Antennas and Free-Space Propagation

Exercise 1: If the effective area of an antenna is 1 m^2, what is the path loss, in dB, at a distance of 100 m? At the distance to a geo-stationary satellite ($\approx 36,000$ km)? How does it increase (in dB) with distance?

$10\times\log\frac{1}{4\times\pi\times100^2}=$

-50.99209864

$A_e = 1 \ m^2$

$d = 100m$

$$PL = \frac{P_R}{P_T} = \frac{A_e}{4\pi d^2} = \frac{1}{4\pi(100)^2} \approx \frac{1}{12\times10^9}$$

$10\times\log\frac{1}{4\times\pi\times36E6^2}=$

-162.1181487

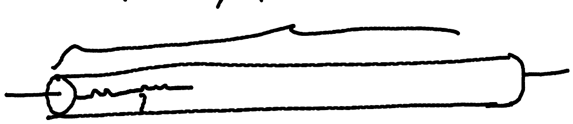
$d = 36\times10^6 \ m$

$PL \propto \frac{1}{d^2}$

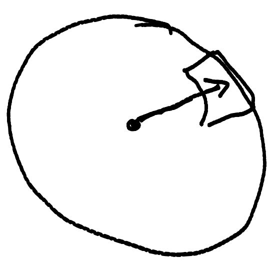
in dB: $\propto 10\log d^{-2}$

attenuation increases by 20 dB per decade.
("order of magnitude").

10 dB/100m



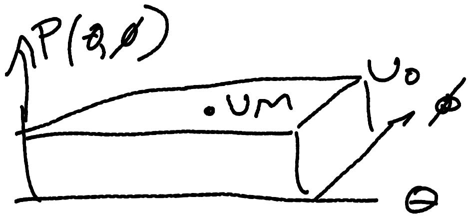
constant increase per m.



" per factor.

Exercise 2: What is the directivity of an isotropic radiator?

$$D = \frac{U_m}{U_0} = 1$$



Exercise 3: For some types of antennas, such as reflectors, the effective aperture can be approximated by the physical area of the antenna[1] . What are the approximate effective aperture and directivity of a 1-m diameter satellite dish antenna receiving signals at ≈ 15 GHz ("Ku-band")?

$\frac{\pi}{4} \approx A_e$

$\frac{A_e}{D} \approx \frac{\lambda^2}{4\pi}$

$\lambda = \frac{3 \times 10^8}{15 \times 10^9}$

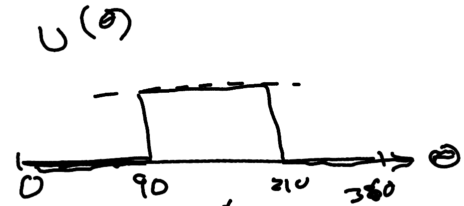
$= 0.02\ m$

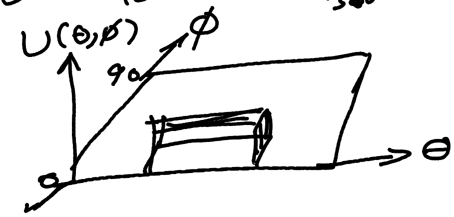
$= 2 \times 10^{-2}$

Exercise 4: What is the maximum value of k?

maximum $k = 1$.

Exercise 5: Another useful approximation relates the gain of an antenna to it's beamwidth. Since a sphere has a surface "solid angle" of 4π steradians (≈ 41253 square degrees), we can approximate the gain by dividing this by the solid angle covered by an ideal (rectangular, "brick-wall") antenna pattern. What is the approximate directivity of an antenna with beamwidths of 15×120 degrees? If the antenna's efficiency is $k = 70\%$, what is the gain?

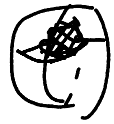




$$D = \frac{41253}{15^\circ \cdot 120^\circ}$$

$$G = D \times 0.7$$

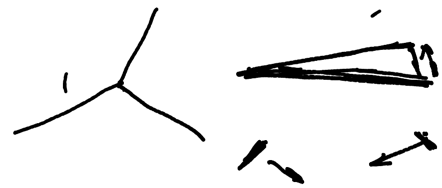
$$\approx 12 \text{ dBi}$$



10log(41253 / 15×120 ×.7)=

1.205281072

12.0



Exercise 6: A point-to-point link uses a transmit power of 1 Watt, transmit and receive antennas with gains of 20dB and operates at 3 GHz. How much power is received by a receiver 300m away?

$$P_R = P_T \, G_T \, G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

$$G_R = G_T = 20 \text{ dB} = 10^{\frac{20}{10}} = 100$$

$$f = 3 \times 10^9 \text{ Hz} \qquad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{m}$$

$$d = 300 \text{ m} \qquad P_T = 1 \text{ W}$$

$$P_R = \underset{W}{1} \cdot 100 \cdot 100 \cdot \left(\frac{0.1}{4\pi\, 300}\right)^2$$

=

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octave:1> pr=1*100*100*(0.1/(4*pi*300))^2
pr =  0.0000070362
octave:2> 10*log10(pr)
ans = -51.527
octave:3>
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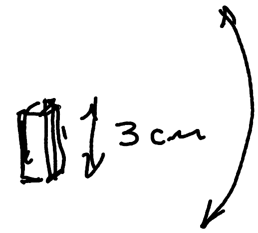
BW

Exercise 7: What is the far-field distance for an cell phone antenna operating at 3 GHz that has a physical size of 1 × 1 × 3 cm? For a 100 m diameter antenna?



$$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1\,m$$

$$L = 0.03\,m$$

$$d \gg \frac{L^2}{\lambda} = \frac{(3\times10^{-2})^2}{0.1} = 9\,mm$$

far-field is distances of e.g. > 9 cm

$$\frac{100^2}{0.1} = \frac{10^4}{10^{-1}} = 10^5\,m = 100\,km.$$

Exercise 8: If we kept the *effective aperture* (not gain) constant at one end of a link (transmitter or receiver), how would the path loss change as a function of frequency? What if we kept it constant at both ends? Is this a feasible approach for mobile systems?

$$P_R = \underbrace{\frac{4\pi A_{e1}}{\lambda^2}}_{G_1} \cdot G_2 \left(\frac{\lambda}{4\pi d}\right)^2 \qquad G = \frac{4\pi A_e}{\lambda^2}$$

P_R is independent of f.

$$\frac{4\pi A_{e1}}{\lambda^2} \cdot \frac{4\pi A_{e2}}{\lambda^2} \left(\frac{\lambda}{4\pi d}\right)^2$$

$$P_R \propto \frac{1}{\lambda^2} \propto f^2$$

directional antennas not practical for mobile systems.