:

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots$$

and the equivalent noise temperature is given by:

$$T = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots$$

The significance of these formulas is that the noise figure of the first amplifier has the most impact on the overall system noise figure. Thus the first ("front end") amplifier in a receiver should be chosen to have the lowest possible noise figure and enough gain to reduce the impact of the noise figure of subsequent stages.

Exercise 6: A What is the system noise figure of a receiver that consists of a 10 dB amplifier with 3 dB noise figure followed by a mixer with a 6 dB loss and an IF amplifier with a 20dB gain and a noise figure of 10 dB?

Cascade IP3

When multiple amplifiers are connected in series (cascade) the signal level at the input to the second amplifier is higher than the level at the input to the first stage. We would thus expect that the IP3 of the cascade would be determined primarily by the IP3 of the final amplifier.

It is possible to show that the input IP3 (in linear units) of a cascade of amplifiers with gains $G_1, G_2, \ldots$ and input IP3's $I_1, I_2 \ldots$ is:

$$\frac{1}{IIP3} = \frac{1}{I_1} + \frac{G_1}{I_2} + \frac{G_1 G_2}{I_3} + \ldots$$

When the gains are significant, the IIP3 of the last stage predominates.