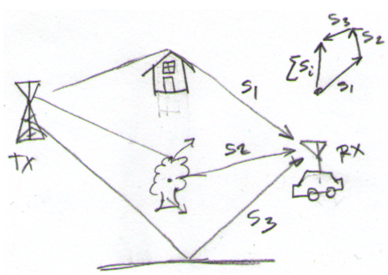
Multipath Fading

Introduction

The most significant advantage of wireless communication systems is their ability to communicate with portable and mobile devices. In many cases this requires that the radio signals propagate over multiple non-line-of-sight (NLOS) paths:



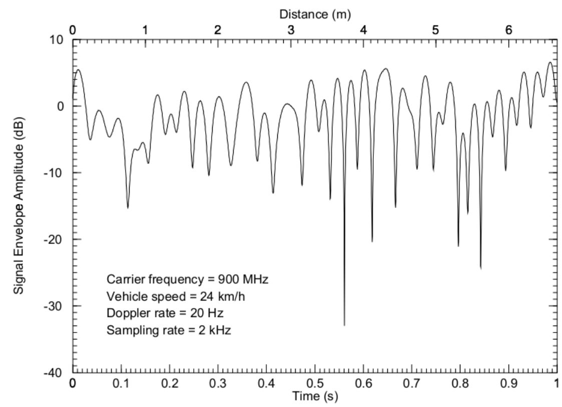
and the received signal is the sum of multiple delayed and attenuated copies of the transmitted signal. The most significant indirect propagation mechanisms include diffraction (e.g. over rooftops), scattering (e.g. through foliage) and reflection (e.g. from the ground).

Modern wireless systems operate in the VHF (30 MHz to 300 MHz) and UHF (300 MHz to 3 GHZ) range. The corresponding wavelengths range from 10 m to 10 cm.

As the receiver moves, the path lengths and thus the phase shifts of each of these signals will change. The exact phase change will depend on the geometry of the propagation paths and is unpredictable.

The multiple components combine at the receiving antenna. The resulting (vector) sum is a signal whose amplitude and phase can change significantly over distances on the order of a few wavelengths.

Since wireless systems often include moving transmitters and receivers (or moving objects that interact with the signal), the received signal amplitude and phase will vary with time. The following plot shows an example of magnitude versus time for such a signal:



Recovering information from a signal that varies upredictably in amplitude and phase over time is the key challenge in designing wireless communication systems.

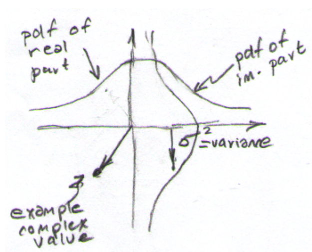
This type of fading is known by various names:

- *small-scale fading*: because it changes over distances on the order of a wavelength; these distances are small relative to the path length
- *fast fading*: because it changes faster than other fading effects such as distance-dependent effects or shadowing by buildings
- *"multipath" fading*: because this sort of fading only happens when there are multiple propagation paths
- *"Rayleigh"* fading: because this is the most common resulting statistical distribution of the envelope (the magnitude of the complex envelope representation of a signal)

Envelope Distribution

When there are a large number of paths, the probability distribution of the amplitude of the received signal can be derived by decomposing the (complex) vector sum of the different signal components into real and imaginary parts. According to the Central Limit Theorem the real and imaginary components will then be normally distributed since each is the sum of a large number of independent random variables (r.v.'s).

The probability density function (pdf) of the magnitude of a complex r.v. whose real and imaginary components are normally distributed is Rayleigh distributed. The following diagram draw the pdfs of the real and imaginary components on quadrature axis and a sample value drawn from the distribution:



The Rayleigh pdf has the form:

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) & r \geq 0 \\ 0 & r < 0 \end{cases}$$

The mean (first moment) of the Rayleigh distribution is

$$\bar{r} = \int_0^\infty r \cdot p(r) dr = \sigma\sqrt{\frac{\pi}{2}} \approx 1.25\sigma$$

