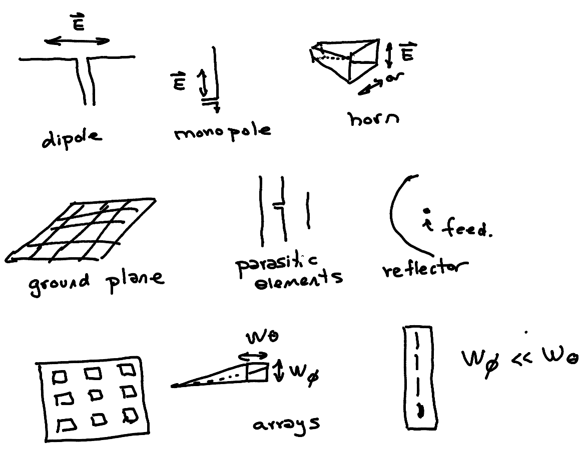
Antennas and Free-Space Propagation

Antennas

As predicted by Maxwell and demonstrated by Hertz, time-varying electromagnetic fields can propagate through free space[1]. Propagation of EM fields at frequencies between 3 kHz and 3 THz can be used for wireless communication.

Antennas are passive devices that couple RF currents flowing along conductors to electromagnetic fields propagating through space.

Some common antennas include dipoles, monopoles, and horns. These are often combined with ground planes, parasitic elements or reflectors. Antennas are often connected together in arrays.



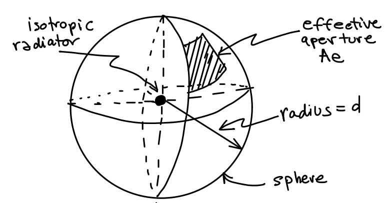
The propagating EM field has perpendicular electric and magnetic fields that vary with time (at the RF frequency) and distance (at about 3×10^8 m/s). The EM fields are also perpendicular to the direction of propagation. The polarization of the EM field refers to the orientation of the peak electric field. The polarization can be fixed (linear polarization, typically vertical or horizontal) or can vary with time and space (right-hand or left-hand circular polarization).

Part of the power fed into an antenna radiates into space and is considered to be absorbed by a hypothetical "radiation resistance" while part is absorbed by the antenna itself and dissipated as heat.

[1]This "action at a distance" was a surprising phenomena although its applications are now so common it no longer appears so.

Effective Area

Consider an isotropic source – one that transmits equally in all directions – transmitting power P_T at the center of a sphere of radius d (distance) and a receiving antenna of area A_e that collects all of the power incident ("shining") on it:



Since the transmit power is equally distributed over the surface of the sphere (of area $= 4\pi d^2$), the ratio of the received power to transmitted power, the "path loss," is:

$$\frac{P_R}{P_T} = \frac{A_e}{4\pi d^2}$$

Exercise 1: If the effective area of an antenna is 1 m^2, what is the path loss, in dB, at a distance of 100 m? At the distance to a geostationary satellite ($\approx 36,000$ km)? How does it increase (in dB) with distance?

Pattern. The power radiated from an antenna as a function of direction is called the antenna pattern. The antenna pattern is typically specified in spherical coordinates (azimuth, θ and elevation, ϕ).

Reciprocity. For linear (passive) antennas and linear propagation media (e.g. air) the coupling between fields and currents is independent of the direction of energy flow. Thus antenna patterns are reciprocal – the pattern of an antenna is the same whether it is transmitting or receiving.

Directivity

The directivity (D) of an antenna is the the ratio of the maximum power density (U_m) to the average power density (U_0):

$$D = \frac{U_m}{U_0} \,.$$

Exercise 2: What is the directivity of an isotropic radiator?

Consider the power received at a point in space. It increases proportionately to both the directivity and the effective area. Thus we expect that effective area and directivity will be proportional, that is,

$$\frac{A_e}{D} = \text{constant}$$

for *any* antenna. To find this constant, we can analyze *any* antenna for which A_e and D can both be conveniently derived.

