Solutions to Final Exam

Question 1

Assuming the same transmit and receive antennas are used, what antenna gains would be required to achieve a received signal level of −80 dBm over a 10 km LOS path at a frequency of 2.4 GHz if the transmit power is 20 dBm? Give your answer in dBi.

Solution

Using the Friis equation:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

setting $G_T = G_R = G_A$ and solving for G_A:

$$G_A = \sqrt{\frac{P_R}{P_T}} \left(\frac{4\pi d}{\lambda}\right)$$

Substituting $P_R = 10^{-80/10}$ mW, $P_T = 10^{20/10}$ mW, $\lambda = {}^{3\times10^8}/_{2.4\times10^9}$ m, $d = 10^4$ m, we find $G_A \boxed{\approx 10 \text{ dB}}$.

Question 2

You observe a pseudo-random bit pattern that you believe is from a ML PRBS generator. The longest sequence of consecutive 1's is 24 bits long. What would you expect to be the period of the sequence? Give your answer in bits.

Solution

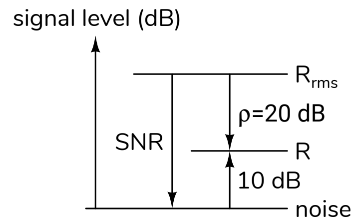
One of the properties of an m-sequence is that there is one run of length m ones (and all other run lengths are shorter). Thus $m = 24$ and the period is $2^m - 1 = 2^{24} - 1 = \boxed{16,777,215 \text{ bits}}$.

Question 3

A signal with a Rayleigh-distributed amplitude is received along with constant-power additive noise. You require that the signal be faded less than 1% of the time. A fade is defined as an SNR of less than 10 dB. What average SNR is required? *Hint: Since the noise level is constant, the SNR is proportional to the signal power.*

Solution

The following diagram describes the relationship (in dB) between the noise level, the average signal power level (R_{rms}), the fading threshold (R) and the SNR:



A Rayleigh random variable has a CDF:

$$P(r \le R) = 1 - e^{-\rho^2}$$

Solving for $\rho = R/R_{\text{rms}}$ when the probability of a fade is 1%:

$$\begin{aligned}\rho &= \sqrt{-\ln(1 - P(r \le R))} \\ &= \sqrt{-\ln(1 - 0.01)} \\ &\approx -20 \text{ dB}\end{aligned}$$

Since the fading threshold R is at 10 dB above the noise, the mean signal power must be 20 dB above this, at $\approx 10 + 20$ $\boxed{\approx 30 \text{ dB SNR}}$.

Question 4

A signal is received by a moving receiver over a NLOS channel and the directions of arrival are uniformly distributed as in Clarke's model. The signal is faded 10% of the time. You measure an average of 10 fades per second. What is the average fade duration?

Solution

As shown in one of the exercises, for this model the fraction of time the signal is below a threshold, $(P(r \le$

R)) is equal to the product of the level crossing rate (N_R) and the average duration below that level ($\overline{\tau}$):

$$P(r \leq R) = N_R \cdot \overline{\tau}$$

and we can solve for the average fade duration:

$$\overline{\tau} = P(r \leq R)/N_R = 0.1/10 = \boxed{10 \text{ ms}}$$

Question 5

You set up a test transmitter in a warehouse and make some measurements of the received signal strength. The mean received signal level at a distance of 10 m is −40 dBm. The path loss exponent appears to be 3.5. The standard deviation of the path loss (due to shadow fading) appears to be 9 dB. You need to achieve a signal level of more than −80 dBm at distances of up to 100 m with 95% probability.

Assuming a power law is an appropriate model for the mean signal level and a log-normal model is a reasonable model for probability distribution of path loss due to shadowing, by how much do you need to increase (or decrease) the transmit power?

Hint: the probability that normally-distributed random variable has a value 1.65 standard deviations above the mean is about 5%.