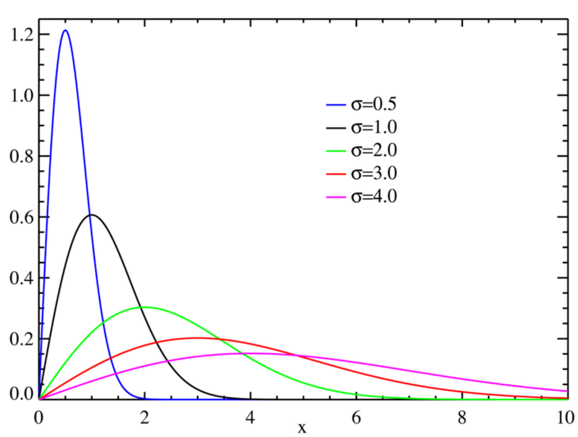
and its variance (second central moment) is

$$\sigma_r^2 = \int_0^\infty (r - \bar{r}) \cdot p(r) dr = \sigma^2\left(2 - \frac{\pi}{2}\right) \approx 0.43\sigma^2 \ .$$

Note that r is the magnitude (or "envelope") of the complex signal, not the signal itself. Since the magnitude cannot be negative, r is always positive.

The Rayleigh distribution has one parameter, σ^2, which is the variance of each component (real or imaginary) of the signal.

The following plot of the Rayleigh pdf is from the Wikipedia article:



The cumulative distribution (CDF) of a Rayleigh random variable can be written as:

$$P(r \leq R) = \int_0^R p(r)\, dr = 1 - e^{-\rho^2}$$

where

$$\rho = R/R_{rms}$$

is the threshold normalized by $R_{rms} = \sigma\sqrt{2}$ which is the square root of the second (non-central) moment – the square root of the power of the received signal.

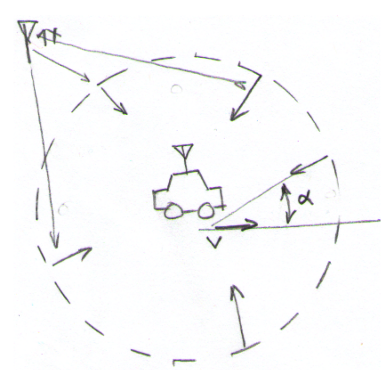
Exercise 1: Show that R_{rms}^2 is the mean power of the signal.

Exercise 2: What fraction of the time is a Rayleigh-distributed signal 10dB below the mean? 20dB? 30dB? This is a useful result to remember.

Clarke Flat-Fading Model

Clarke developed a simple model whose predictions agree reasonably well with the statistics of many NLOS channels.

The model consists of a large number of signal paths arriving from directions that are uniformly distributed in a circle around a moving receiver. Each path has equal delay and loss (amplitude) but random phase (0 to 2π).

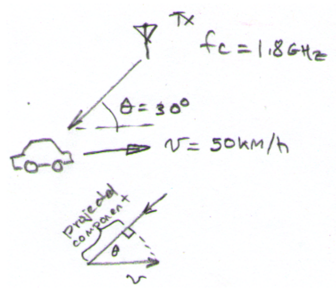


Doppler Spectrum

When a path length changes because the transmitter, receiver and/or the scattering objects are moving relative to each other the received phase will seem to change at a constant rate. This will appear as a frequency shift called a "Doppler" shift and is given by:

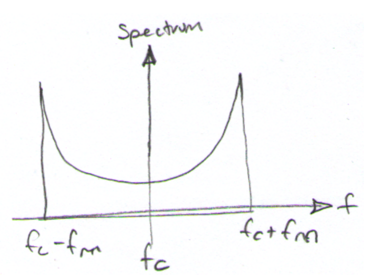
$$f_D = \frac{v}{\lambda} = \frac{v}{c} f_c$$

where c is velocity of light, v is the rate of change of path length, and f_c is the frequency of the signal with wavelength λ.



Exercise 3: A receiver in a car receives a 1.8 GHz signal while travelling on a road at 50 km/h. The road is at an angle of 30 degrees relative to the direction of arrival of the signal. What is the velocity relative to the direction of arrival of the signal? By how much does the path length change each second (in meters)? In wavelengths? What is the Doppler shift?

The signal will be spread in frequency due to the Doppler shifts of the (infinite number of) components. Each component will have a Doppler shift proportional to the cosine of the angle α relative to the direction of motion (see diagram above). Assuming an omnidirectional antenna the Doppler spectrum has a “bathtub” shape extending over the range $f_c \pm f_m$ where f_m is the maximum Doppler shift ($f_m = f_c v/c$):



Level Crossing Rate and Mean Fade Duration

From the power spectrum it is possible to derive two useful time-domain statistics. The level crossing rate is the rate at which the received signal level crosses a threshold ρ in one direction. The level crossing rate is:

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

and the average fade duration is:

$$\overline{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

Exercise 4: How often will the signal drop 10dB below the mean if the carrier frequency is 1.8 GHz and the velocity is 100 km/h? On average, how long will each of these fades last?

