Solutions to Midterm Exam 2

Question 1

A company is claiming that they have a modulation and FEC technique that will allow nearly error-free data transmission at a rate of 80 Mb/s over a 20 MHz channel that has an SNR of 5 dB with white, Gaussian noise.

Give a brief explanation of why this is, or is not, credible. Show your calculations.

Answer

The Shanon capacity for an AWGN channel is $C = B\log_2(1 + S/N)$. For this system $B = 20 \times 10^6$ and $S/N = 10^{5/10} \approx 3.2$ so $C \approx 40$ Mb/s. The company is thus claiming that their system is able to transmit data error-free at a rate greater than the channel capacity. This violates the Shanon capacity theorem and so the claim is not credible.

[Such claims are still made. Typically they involve using some sort of compression to reduce the information rate transmitted over the channel.]

Question 2

The generator matrix for a code is:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(a) What are n and k?

(b) What is the transmitted codeword when the data word is [1 0 1]?

(c) What is the parity check matrix for this code?

Answer

(a) The dimensions of the generator matrix are $k\times n$ and the matrix is 3×5 thus $k = 3$ and $n = 5$.

(b) The transmitted codeword [1 0 1] G which evaluates to [1 0 1 0 1] (the sums are modulo-2).

(c) The parity check matrix is:

$$H = [P^T \,|\, I_{n-k}]$$

which in this case is:

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Question 3

A code uses the following parity check matrix:

$$H = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

What is the minimum distance of the code? Show your work.

Answer

The corresponding generator matrix is:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

There are four possible data vectors: [0 0], [0 1], [1 0] and [1 1] with corresponding codewords [0 0 0 0], [0 1 1 1], [1 0 0 1] and [1 1 1 0].

The distances between these codewords are shown in the table below:

	0000	0111	1001	1110
0000	0	3	2	3
0111		0	3	2
1001			0	3
1110				0

from which we can see that the minimum (Hamming) distance of the code is 2.

Question 4

A communication system uses a rate-3/4 FEC code to achieve an error rate of 1×10^{-5} at an SNR of 10 dB. An SNR of 16 dB is required to achieve this error rate if FEC is not used. In both cases the channel bandwidth is 500 kHz and the transmitted data rate, including any FEC, is 1 Mb/s.

What is the coding gain as defined by the reduction in E_b/N_0 required to obtain an error rate of 1×10^{-5}?

Answer

We can calculate

$$\frac{E_b}{N_0} = \frac{S/R}{N/B} = \frac{S}{N} \cdot \frac{B}{R}$$

where S/N is the SNR, R is the information bit rate (before adding FEC) and B is the bandwidth (given as 500 kHz).

With FEC R is 750 kbps because of the rate-3/4 code and the SNR is 10 dB (10). Without FEC R is 1 Mb/s and the SNR is 16 dB (40).

The ratio of E_b/N_0 without FEC to that with FEC is thus:

$$\frac{40 \cdot \frac{B}{1000 \times 10^3}}{10 \cdot \frac{B}{750 \times 10^3}} = 4 \times \frac{3}{4} = 3$$

Thus the coding gain is 3 or 4.8 dB.

Question 5

An OFDM system uses 1024 subcarriers and a sampling rate of 10 MHz. It operates over a channel with a delay spread of 10 microseconds. The guard time duration is twice the channel delay spread. Only 612 of the 1024 the subcarriers are transmitted and each of these is modulated using QPSK modulation (2 bits per subcarrier).

(a) What is the total duration of each OFDM symbol, including the guard time?

(b) How many bits are transmitted per second?

(c) What is the approximate bandwidth of the signal?

(d) What is the spectral efficiency of this system in bits per second per Hz?

Answer

(a) The guard time is twice the delay spread or $2 \times 10\ \mu$s. If there are $N = 1024$ subcarriers there are also 1024 samples per OFDM symbol. At a sampling rate of 10 MHz, the OFDM symbol duration is $1024/10 \times 10^6 = 102.4\ \mu$s. The total duration of each OFDM symbol, including the guard time is thus $102.4 + 20 = 122.4\ \mu$s.

(b) If we transmit 2 bits in each of 612 subcarriers, we transmit 1224 bits in 122.4 μs or 10 Mb/s.

(c) The bandwidth of the signal is approximately the subcarrier spacing times the number of subcarriers used. The subcarrier spacing is the inverse of the OFDM symbol duration. Thus the bandwidth is approximately $612 \times 10 \times 10^6/1024 \approx 6$ MHz.

(d) The spectral efficiency is $\approx \frac{10 \text{ Mb/s}}{6 \text{ MHz}} \approx 1.67$ b/s/Hz.