Solution

The power law model in dB is:

$$PL(d)_{\text{dB}} = PL(d_0)_{\text{dB}} + 10n \log\left(\frac{d}{d_0}\right)$$

and for $d_0 = 10$ m, $d = 100$ m, $n = 3.5$:

$$\begin{aligned} PL(100 \text{ m}) &= PL(10 \text{ m}) + 10 \cdot 3.5 \log\left(\frac{100}{10}\right) \\ &= PL(10 \text{ m}) + 35 \text{ dB} \end{aligned}$$

and the mean signal level at 100 m will be $-40-35 = -75$ dBm and the transmit power can be reduced by 5 dB.

If the shadow fading has a log-normal distribution with a standard deviation of $\sigma = 9$ dB, then 95% of the time attenuation will be $1.65\sigma = 14.9$ dB less than the mean (as given in the hint) and 5% of the time it will be more. Thus the transmit power must be increased by 14.9 dB.

Combining the two effects, the transmit power must be increased by $14.9 - 5 \boxed{\approx 10 \text{ dB}}$.

Question 6

A wireless system uses selection diversity. Without diversity the probability of a fade is 20%. How many independently-fading branches would be required to reduce the probability of a fade to less than 1%? Your answer should be an integer.

Solution

If the probability that one branch is faded is p then the probability that M independently-fading diversity branches are faded is p^M. For $p = 0.2$ p^2 is 4% while p^3 is 0.8% so $\boxed{\text{3 branches}}$ are needed.

Question 7

You need to add FEC to a existing communication system to reduce the error rate to a very low value. You only have access to the binary data at the input and output of the system. The system transmits and receives bits at a rate of 100 kb/s and the BER without FEC is 8×10^{-3}. What is fastest possible rate at which you can hope to transmit information over this channel with an arbitrarily low error rate?

Solution

Since the system transmits binary data and the error rates are symmetrical, the BSC channel model with $p = 8 \times 10^{-3}$ is appropriate. The capacity (per bit) is:

$$\begin{aligned} C &= 1 - (-p \log_2 p - (1-p) \log_2 (1-p)) \\ &\approx 0.93 \text{ bits/bit} \end{aligned}$$

Since the bit rates is 100 kb/s the capacity in bits/second is $100 \times 10^3 \times 0.93 \boxed{\approx 93 \text{ kb/s}}$. By definition this is the fastest rate at which information can be transmitted over the channel with an arbitrarily low error rate.

Question 8

You have decided to use a Reed-Solomon code using symbols from GF(64) in a communication system. From simulations you've determined that you need to correct errors in at least 10% of the symbols.

What is the block size in units of symbols? How many parity symbols should you use in each block? Your answers should be integers.

Solution

RS codes use blocks of length $n = 2^m - 1$ symbols where m is the number of bits per symbol. For symbols from GF(64) $m = \log_2(64) = 6$ and blocks are $n = 2^6 - 1 = \boxed{63 \text{ symbols}}$ long.

10% of this is 6.3 which rounds up to 7 symbols. A RS code can correct $\frac{(n-k)}{2}$ symbol errors so setting this equal to 7 we find $k = n - 7 \times 2 = 63 - 14 = 49$. The number of parity symbols required is $n - k = \boxed{14 \text{ symbols}}$.
[To correct an *average* of 6.3 errors per frame rather than 6.3 in each frame then we would need $\lceil 2 \times 6.3 \rceil = \boxed{13}$ symbols.]

Question 9

You measure the SNR at the IF of a receiver as 10 dB. The noise bandwidth is 350 kHz. The bit rate is 1 Mb/s. No FEC is used. What is the E_b/N_0? Give your answer in dB.

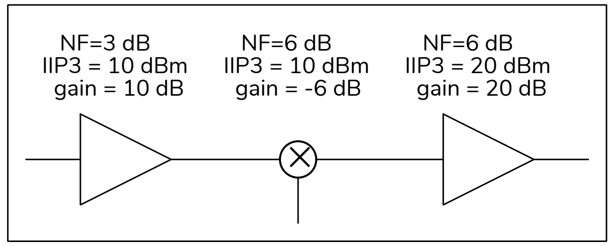
Solution

The energy per bit, E_b, is computed as the signal power divided by the bit rate: $E_b = S/f_b$. The noise power spectral density, N_0, is computed as the noise power divided by the noise bandwidth: $N_0 = N/W$. $E_b/N_0$is thus:

$$
\begin{aligned}
\frac{E_b}{N_0} &= \frac{S}{N} \cdot \frac{W}{f_b} \\
&= 10^{10/10} \cdot \frac{350^3}{1 \times 10^6} \\
&= 3.5 \boxed{\approx 5.4 \text{ dB}}
\end{aligned}
$$

Question 10

The following diagram shows the front end of a receiver. The input IP3 (IIP3), noise figure (NF) and gain of each stage are as shown. What are the overall (cascade) input IP3 and NF?