Exercise 5: What is the product of N_R and $\overline{\tau}$? How does this compare to $P(r \leq R)$? Why?

Ricean Fading

A common situation is that there are both LOS and NLOS components. In this case the received signal is the sum of a constant component and a Rayleigh-distributed component. The resulting probability distribution is Ricean. The ratio of the powers of the direct and Rayleigh components is given by the parameter called the “Ricean K factor”:

$$K(dB) = 10 \log \frac{A^2}{2\sigma^2}$$

where A is the amplitude of the direct (LOS) component.

Dispersive Fading

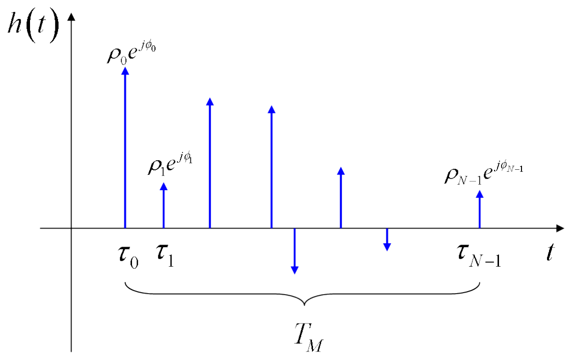
If the delays of the paths differ by a significant fraction of the symbol period then the signal will be distorted by inter-symbol interference (ISI). Otherwise, the signal level may be affected but the waveform will not be significantly distorted.

Another way to look at this distinction is to consider the situation in the frequency domain. Since the phase shift is the product of frequency and delay, the signal level resulting from multipath propagation will also be vary with both frequency and delay. If the bandwidth of the signal is significantly smaller than the variation with frequency of the path loss then the signal will not be distorted, otherwise it will.

The situation where the fading does not affect the signal’s spectrum is called frequency-flat (or just “flat”) fading; the other situation is called “frequency-selective” fading. The channel that exhibits the latter type of fading is also called a “dispersive” channel because the signal is dispersed (spread) in time.

Measures of Dispersion

A multipath channel has an impulse response consisting of one impulse for each multipath component. Each has a corresponding delay and amplitude (example from Wikipedia):



We can quantify the time dispersion of the signal using various metrics. Two popular ones are the first and second central moments of the "power delay profile," $P(\tau) = h^2(\tau)$, the square of the impulse response. These two statistics are called the "mean excess delay":

$$\bar{\tau} = \sum p(\tau)\tau$$

and the "RMS delay spread" of the channel:

$$\sigma = \sqrt{\sum p(\tau)(\tau - \bar{\tau})^2}$$

where $p(\tau)$ is the normalized power delay profile:

$$p(\tau) = \frac{P(\tau)}{\sum P(\tau)} .$$

The minimum observed delay has no effect on dispersion and should be subtracted out when computing $\bar{\tau}$.

Exercise 6: A channel has three multipath components with delays of 1, 2 and 3 μs and amplitudes of 10, 6 and 0 dBm respectively. What are the excess delays, the power delay profile, the normalized power delay profile, the mean excess delay and the RMS delay spread?

Similarly, the frequency response of the channel will not, in general, not have a well-defined shape however we can define a quantity called the "coherence bandwidth" which can be used to quantify the frequency selectivity of the channel. The coherence bandwidth is the frequency range over which the fading at two frequencies are well-correlated.

It is also possible to define a "coherence time", related to the Doppler rate, during which the fading on the channel is well-correlated.

Note that the frequency selectivity (dispersiveness) of the channel and how the channel changes with time (the fading in time) are independent of each other. The former is a function of the path lengths (and thus delays) while the latter is a function of velocity of objects. Thus it is possible to have a dispersive channel that does not experience fading and flat-fading (non-dispersive) channel. The nature of the channel and its effect on the signal thus depends on the propagation environment and motion through it.

Exercise 7: Imagine a receiver traveling in a straight line towards a transmitter but with no LOS path. How could you arrange reflecting objects such that there was no time dispersion (flat fading)? What arrangement would result in no time-varying fading? Neither?