

Solutions to Quiz 1

Question 1

A vehicle is receiving a Rayleigh-fading signal that has a mean received signal level of -60 dBm. The received signal is considered faded when it is below -76 dBm. What fraction of the time is the signal faded?

Answer

The cumulative distribution (CDF) of a Rayleigh random variable,

$$P(r \leq R) = 1 - e^{-\rho^2}$$

where

$$\rho = R/R_{rms}$$

In this case the fading threshold is $R = -76$ dBm and the mean signal level is $R_{rms} = -60$ dBm,

$$\rho = 10^{(-76 - (-60))/20} \approx 0.16$$

and the probability that the signal is faded is $1 - e^{-\rho^2} \approx 2.5 \times 10^{-2}$.

Using Matlab (Octave, actually) as a calculator:

```
octave:4> rho=10^((-76-(-60))/20)
rho = 0.15849
octave:5> 1-exp(-rho^2)
ans = 0.024806
```

Question 2

The same vehicle as above is travelling at 100 km/h and receiving a signal at a frequency of 1900 MHz. It is receiving data at a rate of 100 kb/s. Assuming Clarke's multipath model applies, how many bits, on average, are transmitted during each fade? Assume the same mean and 'faded' signal levels as in the previous question.

Answer

The average fade duration for Clarke's model is:

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

where

$$f_D = \frac{v}{\lambda} = \frac{v}{c} f_c$$

ρ is as before,

$$f_m = f_D = \frac{100 \times 10^3 / 3600}{3 \times 10^8} \cdot 1.9 \times 10^9 \approx 176 \text{ Hz}$$

and

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} \approx 0.36 \text{ ms}$$

At a bit rate of 100 kb/s the average number of bits transmitted each fade is thus $\approx 0.36 \times 10^{-3} \cdot 100 \times 10^3 = 36$ bits, calculated as shown below:

```
octave:14> fm=(100e3/3600)/3e8*1900e6
fm = 175.93
octave:15> taubar=(exp(rho^2)-1)/(rho*fm*sqrt(2*pi))
taubar = 0.00036395
octave:16> taubar*100e3
ans = 36.395
```

Question 3

A signal arrives at a portable receiver over three paths with the following path lengths and attenuations:

- 300 m, 60 dB
- 100 m, 54 dB
- 50 m, 50 dB

For example, a signal transmitted with a power of 0 dBm would be received over the first path $1 \mu\text{s}$ later at a level of -60 dBm.

- What is the mean excess delay of this channel?
- What is the RMS delay spread?

Answer

The mean excess delay is defined as:

$$\bar{\tau} = \sum p(\tau)\tau$$

where

$$p(\tau) = \frac{P(\tau)}{\sum P(\tau)}$$

and τ_i are the excess delays.

The “RMS delay spread” of the channel is

$$\sigma = \sqrt{\sum p(\tau)(\tau - \bar{\tau})^2}$$

The (absolute) delays can be computed from the path lengths as d/c . These are converted to excess delays (τ) by subtracting the minimum delay. The un-normalized signal levels (P) can be computed as the negatives (in dB) of the attenuations. Note that the actual signal powers don't affect the values of mean delay and the delay spread because these use are computed using the normalized power profile (total power = 1).

The mean excess delay and RMS delay spreads can be computed using a spreadsheet:

d	delay	tau	att (dB)	P	p	p(tau)*tau		p(tau)*(tau-taubar)^2
300	1E-6	833E-9	60E+0	1E-6	67E-3	56E-9		36E-15
100	333E-9	167E-9	54E+0	4E-6	266E-3	44E-9		1E-15
50	167E-9	0E+0	50E+0	10E-6	668E-3	0E+0		7E-15
<i>min:</i>	167E-9		<i>sum:</i>	15E-6	<i>mean excess delay:</i>	100E-9	<i>rms delay spread:</i>	209E-9

or the following Octave script:

```
format short eng
att=[60, 54, 50];
d=[300,100,50];
tau=(d-min(d))/3e8;
P=10.^(-att/10);
p=P/sum(P);
taubar=sum(p.*tau)
sigma=sqrt(sum(p.*(tau-taubar).^2))
```

which outputs:

```
taubar = 99.9158e-009
sigma = 209.1729e-009
```