We can do this for an electrically-short[2] dipole antenna. From the current distribution we can compute the electric field ($E(\theta, \phi)$) by integrating the fields produced by infinitesimally small electric dipoles. This allows us to determine both the peak and average power densities and from this the directivity. For the electrically-short dipole

$$D_{\text{short dipole}} = 3/2$$

From the field distribution and using reciprocity we can also derive the power at the antenna terminals resulting from this power density and from this obtain the effective area which is found to be

$$A_{e\text{ short dipole}} = \frac{3\lambda^2}{8\pi}$$

And from this we find that

$$\frac{A_e}{D} = \frac{\lambda^2}{4\pi}$$

and based on our reasoning above this ratio applies to any antenna, not just a short dipole.

Exercise 3: For some types of antennas, such as reflectors, the effective aperture can be approximated by the physical area of the antenna[3] . What are the approximate effective aperture and directivity of a 1-m diameter satellite dish antenna receiving signals at $\approx$ 15 GHz ("Ku-band")?

[2]Short relative to the wavelength.

[3]However, for many antennas, such are wire antennas, the effective area is *not* related to the physical area.

Gain

Measuring directivity requires measuring the power density in all directions in order to compute the average. A more practical measurement is the *gain* of an antenna which is the ratio of the maximum power density to the power density of a lossless reference antenna U_r, typically an ideal (lossless) isotropic radiator:

$$G = \frac{U_m}{U_r}$$

The ratio of gain to directivity:

$$\frac{G}{D} = \frac{\frac{U_m}{U_r}}{\frac{U_m}{U_0}} = \frac{U_0}{U_r} = k$$

is the antenna's *efficiency*: the ratio of the average radiated power of the real antenna to the average radiated power of an ideal isotropic source. This difference is due to resistive losses in the antenna.

Exercise 4: What is the maximum value of k?

Antenna gain, like most quantities in communications is usually specified in dB. If the reference antenna is an lossless isotropic antenna the units are specified as dBi.

We can now relate the effective area of a (lossy) antenna to its gain:

$$G = \frac{4\pi A_e}{\lambda^2}$$

Exercise 5: Another useful approximation relates the gain of an antenna to it's beamwidth. Since a sphere has a surface "solid angle" of 4π steradians ($\approx$ 41253 square degrees), we can approximate the gain by dividing this by the solid angle covered by an ideal (rectangular, "brick-wall") antenna pattern. What is the approximate directivity of an antenna with beamwidths of 15×120 degrees? If the antenna's efficiency is $k = 70\%$, what is the gain?

Friis Equation

Substituting G for A_e in the equation for path loss and solving for the received power we get the Friis equation:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

where P_R and P_T are the received and transmitted powers, G_R is the gain of the receive antenna, λ is the wavelength and d is the distance from transmitter to receiver. The additional term G_T is used to account for the common case of a non-isotropic transmit antenna that increases the transmit power density by a factor (gain) G_T in the direction of the receiver compared to an isotropic radiator.

Exercise 6: A point-to-point link uses a transmit power of 1 Watt, transmit and receive antennas with gains of 20dB and operates at 3 GHz. How much power is received by a receiver 300m away?

This equation only applies at distances that are in the "far field" where the field strength is uniform. The far field is considered to be at distances $d >> L^2/\lambda$ where L is the largest dimension of the antenna.

Exercise 7: What is the far-field distance for an cell phone antenna operating at 3 GHz that has a physical size of $1 \times 1 \times 3$ cm? For a 100 m diameter antenna?

Loss vs Frequency

Consider the relationship between antenna gain and effective aperture. Note that for a fixed gain the effective aperture increases with the square of the wavelength ($A_e \propto \lambda^2$). Put another way, for a given antenna gain the effective area, or power collected, decreases with the square of the frequency.

This is the reason for the frequency dependence of the propagation loss given by the last factor of Friis equation.

However, it's important to understand that the reason the propagation loss appears to increase with frequency is simply because, for an antenna with a fixed gain, the effective aperture decreases as the frequency increases. Propagation loss is ***not*** a result of power being absorbed by the medium through which the signal is propagating.

Exercise 8: If we kept the *effective aperture* (not gain) constant at one end of a link (transmitter or receiver), how would the path loss change as a function of frequency? What if we kept it constant at both ends? Is this a feasible approach for mobile systems?