Solutions to Midterm Exam

Exam 1

Question 1

For free-space propagation the received power is given by the Friis equation:

$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

where the wavelength is

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{900\times 10^6} = 1/3 \text{ m},$$

and the antenna gains are:

$$G_T = G_R = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi\cdot 10^{-2}}{(1/3)^2}.$$

There were two versions of the question. For $d = 1$ km and $P_T = 100$ mW,

$$P_R = 100\left(\frac{4\pi\cdot 10^{-2}}{(1/3)^2}\right)^2\left(\frac{1/3}{4\pi 10^3}\right)^2 = 90\times 10^{-9} \text{ mW}$$

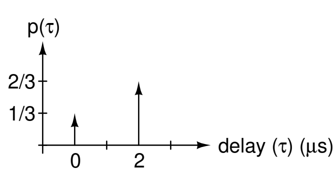
and for $d = 100$ m and $P_T = 10$ mW,

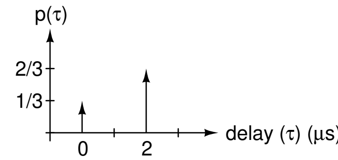
$$P_R = 10\left(\frac{4\pi\cdot 10^{-2}}{(1/3)^2}\right)^2\left(\frac{1/3}{4\pi 10^2}\right)^2 = 900\times 10^{-9} \text{ mW}$$

which are -70.5 and -60.5 dBm respectively.

Question 2

After converting the power-delay profiles to power (mW), subtracting the minimum delays and normalizing to a total power of 1, the normalized power-delay profiles are:





(a) The mean excess delay is the first moment of the normalized distribution:

$$\bar{\tau} = \sum p(\tau)\tau = \frac{1}{3}\cdot 0 + \frac{2}{3}\cdot 2 = \frac{4}{3}$$

(b) The RMS delay spread is the second central moment of the excess delay distribution:

$$\begin{aligned}\sigma &= \sqrt{\sum p(\tau)(\tau-\bar{\tau})^2}\\ &= \sqrt{\frac{1}{3}\cdot\left(0-\frac{4}{3}\right)^2 + \frac{2}{3}\cdot\left(2-\frac{4}{3}\right)^2}\\ &= \sqrt{\frac{16}{27}+\frac{8}{27}} = \sqrt{\frac{24}{27}} \approx 0.94\ \mu\text{s}\end{aligned}$$

Exam 2

Question 1

As described in the hint, reading from this memory is similar to transmitting data over a Binary Symmetric Channel (BSC).

The question asks for the amount of information that can be read from the memory with an arbitrarily low error rate. By definition, this is the capacity of the channel.

The capacity of the BSC is measured in information bits per channel use. Each bit retrieved from a memory cell is one channel use so the error-free information that can be stored will be the number of cells multiplied by the channel capacity.

The capacity of the BSC is:

$$C = 1 - (-p\log_2 p - (1-p)\log_2(1-p))$$

There were two version of this question, one for $p = 2^{-8}$ and one for $p = 2^{-10}$. Computing the capacity:

```
octave:5> p=2^-8
p =  0.0039062
octave:6> c=1-(-p*log2(p) - (1-p)*log2(1-p))
c =  0.96313
octave:7> p=2^-10
p =  0.00097656
octave:8> c=1-(-p*log2(p) - (1-p)*log2(1-p))
c =  0.98883
```

Thus the memory can store about 963 kbits or 988 kbits of information for error probabilities of 2^{-8} and 2^{-10} respectively.

Question 2

(a) As given in the question, $n = 7$, $k = 4$ and $n - k = 7 - 4 = 3$[1]. The parity check matrix should have dimensions of $n - k$ by n so $n - k = 3$ and $n = 7$ (and $k = 4$).

(b) To compute the transmitted codewords we find the Generator matrix as:

$$G = \left[I_k \,|\, P \right]$$

where P is the transpose of the first four colums of H:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

There were two versions of this question. For data bits $d = [1, 1, 0, 0]$ the transmitted codeword is:

$$\begin{aligned} x &= dG \\ &= \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

and for $d = [0, 0, 1, 1]$,

$$x = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

[1] I accidentally made this question trivial by giving (n, k) in the question.