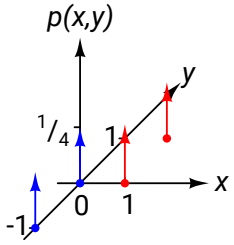
Solutions to Quiz 2

-

Question 1

The input to a channel can have the value 0 or 1. The output can have the values −1, 0 or 1. The joint probability density is shown in the following table and (corrected) diagram:

X	Y		
	−1	0	1
0	1/4	1/4	0
1	0	1/4	1/4



For example the joint probability $p(x = 0, y = 1) = 0$ and $p(x = 1, y = 1) = 1/4$ and the marginal probability $p(x = 0) = 1/2$.

What is the mutual information between the input and output of this channel in bits per channel use?

Answer

The mutual information is defined as:

$$\mathrm{I}(X;Y) = \sum_{y\in\mathcal{Y}} \sum_{x\in\mathcal{X}} p(x,y) \log_2\left(\frac{p(x,y)}{p(x)\,p(y)}\right) \quad \frac{\text{bits}}{\text{channel use}}$$

The table in Figure 1 summarizes the calculation showing that the mutual information is 0.5 bits per channel use.

Question 2

The input to (another) channel can be 0 or 1 and the output can be 0 or 1. The probability of error (that a 0 is received as a 1 or vice-versa) is 0.1. What is the capacity of this channel in "information bits per bit transmitted over the channel"?

Answer

From the description this is a binary symmetric channel (BSC) with $p = 0.1$. The capacity of the BSC is given by:

$$C = 1 - (-p \log_2 p - (1-p)\log_2(1-p))$$

which we can calculate in Matlab as:

```
octave:5> p=0.1
p =  0.10000
octave:6> 1-(-p*\log_2(p)-(1-p)*log2(1-p))
ans =  0.53100
```

Question 3

A message is transmitted together with a CRC computed using the simplified algorithm described in the lecture notes. The generator polynomial is x^3+1. The message received, including the CRC, is **10101111**. Does the CRC indicate the received message has an error? Show your work.

Answer

We can divide the message polynomial by the generator polynomial and check that the remainder is zero:

```
     ___10111_
1001|10101111
     1001
     ----
      0111
      ----
       1111
       1001
       ----
        1101
        1001
        ----
         1001
         1001
         ----
          000
```

Since the remainder is zero, the CRC does not indicate that the message has an error.

x	y	$p(x,y)$	$p(x)$	$p(y)$	$p(x,y)\log_2\left(\frac{p(x,y)}{p(x)\,p(y)}\right)$		
0	-1	$1/4$	$1/2$	$1/4$	$1/4\log_2(1/4/(1/2\cdot 1/4))$	$=$	$1/4$
0	0	$1/4$	$1/2$	$1/2$	$1/4\log_2(1/4/(1/2\cdot 1/2))$	$=$	0
1	0	$1/4$	$1/2$	$1/2$	$1/4\log_2(1/4/(1/2\cdot 1/2))$	$=$	0
1	$+1$	$1/4$	$1/2$	$1/4$	$1/4\log_2(1/4/(1/2\cdot 1/4))$	$=$	$1/4$
					sum	$=$	$1/2$

Figure 1: Calculation of Mutual Information (Question 1).