Solution

In this problem $G_1 = 10^{10/10} = 10$, $G_2 = 10^{-6/10} = 0.25$, $G_3 = 10^{20/10} = 100$, $I_1 = 10^{10/10} = 10$, $I_2 = 10^{10/10} = 10$, $I_3 = 10^{20/10} = 100$, $F_1 = 10^{3/10} = 2$, $F_2 = 10^{6/10} = 4$, $F_3 = 10^{6/10} = 4$,

The cascade input IP3 can be calculated as:

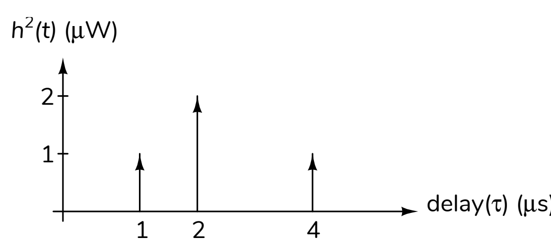
$$
\begin{aligned}
IIP3 &= \left(\frac{1}{I_1} + \frac{G_1}{I_2} + \frac{G_1 G_2}{I_3}\right)^{-1} \\
&= \left(\frac{1}{10} + \frac{10}{10} + \frac{10 \cdot 0.25}{100}\right)^{-1} \\
&\approx \boxed{0.89 \text{ mW} \approx -0.51 \text{ dBm}}
\end{aligned}
$$

The cascade noise figure can be calculated as:

$$
\begin{aligned}
F &= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots \\
&= 2 + \frac{4-1}{10} + \frac{4-1}{10 \cdot 0.25} \\
&= \boxed{3.5 \approx 5.4 \text{ dB}}
\end{aligned}
$$

Question 11

You measure the impulse response of a channel to be as shown below:



How would you report the (excess) delay spread of this channel? Give the value and the units.

Solution

The following table shows the delays, (t_i), excess delays (τ), powers ($P(\tau)$), normalized powers ($p(\tau)$), the products used to compute the mean delay ($\tau p(\tau)$), and the product used to compute the delay spread, ($p(\tau)(\tau-\bar{\tau})^2$). The sums of the columns used to compute $p(\tau)$, $\bar{\tau}$ and σ^2 are shown on the last row.

t	τ	$P(\tau)$	$p(\tau)$	$\tau p(\tau)$	$p(\tau)(\tau-\bar{\tau})^2$
1	0	1	0.25	0	0.391
2	1	2	0.5	0.5	0.031
4	3	1	0.25	0.75	0.765
		4		$\bar{\tau}$ =1.25	σ^2 =1.18

The delay spread of the channel is:

$$\begin{aligned}\sigma &= \sqrt{\sum p(\tau)(\tau-\bar{\tau})^2} \\ &= \sqrt{1.18} = \boxed{1.1\ \mu\text{s}}\end{aligned}$$

Question 12

You are considering using MIMO to improve the data rate of a communication system. You measure the channel matrix using arrays of 5 transmit and 5 receive antennas. You compute the eigenvalues of the channel matrix and find that in most cases this matrix has 3 significant (non-zero) eigenvalues. By how much would you expect to be able to improve the data rate of the system using these antennas? Explain briefly.

Solution

If a matrix has N non-zero eigenvalues then it represents a system of N linearly independent equations.

In this question the channel matrix has $N = 3$ non-zero eigenvalues and it should be possible to transmit $N = 3$ independent data streams thus improving the data rate by a factor of $\boxed{3}$.